Optimal Fiscal Policy with Endogenous Disaster Risk: How to Finance Defense Spending?*

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Abstract

How should optimal policy manage disaster risk? We study an optimal fiscal policy problem with defense capital that both deters war and insures against wartime spending needs. Calibrating the war-risk channel with the Geopolitical Risk Index, we show analytically and quantitatively that optimal defense financing relies heavily on debt. Borrowing lowers current tax distortions; although it raises future distortions, defense investment reduces the probability that costly war states occur. Heightened geopolitical risk therefore calls for larger debt-financed defense spending and delayed taxation compared to financing other types of spending. Results are robust to optimal monetary policy and preemptive-strike incentives.

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1 Introduction

How should fiscal policy optimally manage disaster risks? In this paper, we answer this question by means of a Ramsey problem where the planner faces the risk of war, which we model as an economic disaster. The planner can preemptively invest in defense capital to mitigate both the disaster probability and its damage. The question is important from both theoretical and policy points of view. Theoretically, disaster risk models have demonstrated major success in explaining stock market moments (e.g., Rietz, 1988; Barro, 2006) and business cycles, as in Gourio (2012). Yet, there is little theoretical guidance on how disaster risk affects optimal policy and how policymakers should manage these risks. Policy-wise, a recent rise in geopolitical risk followed by increasing calls to boost defense spending begs the question: how should it be financed? Intuitively, higher defense spending acts as deterrence and creates future benefits by making disasters less likely at the cost of increasing tax distortions in the present. Optimal policy calls for spreading the benefits and smoothing taxes over time. Borrowing helps to achieve these objectives by reducing present tax distortions while, at the same time, making the disaster state distortions exceedingly costly as debt needs to be repaid in either state. The optimal debt level is determined by the balance between the current tax smoothing benefits and the expected cost of distortions in the disaster state, while taking into account that government investment in defense capital reduces the probability of disaster. We find that the optimal defense financing mix is heavily tilted toward debt compared to financing other types of government spending.

We study the Ramsey problem where the planner can issue non-state-contingent debt and levy distortionary labor taxes, building on Aiyagari, Marcet, Sargent, and Seppala (2002). Besides standard stochastic expenditure shocks, the planner faces time-varying disaster risk in the form of exceptionally high expenditure needs and drops in productivity. In addition to standard policy instruments, the planner can invest in defense capital, an additional stock variable that mitigates disaster risk. Defense capital is valuable for two purposes. First, in its deterrence role, it mitigates the disaster probability. Second, defense capital stock and defense investment can be used to absorb some of the spending needs in the disaster state. We refer to the former as the *deterrence*, or the disaster risk management, motive, and to the latter as the *insurance* motive. The insurance benefits of defense capital endow the defense capital stock with properties similar to those of a state-contingent asset. Our key conceptual contribution is to endogenize the probability distribution over future fiscal states. In standard optimal policy problems, debt smooths distortions across exogenously given states of nature. Here, the government finances an investment that changes the distribution of those states by reducing the likelihood of future disasters. Consequently, optimal debt management is no longer solely about allocating distortions across states, but also about financing investments that reshape the risks the government faces. The analysis proceeds in two steps.

First, after laying out the full infinite-horizon quantitative model, we study a two-period version of the model to gain analytical insights in Section 4. Such a simplification allows us to highlight the key trade-offs and to isolate the role of each channel separately. We show that borrowing to finance defense investment has different implications for bond prices than borrowing to finance standard government expenditure. In particular, both *deterrence* and *insurance* motives exert negative pressure on bond prices. Intuitively, the *deterrence* aspect of defense investment decreases household precautionary saving motives, and the *insurance* aspect raises expected future consumption, decreasing the need to save due to intertemporal consumption smoothing. Abstracting from these price effects, we then study what defense spending implies for tax smoothing. By accumulating assets, the planner builds a cushion in case a disaster occurs in the future. Such reserves enable the planner to smooth taxes in all future states but require increasing taxes in the current period. Hence, such policy implies tax smoothing across states. In contrast, investment in defense, if financed through borrowing, does not require increasing current taxes and allows the planner to mitigate the disaster risk. Yet, the debt needs to be repaid independently of the state realized in the future, which means that smoothing of distortions in the disaster state needs to be sacrificed. We show that optimal financing of defense spending sacrifices smoothing across states to favor smoothing over time. In other words, defense spending through borrowing minimizes the disaster probability but makes the disaster more severe for households. Finally, we show that the optimal financing mix of defense spending involves more borrowing than financing of other types of government expenditure. The intuition comes from the optimality condition that equates the excess burden of taxation today to the expected excess burden of taxation tomorrow. Debt issuance increases the expected excess burden of future taxation, as higher future taxes will be needed to repay debt. However, because of the *deterrence* channel, defense spending makes the disaster state less likely, and the expected excess burden of taxation increases by less than when debt is issued to finance other types of government spending. Consequently, through the optimality condition of the planner, the current excess burden of taxation increases less, implying lower taxes and higher debt than when debt is used to finance other types of government spending.

We then compare the Ramsey and the first-best allocations in an infinite-horizon model. We show that tax distortions increase in the war states and, consequently, the *deterrence* aspect becomes quantitatively stronger in the Ramsey setting. Therefore, we show that absent the effects on bond prices, the Ramsey planner over-invests in defense relative to the first-best. When defense spending moves bond prices, their effect depends on whether the planner issues debt or accumulates assets. We show that by issuing debt, the planner makes use of the negative price effects of defense spending and can potentially achieve the first-best allocation of defense while incurring distortionary taxes.

Next, we analyze optimal defense spending dynamics in an infinite-horizon model solved nonlinearly and globally using a neural network-based algorithm proposed in Valaitis and Villa (2024). Specifically, in Section 5, we study our baseline model dynamics, comparing it to counterfactual models where we first switch off the *insurance* channel and then, in addition, switch off the *deterrence* channel. In this case, the problem collapses to a standard policy problem under incomplete asset markets as in Aiyagari, Marcet, Sargent, and Seppala (2002), with the only difference being that our setting contains uncertainty shocks and disasters still happen but are outside of the planner's control. We refer to this as the standard model. To quantify the importance of defense capital, we estimate the *deterrence* channel using the *threats* and *acts* series of the Geopolitical Risk Index by Caldara and Iacoviello (2022). We also verify that the calibrated model fares well in terms of untargeted moments and in terms of responses to spending shocks, which we compare to the local projection estimates using a similar methodology as in Ramey and Zubairy (2018). The quantitative model allows us to test whether the analytical predictions from the two-period model survive in the infinite-horizon setting as well as to gain additional insights.

Even though the planner can invest in defense to mitigate disaster risk, ex ante, it is not obvious if it is optimal to do so and which of the channels is more relevant. Instead, the planner may simply accumulate assets to be able to smooth tax distortions arising from spending needs in the disaster state. Long-run averages show that, in our baseline calibration, it is optimal to invest significant amounts of output into defense, slightly below the levels currently observed in the US. When we consider the case without the no-lending constraint, we recover the results of Aiyagari, Marcet, Sargent, and Seppala (2002) that it is optimal to accumulate assets in the long run, although such accumulation is slower in the baseline compared to the standard model. Once we impose the no-lending constraint, we find significantly higher long-run debt levels in the standard model. While average debt is close to the US post-war average of 58% in the baseline model, it falls to 42% in the standard model with the same average war probability. Using the generalized impulse responses, we show that this difference in average debt is driven by how the planner responds to uncertainty shocks in the two models. In the standard model, the planner responds to such shocks by saving out of precautionary motives, which is accompanied by a modest increase in taxes and a fall in consumption. In our baseline model and the benchmark without the *insurance* motive, the planner responds by issuing large amounts of debt to finance defense investments. This entails a large fall in current household consumption and utility, which the planner finds optimal as the policy allows it to minimize risks of even larger shocks in the future. At the same time, the tax smoothing considerations imply that once an additional defense unit lowers the future war probability, it also makes the expected tax distortions less likely, and then debt is used to bring these future tax smoothing benefits into the present.

In light of these findings, in Section 5.3 we then investigate the implications of the *deterrence* channel for tax smoothing by solving the model with an increasingly stronger *deterrence* motive. As this motive becomes stronger, consistent with our theoretical results, debt becomes more volatile and there is more borrowing in normal states, which is used to finance an increasingly relevant defense investment. As a consequence, wars tend to occur when debt levels are already high, which inhibits tax smoothing across the normal and the war states. We show that as *deterrence* becomes more powerful, the planner borrows increasingly more when responding to uncertainty shocks, and in doing so it optimally sacrifices tax smoothing across states. Motivated by this finding, we consider a policy application where we ask whether it is optimal to allow for large primary deficits if they are used to fund defense. We answer this by comparing the generalized impulse response of debt and taxes to a government spending shock and to an uncertainty shock that induces the same response path in defense investment. Results show that financing defense entails up to ten times as large deficits and significant backloading of tax distortions. Hence, our results suggest that accounting for the *deterrence* and *insurance* aspects is highly relevant when deciding the optimal financing mix.

In the last section, we extend the model to allow for optimal monetary policy and a preemptive strike. The monetary policy extension is motivated by the observation that governments tend to use inflation during wars and also tend to inflate the wartime debt (Acalin and Ball, 2026; Hall and Sargent, 2011). To address this, we introduce a New Keynesian block by making the debt nominal and allowing the planner to inflate the debt away. In this case, we assume the government runs joint optimal fiscal and monetary policy. The use of inflation during wars could potentially make them less severe from the fiscal point of view and would diminish the importance of both the *deterrence* and *insurance* aspects. At the same time, we introduce nominal rigidities by making prices sticky using Rotemberg adjustment costs. We find that the costs of nominal rigidities dominate the state-contingency benefits of inflating the debt away. As a consequence, our main results of borrowing to finance defense hold in this extension. The second extension is motivated by the idea that a stronger military may increase the government's temptation to start a war (Eisenhower, 1961; Mearsheimer, 2001), and so defense investment may not always lead to a lower conflict probability. We incorporate this possibility by allowing the home government to launch a preemptive strike, which entails lower war costs than being attacked by the foreign government. Additionally, we allow for the costs of such preemptive strikes to decrease as the home country accumulates more defense capital. After calibrating the channel using the evidence from Federle, Meier, Müller, Mutschler, and Schularick (2026), we find that such preemptive strikes are infrequent and only happen during extreme and abrupt increases in geopolitical risk. Hence, the preemptive strike option does not alter our main results. Nevertheless, when such preemptive strikes occur, we observe a large military buildup preceding the preemptive strike and decumulation of military capital afterward. We show that, besides the strike, such buildups further deter foreign attacks.

Related Literature

This paper builds on the optimal fiscal policy literature (Lucas and Stokey, 1983; Barro, 1979). Specifically, it considers the Ramsey problem without state-contingent debt and under full commitment, following Aiyagari, Marcet, Sargent, and Seppala (2002). We contribute to the literature by merging the optimal fiscal policy approach with the disaster risk literature (Rietz, 1988; Barro, 2006, 2009) by allowing the Ramsey planner to invest in a stock variable that mitigates the disaster risk. Our framework nests Aiyagari, Marcet, Sargent, and Seppala (2002) as a special case when the planner cannot affect the disaster probability.

Our paper is also related to work on optimal policies when the planner uses policy tools to affect the actual (or perceived) event probabilities. A series of papers have considered optimal fiscal policy design under ambiguity-averse agents (Karantounias, 2013; Ferriere and Karantounias, 2019; Karantounias, 2023; Michelacci and Paciello, 2019; Benigno and Paciello, 2014), in which case the planner has incentives to use policy tools to affect the perceived worst-case belief of the agents. The latter two papers (Michelacci and Paciello, 2019; Benigno and Paciello, 2014) study how monetary policy is affected by ambiguity-averse agents who endogenously form worst-case beliefs. Our focus is on fiscal policy. Karantounias (2013) considers a setting where agents have doubts about the probability model of government expenditures and a planner who trusts the model and acts paternalistically using contingent taxes to manage the endogenous probability of a particular state to affect prices of contingent claims. Karantounias (2023) considers a more general setting where both the agents and the planner have doubts about the true model. Ferriere and Karantounias (2019) instead consider a setting with ambiguity-averse agents and endogenous government expenditure, where the planner uses state-contingent taxes to affect the agents' perceived probability distribution and, consequently, the prices of state-contingent bonds. All these papers consider how the planner can use standard policy tools to manipulate agents' beliefs in a setting with state-contingent debt, building on Lucas and Stokey (1983). We, instead, consider the setting with non-state-contingent debt following Aiyagari, Marcet, Sargent, and Seppala (2002) and ask whether the planner should resort to the standard policy tools or invest in the stock variable that affects the disaster probability. Our paper also shares similarities with the carbon taxation literature where a reduction in emissions reduces the future economic damage or the risk of future extreme events.[1] This literature tends to study first-best policies and, with few exceptions, focuses only on the abatement policies, which are analogous to our *insurance* channel. Nevertheless, it is possible to see our work as more general and applicable to answer other questions, such as the optimal financing of green technologies.

Another closely related paper is Niemann and Pichler (2011), who consider optimal fiscal policies under disaster risk when the planner issues non-state-contingent debt. They compare and contrast policies under full commitment and no commitment to future policies. They show that, under full commitment, the planner mainly uses debt while, under no commitment, an increase in debt leads to rising inflation expectations, rendering debt issuance costly. Consequently, the planner issues little debt and resorts to using distortionary taxes. In their paper, disaster risk is exogenous, and the planner has no policy tools to affect its probability. Hence, our setting nests the full commitment case of Niemann and Pichler (2011) as a limiting case when the *deterrence* channel is absent.

The paper is also related to the recent work on the economics of wars (Federle, Meier, Müller, Mutschler, and Schularick, 2026; Levine and Ohanian, 2024; Pflueger and Yared, 2024; Benmelech and Monteiro, 2025; Boullot, Cahn, Challe, and Matheron, 2026; Antonova, Luetticke, and Müller, 2026; Gorodnichenko and Vasudevan, 2026). Federle, Meier, Müller, Mutschler, and Schularick (2026) document the economic cost of wars and show how they differ between war sites and neighboring countries. Levine and Ohanian (2024), in a game-theoretic setting, characterize the equilibrium dynamics of deterrence and appeasement. Similar to us, Pflueger and Yared (2024) propose a model where geopolitical risk drives the interaction between military investment and bond prices. In their setting, wars occur with exogenous probability and military investment affects the likelihood of winning the war and consequently the default spreads. Benmelech and Monteiro (2025) empirically establish that an increase in military spending decreases the war probability in the medium run and this effect is stronger for democracies. On the more quantitative end, Boullot, Cahn, Challe, and Matheron (2026) use an incomplete

[1] See Nordhaus (2008), Golosov, Hassler, Krusell, and Tsyvinski (2014), Cai and Lontzek (2019), Hong, Wang, and Yang (2023), Barrage (2019), Douenne, Hummel, and Pedroni (2026), among many others.

markets overlapping generations model to study various means to finance a large and permanent increase in spending. Antonova, Luetticke, and Müller (2026) use a heterogeneous agents New Keynesian (HANK) model to study the effectiveness of defense spending taking military production bottlenecks into account. We, instead, focus on the deterrence channel of military spending and how this determines the optimal financing mix between debt and distortionary taxes.

The paper is organized as follows. Section 2 provides empirical context. Section 3 presents the full infinite-horizon model. Section 4 provides analytical results from a two-period model. Section 5 contains the quantitative results and the policy application. Section 6 extends the model to monetary policy and the possibility of attacking first. Section 7 concludes.

2 Context

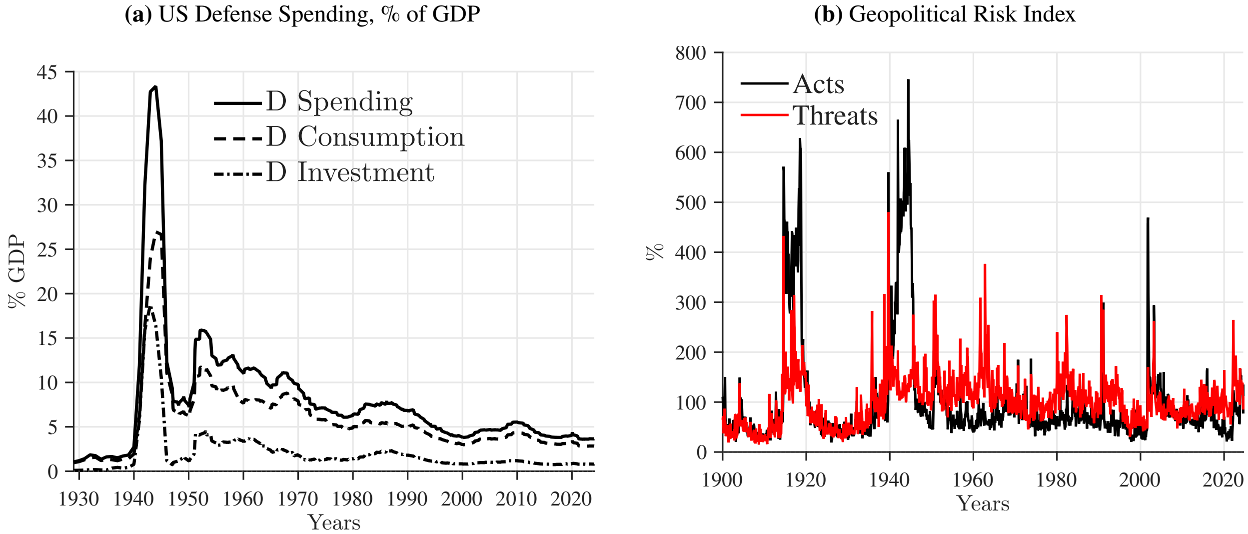
In this section, we provide empirical evidence that we later use as guidance to motivate certain modeling choices. First, we present the relationship between geopolitical risk, defense capital, and the expected real rate on US bonds. Using the geopolitical risk index data from Caldara and Iacoviello (2022), we show that the expected real rate on US bonds typically falls when ex-ante geopolitical risk rises, consistent with the household saving behavior in the presence of disaster risk. We then show that increases in the difference between ex-post realizations of geopolitical risk and its ex-ante probability are followed by declines in US military capital investment. Second, we provide evidence of key government variables from WWII, which we consider a representative definition of a disaster. We show that a disaster entails a sharp increase in government spending needs that in large part consist of defense spending.

2.1 Geopolitical Risk, Defense Spending and Yields

To provide context, Figure 1 shows the evolution of the US defense capital and geopolitical risk. The left panel of Figure 1 shows the US defense spending as a share of GDP and splits it into consumption and investment components. One can think of the consumption component as salaries and other operating expenditures, and the investment component as the purchase of structures and equipment. We view the investment component as a contributing factor to the overall stock of defense capabilities, an asset that takes years to build, and that depreciates over time absent new investment. This stock also gets depleted during war episodes. The right panel shows the geopolitical risk (GPR) index from Caldara and Iacoviello (2022). The index attempts to capture geopolitical risk, defined as *the threat, realization, and escalation of adverse events associated with wars, terrorism, and any tensions among states and political actors that affect the peaceful course of international relations*. The index is constructed using a machine learning algorithm that computes the share of articles mentioning adverse geopolitical events in leading newspapers published in the United States, the United Kingdom, and Canada. While the aggregate index interprets risk to include both threats and realized events, such as wars, the authors provide separate indices for threats (red line) and acts (black line). We can see that the acts index peaks during the two world wars and the September 11 episode. The threats index is intended to capture ex-ante geopolitical risks and tends to be more stable. Notably, it steadily increased in the 1930s leading to World War II, and it was above the acts index throughout the Cold War period. Simultaneously, US defense spending was meager in the 1930s, increased dramatically during World War II, then remained above 10% of GDP during the height of the Cold War and has been on a declining trend since then.

The left panel of Figure 2 presents the relation between the US defense capital stock and the geopolitical risk more systematically. The black-dashed line plots the evolution of the US defense capital stock as a share of GDP,

Figure 1. Geopolitical Risk and Defense Spending.



Notes: The left panel shows various US annual defense spending measures, all expressed as a share of GDP. The solid line shows total defense spending. The dashed line shows defense spending that goes to consumption. The dot-dashed line shows the defense spending that goes to investment in equipment and structures. Data are sourced from the NIPA Table 3.9.5. The right panel shows the historical geopolitical risk index constructed by Caldara and Iacoviello (2022). The black line shows the geopolitical acts index. The red line shows the geopolitical threats index.

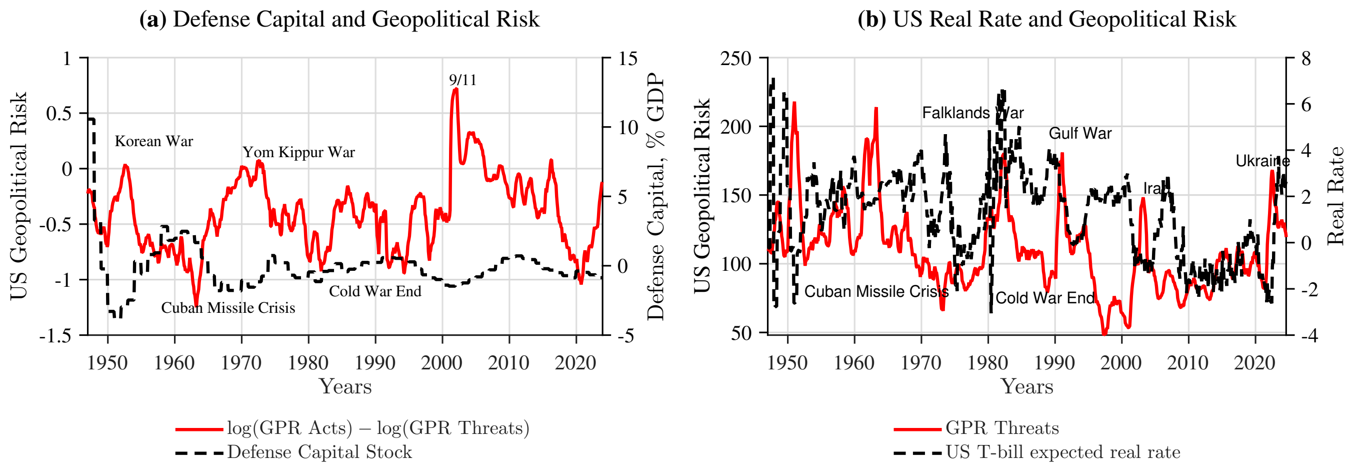
and the red line shows the log difference between the geopolitical risk acts and threats indices. The figure reveals a negative correlation between the two. In the postwar sample, the log difference between acts and threats peaks during the September 11 episode, when the event occurred without a prior increase in underlying geopolitical tensions. It also peaks during war episodes, such as the Korean and Yom Kippur wars. The same difference hits the lowest point during the Cuban missile crisis when no actual disaster followed after the rise in geopolitical tensions. The figure shows that such increases in the difference between acts and threats tend to be preceded by gradual declines in the US defense capital. In contrast, cases with large underlying risks but no geopolitical acts, such as the Cuban missile crisis, then occur when the US defense capital stock is relatively high. We attribute this to the deterrence role of military investment.

The right panel shows that the expected real interest rate on US treasury bills tends to fall in periods when the ex-ante geopolitical risk is high. Notably, this pattern is visible during the Korean War, the Cuban Missile Crisis, the Gulf War, and the 2003 invasion of Iraq, whereas the rate rose when geopolitical risk was at its lowest—around the time of the US withdrawal from Vietnam and during the relative stability of the late 1990s. A similar finding has been documented in Barro (2006), which shows that US bond returns tend to fall during disaster episodes, and is consistent with Hirshleifer, Mai, and Pukthuanthong (2024), showing that wars negatively predict returns on government bonds. This is also consistent with the household precautionary saving effect in the asset pricing models with time-varying disaster risk, e.g. Wachter (2013) and Gourio (2012).

2.2 Disaster Dynamics

Next, we look into economic dynamics during disaster episodes. In particular, we are interested in the macroeconomic dynamics during World War II, which we treat as a representative disaster episode. Figure 3 shows the dynamics of government spending, total factor productivity, government debt, labor tax rate, and defense spending.

Figure 2. Defense Capital, Returns and Geopolitical Risk.



Notes: The right panel shows the Geopolitical risk threats index (GPRHT) (red line) and the expected rate on the US treasury bills. We take the 12-month moving average of the GPRHT series to remove the short-term volatility. Nominal T-bill rate is the 3-Month Treasury Bill Secondary Market Rate from FRED. For 1947-1982 expected inflation is sourced from the Livingston Survey. For 1982-2024, expected inflation is the 1-year ahead expected inflation from the Cleveland Fed. The left panel shows the log difference between the geopolitical risk acts (GPRHA) and threats (GPRHT) indices (red line) and the US defense capital stock. We take the 12-month moving average of both GPRHA and GPRHT series. US defense capital stock series is detrended by subtracting the 20-year moving average. Series comes from the NIPA Fixed Asset Table 7.1.

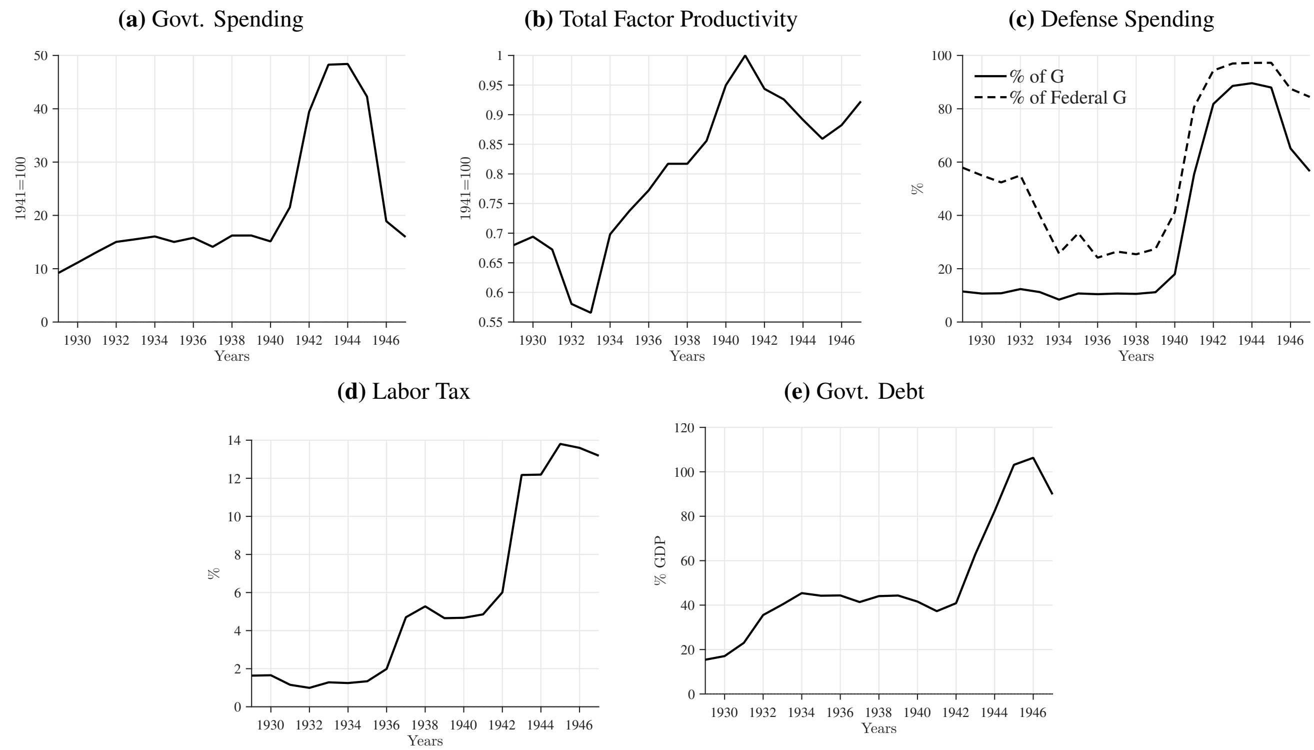
Top panels show that disaster manifests as a simultaneous rise in government expenditure and a drop in total factor productivity. In this particular case, government spending increased from 15% to 30% of GDP and total factor productivity fell by around 15% relative to the initial level in 1941. As Field (2023) argues, this was due to wartime supply chain disruptions, capital and manpower shortages, and the need to adapt production plants for the production of wartime goods. As is natural during wartime, most of the increase in government spending went to defense. Panel (c) shows that defense spending – expressed as a share of total government spending – rose from 10% to 90% of total government spending and to almost 100% of federal government spending. Panels (d) and (e) show that this increase was financed with a mix of taxes and debt. Notably, the United States did not default on its World War II debt in the postwar period, consistent with the observation that postwar defaults have been uncommon among countries in which active hostilities did not take place (Barro, 2006). Inflation, however, substantially reduced the real value of the outstanding debt. In Section 6.3.1, we therefore extend the model to allow the government to issue nominal debt and use inflation as an additional fiscal instrument.

The next section introduces the infinite-horizon model that is consistent with the above-documented empirical facts. Most importantly, we allow defense spending to negatively affect the disaster probability and assume that a large share of the disaster spending needs can be met through defense spending.

3 Model

In this section, we describe an infinite-horizon model in which the Ramsey planner levies distortionary taxes and issues non-state-contingent bonds to finance an exogenous stream of government expenditures. Additionally, it invests in defense capabilities that are useful for *deterrence* and *insurance* purposes. This setting allows us to understand how the optimal defense financing mix differs from financing government expenditures and enables direct comparison with the classic Aiyagari, Marcet, Sargent, and Seppala (2002) results.

Figure 3. Disaster state dynamics.



Notes: The figure shows the dynamics of US government spending, total factor productivity, debt-to-GDP ratio, and labor tax rate around World War II. Government spending, debt, GDP, and labor tax rate come from NIPA Tables. A detailed explanation of the data construction for government spending, debt, GDP, and the labor tax rate can be found in Appendix B.3. Total factor productivity is the series estimated in Field (2023). Defense spending data comes from the NIPA Table 3.9.5.

Environment

Time is discrete and infinite, $t = 0, 1, 2, \ldots$. We consider an economy consisting of a home country and a foreign country. The foreign country chooses whether to engage in conflict after observing the actions of the home government. The home country is a closed economy populated by a continuum of identical households, a continuum of identical firms, and a government. The economy is driven by two stochastic processes: (i) government expenditure in the home country, and (ii) the foreign government's preference for conflict. Allocations and policy variables in the home country are determined by a Ramsey planner who represents the home government. The planner is subject to the constraints imposed by the competitive equilibrium in the home country and the strategic responses of the foreign country. In this sense, the home government, acting through the Ramsey planner, is a Stackelberg leader, while the home country's households and the foreign country are Stackelberg followers. All agents discount the future at a common rate $\beta \in (0, 1)$.

Preferences. Households in the home country rank streams of consumption c_t and leisure l_t according to the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t) \right], \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, and $u(\cdot)$ and $v(\cdot)$ are differentiable functions such that $u_c > 0$, $u_{cc} < 0$, $v_l > 0$, $v_{ll} < 0$.

Technology. Output in the home country is produced by a continuum of measure one of competitive firms with a linear production function $F(z_t, h_t) = z_t \cdot h_t$, where hours worked h are the only input and z is the exogenous labor productivity. Hence, aggregate output is given by $Y_t = F(z_t, h_t)$.

Shocks. There are two types of shocks. First, in the home country, there are government expenditure shocks denoted as g_t, which we assume to be a continuously distributed $AR(1)$ process in logs, i.e. $\ln g_t = \mu_g + \rho_g \ln g_{t-1} + \epsilon_t^g$. Second, we introduce an exogenous variable that governs the foreign country's policy preference for conflict, which in turn determines the risk of war. We label it as ξ_t and we assume that it follows another continuously distributed AR(1) process in logs, i.e. $\ln \xi_t = \mu_\xi + \rho_\xi \ln \xi_{t-1} + \epsilon_t^\xi$. We use $g^t \equiv \{g_0, g_1, ..., g_t\}$ and $\xi^t \equiv \{\xi_0, \xi_1, ..., \xi_t\}$ to denote the histories of the shock realizations up until time t. To simplify notation, we avoid explicitly denoting allocations as functions of histories g^t and ξ^t but it is understood that c_t, l_t and other allocations are measurable with respect to g^t and ξ^t.

Foreign Country. Consider a foreign country that chooses whether or not to engage in war. The decision is made at the end of the period after observing the realization of the shocks and the home country's planner's policies. If a war realizes, it occurs at the beginning of the following period when the foreign country receives the war payoff. We assume that the war payoff depends on the home country's defense capital stock DS and on the foreign government's preference for conflict ξ. The payoff at period t depends on DS_t and ξ_t through a payoff function $V(DS_t, \xi_t)$, such that $\frac{\partial V(\cdot)}{\partial DS} < 0$ and $\frac{\partial V(\cdot)}{\partial \xi} > 0$. In addition to the deterministic component $V(\cdot)$, the decision is affected by an idiosyncratic shock ϵ_t drawn from a standard logistic distribution with cumulative distribution function $F(\epsilon_t) = (1 + e^{-\epsilon_t})^{-1}$. The foreign country chooses to engage in a war if the total payoff is nonnegative:

$$V(DS_t, \xi_t) + \epsilon_t \geq 0.$$

Note that this condition can be rewritten as $\epsilon_t \geq -V(DS_t, \xi_t)$. Hence, given the logistic distribution of ϵ_t, the probability that the foreign country engages in a war is

$$P(\mathcal{I}_{t+1} = 1) \equiv \Pr(\epsilon_t \geq -V(DS_t, \xi_t)) = 1 - F\big(-V(DS_t, \xi_t)\big),$$

where the indicator variable $\mathcal{I}_{t+1}$ assumes the value of 1 in the war state and the value of 0 in the peace state. Using the symmetry property of the standard logistic cumulative distribution, we obtain

$$P(\mathcal{I}_{t+1} = 1) = \frac{1}{1 + e^{-V(DS_t, \xi_t)}}. \tag{2}$$

This is the logistic function that maps the defense stock DS_t and the exogenous variable ξ_t into a probability of war that lies strictly between 0 and 1.

Wars. Following the US World War II experience, we interpret wars as extreme events that happen outside of the home country and are marked by a productivity drop and an additional expenditure need. We denote the realization of government expenditure during the war state as g_t^W and assume that $g_t^W = g_t + g^e$, with g^e being a positive constant. Additionally, wartime productivity falls to a level z^W with $z^W << z$, where productivity z during the normal state is constant.

Deterrence. The planner can choose to invest in the defense capital stock DS_t that, together with the exogenous process ξ_t, determines the disaster probability through equation (2). Indeed, given the foreign country's problem, the probability of a war occurring at time $t+1$ is given by:

$$P(\mathcal{I}_{t+1}=1)=\frac{1}{1+e^{-V(DS_t,\xi_t)}}=P(DS_t,\xi_t),$$

where it is trivial to show that $\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}<0$ and $\frac{\partial P(DS_t,\xi_t)}{\partial \xi_t}>0$, given our assumptions about the gradient of the foreign country's payoff.[2] The planner's incentive to invest in DS_t to affect the probability of the disaster state is at the core of our *deterrence* channel.

Insurance. We assume that a fraction ϕ of the additional government expenditure needed during war g^e can be met by depleting the defense stock. Hence, the additional spending need during the war state, g^e, is related to the undepreciated defense stock, $DS_{t-1}(1-\delta)+D_t$ — where D_t denotes current defense investment — through a function $\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e)$, which captures the idea that at most a fraction ϕg^e of the additional wartime spending can be met through the undepreciated defense capital stock. In principle, this function corresponds to a min operator. Throughout the paper, $\mathcal{S}(\cdot)$ takes the form of a differentiable version of the min operator, which we specify and calibrate in Section 5.1. Since a portion ϕ of the additional financing needs in the war state can be covered using the undepreciated defense stock, the net financing needs in the war state are equal to $g_t^W-\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e)$. The *insurance* channel has a natural interpretation. Typically, a large share of wartime expenditure is defense-related and can be purchased in advance, e.g. ammunition stockpiles. If accumulated in advance, it can be used during wartime without incurring additional costs. At the same time, the stockpiles keep depreciating, and if war does not occur, they are useful only for *deterrence* purposes.

Defense Capital. Defense capital is an endogenous state variable that depreciates at a rate δ and is built up through defense investment (D_t). The accumulated stock may also get depleted during war episodes if the insurance channel is strong, in the sense that ϕ is close to 1. More formally, defense stock evolves according to the following law of motion:

$$DS_t=DS_{t-1}(1-\delta)+D_t-\mathcal{I}_t\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e), \tag{3}$$

where $\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e)$ is the portion of the defense stock used in wartime, when $\mathcal{I}_t=1$, through the *insurance* channel.

Resources. The resource constraint of the economy is given by

$$c_t+D_t+g_t-\mathcal{I}_t\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e)=Y_t=z_t h_t. \tag{4}$$

Note that we normalize the household's time endowment to one, therefore $h_t=1-l_t$.

Household Optimality

Households demand consumption goods, supply labor, and trade real non-contingent government bonds denoted as b_t, respectively. To simplify notation, we avoid explicitly denoting bonds as functions of histories s^{t-1} where $s^t\equiv\{g^{t-1},\xi^{t-1}\}$, but it is understood that b_t is measurable with respect to s^{t-1}. The household budget constraint

[2]Note that all our analytical results only rely on $\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}<0$ and $\frac{\partial P(DS_t,\xi_t)}{\partial \xi_t}>0$ and not on a specific functional form for $P(DS_t,\xi_t)$.

reads

$$q_t b_{t+1} + c_t \leq w_t h_t (1 - \tau_t) + b_t,$$

where w_t is the wage rate, τ_t is the proportional labor tax, and q_t is the bond's price.
Household rationality yields the following standard private sector optimality conditions:

$$q_t = \beta \mathbb{E}_t \frac{u_c(c_{t+1})}{u_c(c_t)}, \tag{5}$$

$$\tau_t = 1 - \frac{v_l(l_t)}{w_t u_c(c_t)}. \tag{6}$$

Government

The government needs to finance the exogenous stream of government spending g_t and the endogenously chosen defense spending D_t using labor income tax and bonds, subject to the following constraint:

$$g_t + D_t + b_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = \tau_t w_t h_t + q_t b_{t+1}. \tag{7}$$

At date t, the government chooses the current tax rate τ_t, defense investment D_t, and next-period debt b_{t+1}, which are measurable with respect to $\{g^t, \xi^t\}$.

Implementability Constraints

We now derive the implementability constraint of the government problem and follow Lucas and Stokey (1983) by taking the primal approach, which allows us to substitute away bond prices and taxes with policy instruments.

The government budget constraint (7) can be combined with the private sector's first-order conditions (5) and (6) to obtain a sequence of recursive implementability constraints for $t = 0, 1, \ldots$ that reads:

$$\forall t : b_t = s_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot b_{t+1} \right], \tag{8}$$

where $u_c(c_t)s_t \equiv u_c(c_t)c_t - v_l(l_t)h_t$ denotes the government's surplus in marginal utility terms, and wage w_t is equal to z_t. Besides, we substitute out leisure and labor everywhere using the resource constraint (4). Also, the notation is such that $b > 0$ indicates a positive amount of government debt and $b < 0$ corresponds to government lending to households. We follow the literature on optimal fiscal policy under incomplete markets (e.g., Aiyagari, Marcet, Sargent, and Seppala, 2002; Faraglia, Marcet, Oikonomou, and Scott, 2019) and we assume that exogenous debt limits $b_t \in [\underline{b}, \overline{b}]$ are in place to prevent Ponzi schemes.

Alternatively, the implementability constraint (8) can be formulated to express the government's liabilities b_t as an expected net present value of surpluses.

$$\forall t : \; b_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right], \tag{9}$$

provided that a transversality condition $\lim_{t \to \infty} u_c(c_{t+1}) b_{t+1} = 0$ is in place.

3.1 The Ramsey Problem under Incomplete Markets

In this subsection, we solve for the time-inconsistent Ramsey plan under incomplete debt markets, following Aiyagari, Marcet, Sargent, and Seppala (2002). Such a problem is nonrecursive as the planner needs to keep track of all the past promises made when deciding on policies at time t. To make the problem recursive, we follow Marcet and Marimon (2019) by introducing an additional co-state variable that summarizes the previous commitments made by the planner. Call the history of shocks $S^t = (g^t, \xi^t)$. The Ramsey planner seeks to maximize household utility (1) by choosing the sequences of plans

$$\{c_t(S^t), l_t(S^t), b_{t+1}(S^t), D_t(S^t), DS_{t+1}(S^t)\}_{t=0}^{T},$$

subject to the implementability constraint (8) with multiplier μ_t, the law of motion for the defense capital (3) with multiplier μ_t^D, the bond bounds $(\overline{b}, \underline{b})$ with multipliers $(\overline{\Lambda}_t, \underline{\Lambda}_t)$, and the defense capital bound with multiplier λ_t^D. The planner takes into account that defense capital affects the probability of war $P(DS, \xi)$ through equation (2). We set the initial conditions to $\mu_0 = b_0 = 0$ and choose DS_0 so that the initial probability of war is 10%. More formally, the recursive Lagrangian of the planner reads:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t) + v(l_t) + \mu_t(\Omega_t + \beta \mathbb{E}_t u_c(c_{t+1}) b_{t+1} - u_c(c_t) b_t) + \\ & \mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t) \\ & - \bar{\Lambda}_t \left(b_{t+1} - \overline{b}\right) + \underline{\Lambda}_t \left(b_{t+1} - \underline{b}\right) + \lambda_t^D D_t \Big\},\end{aligned}$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t) c_t - v_l(l_t) h_t$ is the government primary surplus in marginal utility terms. We use the resource constraint (4) to substitute out l_t. The optimality conditions for DS_t, D_t, and b_{t+1} illustrate the key planner's trade-offs.[3] The first-order condition with respect to bonds is:

$$\mu_t = \mathbb{E}_t(n_{t+1}\mu_{t+1}) + \frac{1}{\beta \mathbb{E}_t \left[u_c(c_{t+1})\right]} \left(\overline{\Lambda}_t - \underline{\Lambda}_t\right), \quad \text{where} \quad n_{t+1} \equiv \frac{u_c(c_{t+1})}{\mathbb{E}_t u_c(c_{t+1})}. \tag{10}$$

Equation (10) corresponds to the standard result of Aiyagari, Marcet, Sargent, and Seppala (2002) that the recursive multiplier μ_t is a risk-adjusted martingale sequence. Because of the non-state contingent nature of government debt, past promises matter for current policies, which introduces persistence in the tax rates and debt. The multiplier μ_t can also be interpreted as the excess burden of taxation. Condition (10) captures that the planner uses tax and debt policies to smooth distortions on average. Using recursive notation we can expand the expression when the bounds do not bind to highlight the role of deterrence in smoothing tax distortions through DS, as shown in equation (11):[4]

$$\begin{aligned}\mu = & P(DS, \xi) \mathbb{E}_{g'|g}[\mathbb{E}_{\xi'|\xi}[n(g', \xi', \mu, b', DS, 1)\mu(g', \xi', \mu, b', DS, 1)]] + \\ & (1 - P(DS, \xi)) \mathbb{E}_{g'|g}[\mathbb{E}_{\xi'|\xi}[n(g', \xi', \mu, b', DS, 0)\mu(g', \xi', \mu, b', DS, 0)]].\end{aligned} \tag{11}$$

In the standard model, the planner has no control over the probabilities of future states and uses tax and debt policies to influence the value of the excess burden of taxation (μ_{t+1}) state by state so that, in expectation, the excess burden of taxation at $t+1$ is the same as at t. We define this as *smoothing across states*. Deterrence through DS introduces an additional channel to achieve the same tax smoothing properties by influencing the

[3] In Appendix A.2 we report all optimality conditions.

[4] The vector of state variables X_t at time t is $X_t \equiv \{g_t, \xi_t, \mu_{t-1}, b_t, DS_{t-1}, \mathcal{I}_t\}$, where $\mathcal{I}_t \in \{0, 1\}$ indicates whether the economy is in the state of war at time t. $\mathcal{I}_t = 0$ stands for the peace state and $\mathcal{I}_t = 1$ stands for the war state.

probabilities of certain states realizing at $t+1$. We define this policy as *smoothing over time*. While ex-ante it is not obvious which type of smoothing is preferred, in Section 4 we investigate analytically how investment in DS affects the excess burden of taxation in various states.

The optimality condition with respect to D_t is

$$\underbrace{\lambda_t^D + \mu_t^D \left(1 - \mathcal{I}_t \frac{\partial S(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t}\right)}_{\text{Marginal Benefit}} = \underbrace{-\mu_t \frac{\partial \Omega_t}{\partial D_t} - v_l(l_t)\frac{\partial l_t}{\partial D_t}}_{\text{Marginal Cost}}, \tag{12}$$

and the optimality condition with respect to DS_t is

$$\mu_t^D = \underbrace{\beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^{g,\xi} \left(U(c_{t+1}^W, l_{t+1}^W) - U(c_{t+1}^N, l_{t+1}^N)\right)}_{\text{Deterrence}} + \underbrace{\beta \mathbb{E}_t \left(\mu_{t+1} \frac{\partial \Omega_{t+1}}{\partial DS_t} + v_l(l_{t+1}) \frac{\partial l_{t+1}}{\partial DS_t}\right)}_{\text{Insurance}} +$$

$$\underbrace{\beta \mu_t b_{t+1} \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^{g,\xi} \left(u_c(c_{t+1}^W) - u_c(c_{t+1}^N)\right)}_{\text{Price Manipulation}} + \underbrace{\beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_{t+1} \partial S(DS_t(1-\delta) + D_{t+1}, \phi g^e))}{\partial DS_t}\right)}_{\text{Future Terms}}. \tag{13}$$

Naturally, these conditions highlight that the optimal investment in DS weights marginal costs and marginal benefits. Marginal costs are contemporaneous and come from the fact that higher D_t decreases the government primary surpluses and makes the implementability constraint more binding. Additionally, the last term in equation (12) captures the resource costs of allocating more labor hours for D_t.

The marginal benefits are shifted in the future, as shown by equation (13). The first is the *deterrence* term, which captures the idea that a higher DS makes the disaster state less likely and, consequently, helps to smooth household consumption. The quantitative importance of this term depends on the degree to which DS can affect the disaster probability, as captured by the gradient of P with respect to DS. Additionally, the term becomes more important if households cannot insure against the disaster, captured by the difference in their utility in normal and disaster states. The second term captures the *insurance* channel. It consists of two terms. The first one captures the idea that higher DS can help to alleviate some of the spending needs in the disaster state and, therefore, it also helps to reduce the future excess burden of taxation. The second one is the potential saving of aggregate resources due to higher DS stock at period $t+1$ in the case of disaster. The third term captures the effect of defense investment on bond prices. A change in bond prices makes the implementability constraint more or less binding and this becomes more relevant when the debt levels are high and when the planner is already significantly constrained, or mathematically when the value of μ_t is high. In the context of this model, high defense investment reduces future risk and therefore reduces the household precautionary saving motive and creates negative pressure on bond prices. The last term captures the benefits from the same three motives in periods beyond $t+1$. Note that *deterrence* becomes irrelevant if we make P invariant to DS and the *insurance* term becomes irrelevant if we set ϕ to 0.

These trade-offs can be contrasted with the first-best condition for DS

$$\mu_t^D = \underbrace{\beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^{g,\xi} \left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N) \right)}_{\text{Deterrence}} + \underbrace{\beta \mathbb{E}_t \left(v_l(l_{t+1}) \frac{\partial l_{t+1}}{\partial DS_t} \right)}_{\text{Insurance}}$$
$$+ \underbrace{\beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_{t+1} \partial S(DS_t(1-\delta) + D_{t+1}, \phi g^e))}{\partial DS_t} \right)}_{\text{Future Terms}}. \tag{14}$$

The main difference is that the social planner does not need to consider effects on bond prices, and its insurance term does not include terms related to the implementability constraint.[5] While the equations (13) and (14) are not directly comparable as the optimal allocations for consumption and labor differ under the Ramsey and the social planner, the following Proposition 1 allows us to compare defense investment in the two cases. Before proceeding with the proposition, define the period welfare loss from distortionary taxation relative to the first-best as $L \equiv U(c_t^{FB}, l_t^{FB}) - U(c_t^R, l_t^R)$. Lemma 1 then establishes that first, the welfare loss increases with distortionary taxes and, second, that the welfare loss is greater in the war state than in the normal state. These results allow us to compare the *deterrence* terms in (13) and (14), and enable us to establish conditions when the Ramsey planner achieves the first-best allocation into defense investment.

Lemma 1 *Define the welfare loss from distortionary Ramsey taxation relative to the first-best in state s as $L_t^s \equiv U(c_t^{s,FB}, l_t^{s,FB}) - U(c_t^{s,R}, l_t^{s,R})$. Then the following statements hold:*

- *Welfare loss L is increasing in distortionary taxes, i.e. $\frac{\partial L_t}{\partial \tau_t} > 0$ for $\tau_t > 0$.*
- *When the elasticity of marginal utility $\varepsilon_u \equiv -\frac{cu''(c)}{u'(c)}$ is constant and satisfies $\varepsilon_u \geq 1$, and assuming that $\tau_t^W > \tau_t^N$, the welfare loss in wartime is greater than in peacetime: $L_t^W > L_t^N$.*

Proof. See Appendix A.3. ■

Proposition 1 ***Defense spending and the First-Best policy.***
Assume that the insurance channel is absent ($\phi = 0$), the defense capital choices are interior, and the relevant objective functions are strictly concave in DS. The Ramsey and first-best policies for DS coincide if the following condition holds for every t:

$$\underbrace{\sum_{j=1}^{\infty} \beta^j (1-\delta)^{j-1} \mathbb{E}_t \left(P_{DS,t+j-1} \mathbb{E}_{t+j-1}^{g,\xi} \left(L_{t+j}^W - L_{t+j}^N \right) \right)}_{\text{P.V. of welfare loss from taxation}} = \underbrace{\sum_{j=1}^{\infty} \beta^j (1-\delta)^{j-1} \mathbb{E}_t \left(\mu_{t+j-1} b_{t+j} P_{DS,t+j-1} \mathbb{E}_{t+j-1}^{g,\xi} \left(u_c(c_{t+j}^{W,R}) - u_c(c_{t+j}^{N,R}) \right) \right)}_{\text{P.V. of Price Manipulation}}$$

where $L_t^s \equiv U(c_t^{s,FB}, l_t^{s,FB}) - U(c_t^{s,R}, l_t^{s,R})$ is the state-specific welfare loss from distortionary taxation.

1. *If $L_t^W > L_t^N$, the Ramsey planner assigns a higher marginal value to deterrence and over-invests in DS when the price-manipulation term is absent.*
2. *If $L_t^W > L_t^N$ and $u_c(c_t^{W,R}) > u_c(c_t^{N,R})$, positive government debt can make the price-manipulation term offset the allocation-gap term, so that the Ramsey and the first-best policies for DS coincide.*

[5] The social planner's problem and all optimality conditions are presented in A.3.

Proof. See Appendix A.3. ■

Proposition 1 establishes that, absent the *insurance* aspect, differences in defense investment in the two cases arise because of the discounted present values of the price manipulation term and the relative welfare loss in wartime and in peacetime, which arises from the differences in the *deterrence* term in the two cases. The two policies then coincide if these two terms exactly offset each other. The first part of the proposition establishes that when the price manipulation term is absent, the Ramsey planner overinvests in DS. Intuitively, because of distortionary taxation, the *deterrence* term is quantitatively larger than under first-best and the planner over-insures against the need to raise distortionary taxes in the war state. The price manipulation term could be irrelevant if the planner does not issue debt or accumulates assets so that $\mu = 0$ or if bond prices do not differ across the two states. The second part of the proposition states the conditions under which the two policies coincide once the price manipulation term is present. Trivially, the two policies coincide if the Ramsey planner accumulates assets and finances all the spending from the interest income, in which case the implementability constraint multiplier μ is zero, and there is no welfare loss from distortionary taxation. When this is not the case, the proposition states that there exists a debt position that can make the price manipulation offset the first term. In this model, where the bond price is driven by household marginal utility, and there is no default risk, issuing debt would enable the Ramsey planner to achieve the first-best allocation to defense spending even while incurring distortionary taxes.

As explained in Appendix A.1.1, by combining the optimality conditions for consumption and leisure, we can express the optimal tax rate as a function of elasticities and, importantly, multipliers and debt levels:

$$\tau_t = \frac{\mu_t(\epsilon_{cc} + \epsilon_{hh})}{1 + \mu_t(1 + \epsilon_{hh})} - \frac{b_t}{c_t}\epsilon_{cc}\frac{(\mu_t - \mu_{t-1})}{1 + \mu_t(1 + \epsilon_{hh})}. \tag{15}$$

Equation (15) highlights how taxes depend on past multipliers and the levels of outstanding debt.[6] While it is the same as in the standard model with exogenous disasters, it still offers insights on how optimal debt management relates to taxes. Under complete markets, $\mu_t = \mu_{t-1} = \mu$ and debt levels become irrelevant. In the standard incomplete markets setting, as in Aiyagari, Marcet, Sargent, and Seppala (2002), the planner is issuing debt to achieve smoothing across states and, therefore, μ_t is a near random walk meaning that μ_t and μ_{t-1} are typically close. This means that tax volatility is not impacted by outstanding debt levels. Under endogenous disaster management, the planner may opt for tax smoothing *over time*, which would aim to reduce the probability of bad states, while allowing the multiplier in those states to be higher than otherwise. Such policy would allow for occasional large differences between μ_t and μ_{t-1}. If the planner simultaneously issues large levels of debt, the policy of smoothing over time then allows for large changes in taxes in some periods.

4 Analysis

To isolate the mechanisms driving the policy choices in anticipation of wars, we consider a two-period version of the model, with dates denoted as t = 0, 1, along with other simplifying assumptions, as discussed below. This streamlined setting allows us to analytically characterize the behavior of multipliers, taxes, and debt in the models with and without *deterrence* and *insurance* channels.

Assumptions. We make the following four assumptions. 1. The economy consists of two periods, denoted as $t = 0, 1$. At $t = 0$, the economy is in a normal state, while the state at $t = 1$ is uncertain. 2. We assume that $\sigma(\epsilon^g) = \sigma(\epsilon^\xi) = 0$, such that $\xi_t = \exp(\mu_\xi/(1-\rho_\xi))$ and g_t is either $\exp(\mu_g/(1-\rho_g))$ or $\exp(\mu_g^W/(1-\rho_g))$

[6] We thank Anastasios Karantounias for showing this tax formula. See Karantounias (2024) for details.

in the normal and the war state, respectively. 3. Household preferences are time-separable with constant Frisch elasticity of labor supply. 4. The economy does not experience a productivity drop in the disaster state, thus $z_t = 1$ for $t \in \{0, 1\}$. Relaxing any of these assumptions still allows for an analytical characterization of the planner's trade-offs but the expressions become too involved thereby offering little additional insights.

Notation. Under these assumptions, there are two states of the world in period 1, namely, war and normal. We use superscripts W and N to denote period 1 variables in the war and normal states, respectively. The planner's implementability constraints then read

$$\begin{aligned}
&\tau_0 h_0 + Q_0 b_1 = g_0 + D_1 + b_0, \text{ at } t = 0,\\
&\tau^W h^W = g^W + b_1 - \mathcal{S}((1-\delta)D_1, \phi g^e), \text{ at } t = 1 \text{ and war},\\
&\tau^N h^N = g^N + b_1, \text{ at } t = 1 \text{ and peace},
\end{aligned}$$

and the bond's optimality condition (10) simplifies to

$$\mu_0 = P(D_1)\mu^W + (1 - P(D_1))\mu^N \tag{16}$$

and, similarly, the bond's price is

$$Q_0 = \beta\left(P(D_1)\frac{u_c(c^W)}{u_c(c_0)} + (1 - P(D_1))\frac{u_c(c^N)}{u_c(c_0)}\right).$$

We begin by analyzing the planner's trade-offs. We consider hypothetical scenarios where all the planner's constraints hold – i.e., we are in a feasible competitive equilibrium – but the economy is not necessarily at the Ramsey equilibrium. First, it is instructive to compare and contrast how defense spending affects bond prices. For a benchmark, consider an increase in g_0 financed with period 0's debt issuance that is to be repaid in period 1. Equation (17) decomposes the effect on bond prices:

$$\frac{\partial Q_0}{\partial g_0} = \underbrace{\frac{\partial Q_0}{\partial c_0}\frac{\partial c_0}{\partial g_0}}_{\text{GE effect}} + \underbrace{P(D_1)\beta\frac{u_{cc}(c^W)}{u_c(c_0)}\left(\frac{\partial c^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial g_0}\right) + (1 - P(D_1))\beta\frac{u_{cc}(c^N)}{u_c(c_0)}\left(\frac{\partial c^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial g_0}\right)}_{\text{Higher } \tau_1}. \tag{17}$$

The first term captures the general equilibrium effect through consumption and labor supply. Through the resource constraint, higher government expenditure requires either a fall in consumption, or higher hours worked, or both. To the extent that both consumption and leisure are normal goods, this term is negative, as lower current consumption means higher marginal utility and hence lower price. The second term captures the effect of higher taxes in period 1. Higher taxes are associated with lower consumption; hence, higher future marginal utility and higher prices. This standard household intertemporal consumption smoothing channel implies that bond prices increase following a debt-financed increase in government spending.

Now consider an analogous debt-financed increase in defense spending D_1. Equation (18) decomposes the effect on bond prices:

$$\frac{\partial Q_0}{\partial D_1} = \underbrace{\frac{\partial Q_0}{\partial c_0}\frac{\partial c_0}{\partial D_1}}_{\text{GE effect}} + \underbrace{P(D_1)\beta\frac{u_{cc}(c^W)}{u_c(c_0)}\left(\frac{\partial c^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial D_1}\right) + (1-P(D_1))\beta\frac{u_{cc}(c^N)}{u_c(c_0)}\left(\frac{\partial c^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial D_1}\right)}_{\text{Higher } \tau_1} +$$
$$\underbrace{\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)}}_{\text{Precautionary Saving}} + \underbrace{\beta P(D_1)\frac{u_{cc}(c^W)}{u_c(c_0)}\frac{\partial c^W}{\partial D_1}}_{\text{Insurance}}. \tag{18}$$

In addition to the terms in equation (17), equation (18) contains both the *deterrence* and *insurance* channels contributing to affect the bond's price.

The *deterrence* channel contained in equation (18) has a natural interpretation. D_1 makes the disaster state less likely, i.e. $P'(D_1) < 0$. Indeed, investment in defense reduces the household's precautionary saving motives and lowers the bond's price. The importance of this term depends on the gradient of disaster probability with respect to D_1 and the difference between marginal utilities of consumption in W and N states. The last term captures the *insurance* channel. Investing in D_1 alleviates spending needs in the war state. This, in turn, lowers τ^W resulting in a larger c^W and, through household intertemporal substitution motive, a lower price. These considerations lead us to formulate the following proposition, which formalizes the differential effect of debt-financed spending on bond prices.

Proposition 2 ***Defense Spending and the Bond's Price.***
Assume that the planner decides to finance an increase in spending by issuing debt. Debt-financed defense spending D_1 exerts a higher negative pressure on the bond's price compared to debt-financed government spending g_0; i.e., $\frac{\partial Q_0}{\partial D_1} < \frac{\partial Q_0}{\partial g_0}$.

Proof. See Appendix A.1.2. ■

Under quasilinear preferences and assuming that b_0 is equal to 0, the optimal tax formula (15) simplifies to

$$\mu^W = \frac{\tau^W}{\epsilon_{hh} - \tau^W(1+\epsilon_{hh})},$$

with analogous expressions for the N state and period 0 variables. It states that the multipliers in a particular state become solely an increasing function of tax rates in that state. This mapping allows us to study the model behavior when both debt and taxes are allowed to adjust optimally. The planner can respond to disaster risks by either accumulating assets or by investing in D_1. If the planner chooses to invest in D_1, it can be done by either issuing debt or using current taxes. Financing through current taxes front-loads tax distortions and, importantly, avoids the risk of exceedingly high tax distortions in the disaster state. In this sense, the policy allows cross-state tax smoothing in period 1 by sacrificing the smoothing between period 0 and period 1. Debt financing does the opposite. Since debt needs to be repaid in period 1 regardless of the state of the world, such financing sacrifices cross-state tax smoothing, while allowing the planner to smooth tax distortions over time. In such a case, there is little change in tax distortions in period 0, while the expected distortions in period 1 move in an ambiguous way as higher D_1 reduces the disaster probability. The other option for the planner is to ignore the *deterrence* motive and to accumulate assets that can be used to smooth tax distortions in the disaster state. Note that if the *deterrence* motive is absent, the planner always insures by accumulating assets rather than investing in D_1 for insurance. The reason is that assets pay off in both states of the world, while D_1 pays off only in one, so it has a lower expected return as an investment.

Proposition 3 states that, in the absence of insurance motives, the optimal mix of defense financing is such that cross-state smoothing of distortions is sacrificed. In other words, it is optimal to finance D_1 with a mix of taxes and debt. The fact that D_1 is financed with debt also means that simultaneous disaster risk management through D_1 and accumulation of assets are not optimal.

Proposition 3 ***Optimal Financing of Defense Spending.***
Assume quasilinear preferences in consumption and no insurance motive, i.e. $\phi = 0$. Optimal financing of defense spending is such that following the increase in D_1, excess burden of taxation increases by more in the disaster state than in the normal state, i.e. $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$. Optimal financing of defense spending sacrifices smoothing across states in favor of smoothing over time.

Proof. See Appendix A.1.3. ■

Proposition 3 assumes no insurance motive. In this case, cross-state smoothing of distortions is sacrificed whenever expenditure financing involves some debt issuance. Hence, this holds for both D_1 and g_0. Proposition 4 then highlights that the optimal mix of D_1 financing involves a larger share of borrowing and more backloading of tax distortions compared to financing g_0. The economics can be understood through the optimality condition for debt (16). Both debt-financed g_0 and D_1 increase the expected excess burden of taxation in period 1, $\mathbb{E}_0(\mu_1)$. However, because deterrence through D_1 makes the disaster state less likely, $\mathbb{E}_0(\mu_1)$ increases by less in response to debt-financed D_1 than debt-financed g_0. Consequently, through the bond optimality condition, optimal μ_0 also increases by less, meaning there is a smaller increase in current taxes and a larger increase in debt. Intuitively, optimal policy calls for spreading the benefits and smoothing taxes over time. Borrowing helps to achieve that by reducing present tax distortions, while at the same time, making the disaster state distortions exceedingly costly as debt needs to be repaid in either state. Optimal borrowing to finance defense is then determined by balancing the current tax-smoothing benefits against the expected cost of distortions in the disaster state. As defense spending minimizes the probability of the disaster states, the expected cost of borrowing in terms of future tax distortions is lower compared to borrowing to finance g_0. In this sense, Proposition 4 states that, in the absence of the *insurance* motive, the optimal financing mix of D_1 sacrifices cross-state tax smoothing by more than the optimal financing mix of government expenditure g_0.

Proposition 4 ***Debt Levels and Defense Spending.***
Assume quasilinear preferences in consumption and no insurance motive, i.e. $\phi = 0$. The optimal level of debt responds more strongly to changes in defense spending than to changes in government expenditure; i.e., $\frac{\partial b_1}{\partial D_1} > \frac{\partial b_1}{\partial g_0}$.

Proof. See Appendix A.1.4. ■

The propositions above analyze the underlying mechanisms of the model when only some of the choice variables are chosen optimally while others are treated as exogenous. It is informative to see how the optimal allocations in the endogenous disasters model compare to the standard model. One way to see the role of debt is to split the Ramsey problem into two steps. In the first step, the planner chooses $\{\tau_0, \tau^N, \tau^W, D_1\}$ given an arbitrary b_1 and in the second step it optimizes b_1 taking into account the optimal responses of taxes and D_1 through a set of implementability constraints and optimality conditions. The second stage optimization would give the following expression:

$$\frac{\partial U_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial b_1}+\beta P(D_1)\frac{\partial U^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial b_1}+\beta(1-P(D_1))\frac{\partial U^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial b_1}+\left(\frac{\partial U_0}{\partial D_1}+\underbrace{\beta\frac{\partial P}{\partial D_1}(U^W-U^N)}_{\text{Deterrence}}+\underbrace{\beta\frac{\partial U^W}{\partial D_1}}_{\text{Insurance}}\right)\frac{\partial D_1}{\partial b_1}=0. \tag{19}$$

The first three terms capture the standard tax-smoothing motives of debt management. In doing this the planner weights the consumption smoothing benefits of lower taxes in period 0 against the costs of higher taxes in period 1. Specifically, the planner weighs the fact that debt needs to be repaid in any state of the world and the large and negative effects of higher τ^W reduce the planner's incentives to issue more debt. The last term captures all the effects related to the optimal response of D_1. It contains the marginal costs of forgone leisure in period 0 and marginal benefits due to the *deterrence* and *insurance* motives. The term is generally positive as long as the optimal D_1 is positive and increasing in b_1. Proposition 5 states that the optimal debt in the endogenous disasters model is at least as high as in the standard model and is strictly higher when the optimal D_1 is greater than zero.

Proposition 5 ***Optimal Debt Level.***
Assume quasilinear preferences and no insurance motive, i.e. $\phi = 0$*. Denote with* b_1 *the optimal debt level and* b_1^* *the optimal debt level when* $\frac{\partial P}{\partial D_1} = 0$*. At the Ramsey optimum,* $b_1 \geq b_1^*$ *and* $b_1 > b_1^*$ *if* $D_1 > 0$*.*

Proof. See Appendix A.1.5. ■

Note that Proposition 5 assumes that the insurance motive is absent. We consider this assumption as a restriction since adding more benefits makes it more likely that D_1 will be positive, and the optimal debt should be even higher in this more general case. Taking stock, using a two-period model we have shown that (i) debt-financed defense spending exerts negative pressure on bond prices (Proposition 2), (ii) the optimal mix of defense financing involves debt (Proposition 3) and (iii) the model with the deterrence channel should have more debt (Propositions 4 and 5). In the next section, we quantitatively evaluate the two-period model insights in a calibrated infinite-horizon model.

5 Quantitative Results

This section presents the quantitative results using the calibrated infinite-horizon model. We show that the qualitative results from the two-period model of Section 4 hold in the infinite-horizon model, in the absence of the assumptions of the two-period model. In addition to that, quantitative results allow us to gain insights that are inaccessible analytically. We first discuss the calibration strategy and then turn to analyzing the results.

5.1 Calibration

The model frequency is annual with $\beta = 0.96$. We parameterize the utility function as follows: $u(c) = log(c)$ and $v(l) = -B\frac{(1-l)^{1+\eta}}{1+\eta}$, with $\eta = 1$ to have a Frisch elasticity $1/\eta = 1$, which is in line with estimates in the literature. We set $B = 16.99$ to match average hours worked of $1/3$ of the time endowment in the first-best case of the N state. The production function is linear $F(z, h) = zh$, where z^N is normalized to a unit value and z^W is calibrated as described in the "Disasters" paragraph.
We calibrate the peacetime government expenditure process using US government expenditure data from 1947

to 2023. We define g_t to include government consumption and investment net of federal defense investment, both reported in NIPA Table 3.9.5. The stochastic properties of g_t, ρ^g and σ^g, are estimated using the linearly detrended and deflated data series. We then set μ_g so that the model-implied government share of output is equal to 13.12%, consistent with the data given our definition of g_t.
Our empirical counterpart of D_t is the US federal defense investment, which includes investment in military equipment, structures and intellectual property products. Consequently, the empirical counterpart of DS_t is the stock of the sum of these categories. We use information from the NIPA Fixed Asset Tables from 1929 to 2023 to obtain the annual depreciation rate and the stock of disaggregated DS_t categories in order to calculate the depreciation rate of each of these categories separately.[7] Calibrated δ is then the weighted average of depreciation rates by each D_t category. This procedure gives an annual depreciation rate of 9.31%.

Disasters. To calibrate parameters related to disasters, we exploit the information contained in the Caldara and Iacoviello (2022) Geopolitical Risk Index. The index can be split into two sub-indices, denoted as *threats* and *acts* indices. The *threats* index captures the underlying geopolitical risk relevant to the US without the actual events realizing. The *acts* index, on the other hand, captures the realizations of actual geopolitical events and higher index values are related to higher event intensity. To identify disasters in our sample, we normalize the *acts* index to be between 0 and 1. We interpret this as disaster probability and we classify as disasters those events where the value of the normalized index is above 0.5. Given this definition, two episodes classify as disasters: World War II and the September 11 episode. To calibrate g^e we compare the US federal government spending in the year before the disaster with the peak of government spending during the disaster. Specifically, in the World War II case, we compare the US federal government spending as a share of GDP in 1940 with the highest value during years 1941 − 1945. Similarly, for the September 11 episode, we compare the US Federal government spending in 2001 with the highest value in years 2002 − 2003, when the US was involved in wars in Iraq and Afghanistan as a consequence of the September 11 acts. To get g^e we then take the average difference of the two episodes. This gives the value of 0.0568 or 17.5 percent of GDP. We calibrate z^W similarly. We compare the US TFP in a year following the disaster episode with the year just before the episode. Specifically, we compare the US TFP in the year 1946 with 1941 and, similarly, the TFP in 2003 with 2001. The calibrated z^W is then the average of the two differences, which is 0.9653. To obtain the TFP data for the pre-World War II as well as for the World War II period we rely on the estimates from Field (2023).

Insurance. To discipline the insurance motive of DS, we draw on the Ukraine Support Tracker compiled by the Kiel Institute for the World Economy.[8] The dataset records military, financial, and humanitarian assistance committed and transferred by governments to Ukraine since the onset of the Russia–Ukraine war. Because an overwhelming fraction of this assistance finances wartime expenditures, we interpret it as a proxy for g^e. We calibrate $\phi = 0.47$, corresponding to the share of military aid in total assistance to Ukraine.

Additionally, solving the model via the system of optimality conditions requires taking the derivative of the function $\mathcal{S}(DS, \phi g^e)$, which is, in principle, a non-differentiable min function. We approximate it using the LogSumExp function:

$$\mathcal{S}(DS, \phi g^e) = \frac{1}{\alpha} \log \left(e^{\alpha DS} + e^{\alpha \phi g^e} \right) \text{ with } \alpha \leq 0,$$

[7] The NIPA Fixed Asset Tables split the defense capital stock in the following categories: (i) Equipment (Aircraft, Missiles, Ships, Vehicles, Electronics, Other equipment), (ii) Structures (Buildings, Military Facilities), and (iii) Intellectual property products (Software, Research and Development).

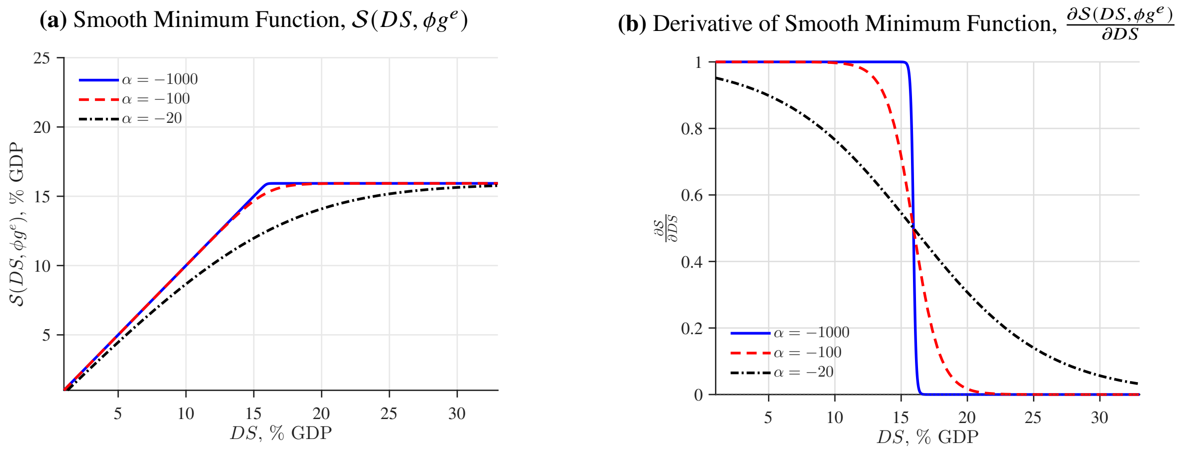
[8] https://www.kielinstitut.de/publications/ukraine-support-tracker-data-6453/

where

$$\lim_{\alpha\to-\infty} \mathcal{S}(DS, \phi g^e) = \min(DS, \phi g^e) \quad \text{and} \quad \lim_{\alpha\to 0} \mathcal{S}(DS, \phi g^e) = \frac{DS + \phi g^e}{2}.$$

A key advantage of LogSumExp over other smooth maximum/minimum operators is that it yields monotonic first derivatives, which is beneficial for numerical implementation. Figure 4 illustrates the function and its derivative for different values of α.

Figure 4. LogSumExp at different values of α.



Deterrence. To estimate the parameters of the ξ_t process, we exploit the *threats* component of the Caldara and Iacoviello (2022) Geopolitical Risk Index. According to the authors, the *threats* index captures developments outside US control—such as war threats, military buildups, or terrorist activity—that elevate the risk of actual events. This aligns closely with our definition of ξ_t. We estimate the AR(1) parameters of $\log(\xi_t)$ directly from the *threats* index data, yielding values of -0.2528, 0.8483, and 0.2446 for μ_ξ, ρ_ξ, and σ_ξ, respectively.

We then use the Caldara and Iacoviello (2022) Geopolitical Risk Index and U.S. defense capital stock data to estimate the *deterrence* motive. Specifically, we parameterize the foreign country's payoff as $V(DS, \xi) = \beta_1 + \beta_2 \log(DS) + \beta_3 \log(\xi)$. In this case, the war probability, given by equation (2), takes the following form:

$$P(DS, \xi) = \frac{1}{1 + e^{-\beta_1 - \beta_2 \log(DS) - \beta_3 \log(\xi)}}. \tag{20}$$

We estimate the parameters $\beta_1, \beta_2, \beta_3$ using nonlinear least squares applied to Equation (20). We use the normalized *acts* index as a proxy for $P(DS, \xi)$, the *threats* index as a proxy for ξ, and obtain DS data from the NIPA Fixed Asset Tables. This procedure yields estimates of $\beta_1 = -2.99$, $\beta_2 = -0.76$, and $\beta_3 = 0.87$, where β_1 governs the baseline disaster probability when $DS = 0$, and β_2 captures the sensitivity to changes in defense capital.[9] Note that a negative estimate for β_2 and a positive estimate for β_3 are consistent with our assumptions on the foreign country's payoff, i.e. $\frac{\partial V(\cdot)}{\partial DS} < 0$ and $\frac{\partial V(\cdot)}{\partial \xi} > 0$.

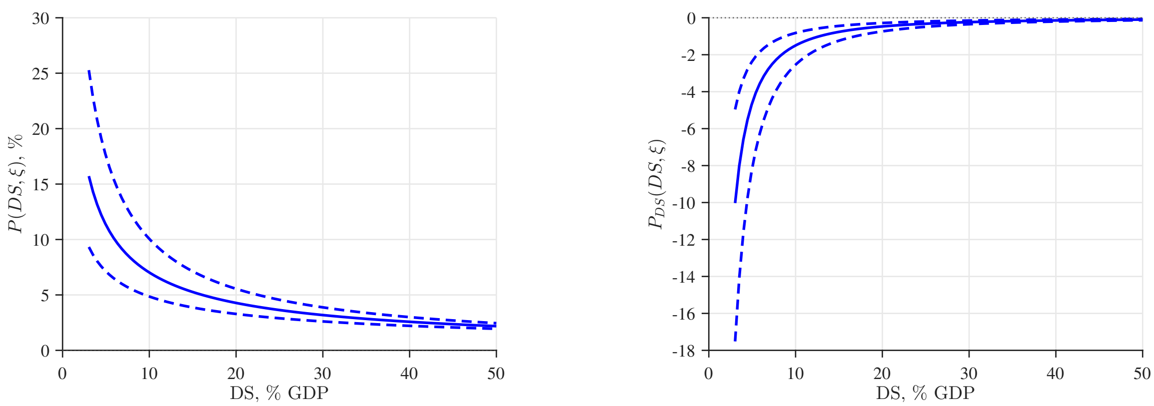
Figure 5 plots $P(DS, \xi)$ and its derivative at the estimated parameter values, as well as 10% confidence bounds for $\hat{\beta}_2$, evaluated at the average value of ξ. The left panel shows that a defense capital stock equal to 5% of GDP corresponds to an 11% disaster probability, while increasing the stock to 15% reduces this probability to below 5%. Lastly, we set the bond limits to 0% and 300% of GDP, effectively not allowing the government to accumulate assets. Table 1 summarizes all parameter estimates.

[9] In Appendix B.2, we provide a detailed explanation of the estimation procedure and the related data transformations. We also explore alternative specifications, which yield robust results.

We solve the model using an algorithm similar in spirit to the Parameterized Expectations Algorithm proposed by den Haan and Marcet (1990). The method relies on stochastic simulation and uses an artificial neural network to approximate integrands of the forward-looking terms in the optimality conditions as functions of the state vector, as proposed in Valaitis and Villa (2024). Stochastic simulation is necessary because the model features six state variables, making grid-based methods computationally infeasible. The presence of disasters introduces highly nonlinear dynamics with large deviations from average values, making the flexible, nonparametric structure of the neural network particularly useful. Model solution details are provided in Appendix A.5.

Figure 5. $P(DS, \xi)$ and $P_{DS}(DS, \xi)$ at the estimated values of β_2.

(a) War Probability — **(b)** Marginal Effect of DS on War Probability



Notes: The figure shows $P(DS, \xi)$ and $P_{DS}(DS, \xi)$ as functions of DS measured as a share of GDP. Solid lines show the functions at the estimated values for β_2 and dashed lines show $P(DS, \xi)$ and $P_{DS}(DS, \xi)$ evaluated at 10% confidence bounds for β_2.

Table 1. Parameter values.

Parameter	Value	Description	Source/target
β	0.96	Discount factor	Annual frequency
γ	1	RRA	Literature
η	1	Inverse Frisch elasticity	Literature
B	16.99	Relative labor disutility	Hours 1/3 of time endowment
$\beta_1, \beta_2, \beta_3$	$-2.99, -0.76, 0.87$	$P(DS, \xi)$ parameters	Own estimates
δ	0.093	Depreciation rate of military capital	NIPA
$z^W - z^N$	0.0347	Output loss during disasters	Own estimates
g^e	0.0568	Govt. disaster spending, 17.5% of GDP	Own estimates
μ_g, ρ_g, σ_g	$-0.1362, 0.9558, 0.0486$	g^N parameters	US govt. spending net of defense investment
$\mu_\xi, \rho_\xi, \sigma_\xi$	$-0.2528, 0.8483, 0.2446$	ξ_t parameters	Geopolitical risk *threats* index
ϕ	0.47	DS and g substitutability	Own estimate
α	-1000	$\mathcal{S}(DS, g^e)$ parameter	To mimic a hard min
$\underline{b}, \overline{b}$	0, 0.99	Bond constraints	0% and 300% of the average GDP.

External Validation. Next, we evaluate how the model compares with US data in terms of untargeted moments, as shown in Table 2. The model-implied averages align closely with the relevant empirical counterparts. For instance, the model implies an average defense investment of 1.41% of GDP, slightly below the 2.13%

observed in the data. Unsurprisingly, the model's average government spending share ($g_t + D_t$) is also very close to the data, at 16.65% versus 15.95%.

Moreover, the model implies a disaster probability of approximately 8.27%, which is only slightly higher than in the data, given the level of defense investment. The results also show that the planner tends to maintain the level of debt average 58.54% of GDP—broadly in-line with the postwar US average of 64%. Importantly, these outcomes are not hardwired into the model: the planner could choose not to invest in defense and accept a much higher war probability.

The model-implied defense capital stock averages 7.88% of GDP, which is significantly lower than the historical US average since 1929. This discrepancy reflects an implication of the model's *insurance* motive: disaster-time defense spending substitutes for emergency fiscal needs but does not contribute to the future defense capital stock unless D_t in the disaster state exceeds g^e, as shown in Equation (3). In contrast, US data suggest that much of the military capital accumulated during World War II was retained. This explains why the model matches the defense investment rate but falls short in reproducing the average defense capital stock. Nevertheless, our number is in the ballpark of the current US defense capital stock of 8% of GDP.

Table 2. Data and Model Untargeted Moments.

Description	Moments	Model	Data
Defense Stock to GDP, %	$\mathbb{E}(DS_t/Y_t)$	7.88	18.50
Defense Investment to GDP, %	$\mathbb{E}(D_t/Y_t)$	1.41	2.13
Gov. Spending to GDP, %	$\mathbb{E}(g_t + D_t)$	16.65	15.95
Disaster Probability, %	$\mathbb{E}(P(DS_{t-1}(1-\delta) + D_t, \xi_t))$	8.27	7.30
Debt to GDP, %	$\mathbb{E}(b_t/Y_t)$	58.54	64.02
Tax Rate, %	$\mathbb{E}(\tau_t)$	18.87	17.21

Notes: The table shows the model-implied moments and the US empirical counterparts. Model moments are obtained from 5000 simulations of 1000 periods. We drop the initial 200 periods in our calculations. US data moments use years 1929-2023. Debt to GDP data uses years 1939-2023. Government spending and defense data come from the BEA NIPA Table 3.9.5 and the BEA Fixed Asset Table 7.1. The disaster probability follows the calibration procedure exploiting the Caldara and Iacoviello (2022) geopolitical risk index. The average tax rate uses the labor tax rate calculated in Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015).

In order to assess how the model performs in terms of the dynamic responses of endogenous variables, we compute impulse responses to military spending shocks in US data using the local-projection method of Òscar Jordà (2005), as implemented by Ramey and Zubairy (2018). The authors use a narrative approach to identify news about future changes in military spending, which are plausibly exogenous to the current state of the economy. These shocks are then used to isolate exogenous changes in government spending.

We use the same sample period as Ramey and Zubairy (2018), from 1929 to 2015, and estimate impulse responses via local projections including four lags of the endogenous variables as controls.[10] We focus on the empirical responses of labor taxes, debt, and government spending, shown in Figure 6. By regressing changes in these variables on the military news shock, we estimate their impulse responses to exogenous government spending changes.

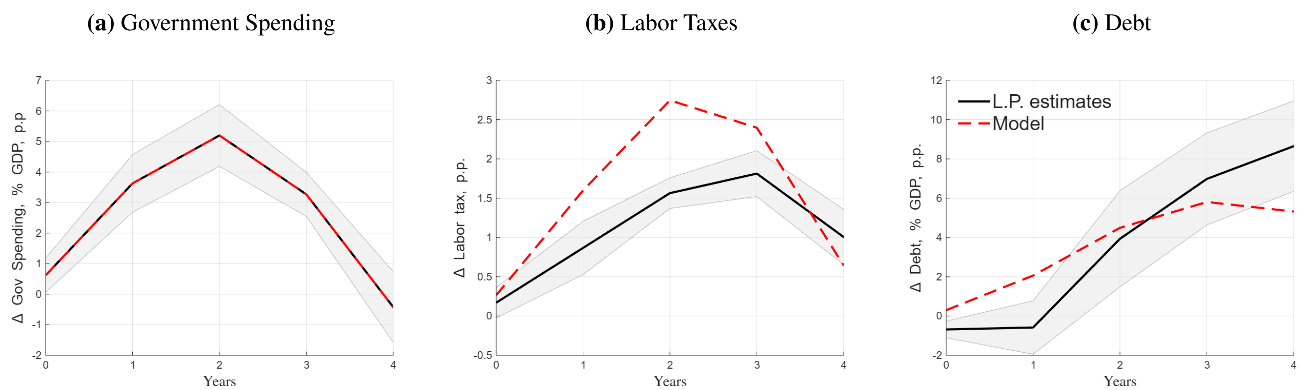
To compare the model with the data, we scale the military news shocks so that the peak response of government spending is approximately 5% of GDP, corresponding to two standard deviations of the estimated AR(1) government spending process used in the calibration. We then solve the model for the sequence of

[10]Appendix B.4 provides a detailed description of the estimation procedure.

exogenous government spending shocks that replicates the empirical government spending response.

Figure 6 plots the empirical impulse responses (solid black line) alongside the model-implied responses (dashed red line). The model replicates the qualitative patterns observed in the data: the tax response is hump-shaped, while the debt response is persistent. Although the model generates a somewhat larger tax increase, we view the overall alignment as satisfactory, especially since these responses were not directly targeted in the calibration.

Figure 6. Data and Model Responses.



Notes: The figure shows responses to exogenous government spending shocks in the model and in the data. Solid black lines display the results of a local projection estimation on yearly US data from 1929 to 2015, giving the impulse responses to a military spending news shock. Solid lines give point estimates and the ranges are 95% confidence intervals. Dashed-red lines display the model impulse responses to the same government expenditure shock as the local projection response in the data.

5.2 Quantitative Analysis

We now turn to the quantitative analysis using the infinite-horizon model. In addition to helping us understand the quantitative importance of *deterrence* and *insurance* channels in the face of disaster risk, the infinite-horizon model allows us to understand the wartime dynamics as well as how debt is decumulated in the aftermath of wars. We proceed in three main steps. First, we present long-run dynamics obtained over multiple realizations of exogenous processes. Our main focus is on the long-run distributions of endogenous variables. Second, we analyze generalized impulse responses to both g_t and ξ_t shocks. Third, we compare how time versus cross-state smoothing of distortions is affected as *deterrence* becomes stronger. Fourth, we present a policy application where we ask if it is optimal to run higher deficits if the borrowing is to finance defense spending. In Appendix A.6, we also present dynamics during war episodes, again, obtained through long stochastic simulations over multiple realizations of exogenous processes.

In all these experiments, we compare the baseline model against two benchmarks. In the first benchmark, we switch off the *insurance* motive by setting $\phi = 0$ and label this as the *No Insurance* benchmark. The purpose is to identify which of the two motives drives the results and to enable direct comparison with Propositions 3 and 4, which also assume $\phi = 0$. In the second benchmark, we switch off both *deterrence* and *insurance* channels to render investment in D ineffective and label this as the *Exogenous* benchmark. In this case, the economy still experiences ξ_t shocks, but the planner can only respond to such uncertainty shocks using standard policy tools, such as taxes and debt. In this sense, it can be thought of as the standard optimal policy problem in incomplete markets (Aiyagari, Marcet, Sargent, and Seppala, 2002) extended with uncertainty shocks, where disasters still happen but are outside of the planner's control. We also refer to this as the "standard model." In this benchmark, we readjust the mean of the ξ_t process so that the average disaster probability is the same as in the baseline. It

would also be possible to solve a version of the model where we switch off only the *deterrence* channel. However, in this case defense capital becomes equivalent to a state-contingent asset that only pays off in the war state. In such a scenario, it will always be more worthwhile to simply accumulate assets in the form of b_t.
We show that the analytical results from Section 4 hold in the infinite-horizon model. Specifically, regarding Proposition 2, we show in Subsection 5.2.2 that an increase in D_t is indeed related to a lower bond's price compared to the second benchmark. Regarding Propositions 4 and 5, we show in Subsection 5.2.1 that the baseline model has higher levels of debt. In Section 5.2.2, we show that debt responds more strongly to both shocks when *deterrence* motives are present. In Subsection 5.3, we perform an additional exercise, where we resolve the *No Insurance* benchmark by increasing the strength of the *deterrence* motive. This allows us to understand how investment in D_t affects the smoothing of distortions over time and across states. The quantitative results from this exercise are consistent with Proposition 3, suggesting that the optimal policy favors smoothing distortions over time versus across states. Our policy application in Section 5.4 speaks directly to the policy debate on how to finance defense spending. We show that it is optimal to run multiple times larger deficits to finance defense compared to standard government spending g_t.
Besides confirming the analytical results, the quantitative results offer additional insights. The main takeaway is that most of the peacetime investment in DS is due to *deterrence* motives. This channel is also the main force driving the differential effects of bond prices in response to both g_t and ξ_t shocks. On the other hand, both channels are relevant in determining the average debt levels. We proceed by discussing these results in greater detail.

5.2.1 Long-Run Moments

We start by comparing long-run moments across the three models. To allow for a fair comparison, in the *Exogenous* benchmark, we set μ^g so that the peace time's average government spending $\mathbb{E}(g_t^N)$ matches the average total spending $\mathbb{E}(g_t^N + D_t)$ of the baseline model. Since g^N is linear in logs, we also adjust σ^g so that the standard deviation of g^N does not change. Furthermore, we tune μ_ξ so that the average disaster probability in the *Exogenous* benchmark is the same as in the baseline specification.
Table 3 shows the relevant moments. Comparison across the models highlights how the *deterrence* and *insurance* channels shape the dynamics of debt, taxes, defense investment, and geopolitical risk. To isolate the role of each channel separately, we first compare the *No Insurance* and the *Exogenous* benchmarks to isolate the role of *deterrence* and, second, we compare the baseline with *No Insurance* to isolate the role of the *insurance* channel. Finally, we compare the baseline model with the *Exogenous* model.

First, the comparison of the *No Insurance* model (column two) with the *Exogenous* model (column three) model highlights the role of the *deterrence* channel. The optimal level of debt is shaped by two forces. On the one hand, when the planner has the option to invest in defense, it invests significant amounts in it, and, as shown in Proposition 3, it mainly finances it with borrowing. On the other hand, through *deterrence*, the planner is able to minimize the average disaster frequency and the variation in disaster risk. As the planner also tends to borrow significantly during war episodes, all else equal, lower war frequency implies lower average debt. Quantitatively, we find that borrowing for deterrence dominates as the *No Insurance* benchmark has almost 10% to GDP more debt, consistently with Proposition 4.

Second, the comparison of the baseline model (column one) with the *No Insurance* model (column two) highlights the role of the *insurance* channel. The option to deplete the existing DS stock as a substitute for wartime spending effectively makes wars less dramatic episodes from the tax smoothing point of view. This creates an extra incentive to maintain a large stock of DS, but at the same time makes the *deterrence* and *price manipulation* channels quantitatively less relevant, since the terms $\mathbb{E}_t^{g,\xi}\left(U(c_{t+1}^W, l_{t+1}^W) - U(c_{t+1}^N, l_{t+1}^N)\right)$ and

$\mathbb{E}_t^{g,\xi}\left(U_c(c_{t+1}^W) - U_c(c_{t+1}^N)\right)$ in Equation (13) shrink. We observe that weaker deterrence leads the planner to invest less in defense—7.88% rather than 8.32% of GDP—and results in more frequent disasters, 8.3% rather than 7.9%. As a consequence of more frequent wars, the planner accumulates more debt on average than in the *No Insurance* benchmark. Finally, the comparison between the baseline model (column one) and the *Exogenous* model (column three) shows that, once we control for the average war probability, the model with endogenous disasters features more debt, consistently with Proposition 5. At the same time, even though we match the average war probability, the baseline model has endogenously lower disaster risk volatility (2.25 versus 3.05), which allows for greater tax smoothing. This can be seen from higher autocorrelations of debt and taxes in the baseline as the optimal tax-smoothing policy aims to make taxes approximately follow a random walk.

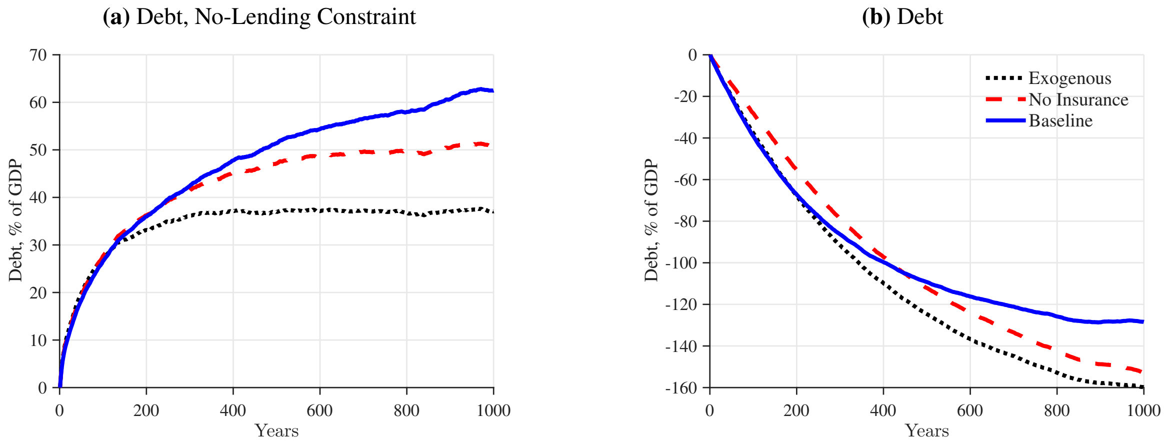
Table 3. Counterfactual Outcomes.

Moments	Baseline	No Insurance	Exogenous
Debt, % GDP			
$\mathbb{E}(b_t/Y_t)$	58.56	51.80	42.00
$\sigma(b_t/Y_t)$	43.21	40.69	34.58
$\rho(b_t/Y_t, b_{t-1}/Y_{t-1})$	0.9833	0.9800	0.9687
Taxes, ppt			
$\mathbb{E}(\tau_t)$	19.23	18.97	18.86
$\sigma(\tau_t)$	3.24	3.18	3.17
$\rho(\tau_t, \tau_{t-1})$	0.96	0.96	0.94
Defense, % GDP			
$\mathbb{E}(DS_t/Y_t)$	7.88	8.32	-
$\mathbb{E}(D_t/Y_t)$	1.41	0.78	-
$\mathbb{E}(D_t/Y_t \vert \mathcal{I}_t = 0)$	0.99	0.84	-
War Probability, %			
$\mathbb{E}(Pr)$	8.3	7.9	8.3
$\sigma(Pr)$	2.25	2.06	3.05

Notes: The table shows the salient long-run moments across the three models. In the *Exogenous* benchmark we adjust the average level of peace-time government expenditure to make it equal to the total peace-time government expenditure $(g + D)$ in the baseline. Moments come from 5000 simulations for 1000 periods. We discard the first 200 periods in our calculations. Bond limits are set to 0 and 300% of GDP.

Next, we turn to analyze the long-run convergence properties. Aiyagari, Marcet, Sargent, and Seppala (2002) show that absent constraints, the Ramsey planner accumulates assets and finances spending using asset income. They prove this analytically in a model with quasi-linear preferences and confirm quantitatively in a model with risk-averse agents. The result relies on the same tax smoothing condition (Equation 10) but requires the expenditure process to be sufficiently homoscedastic, while in our model expenditure process is heteroscedastic as the planner endogenously affects the war probability and therefore the probability distribution from which next period's g is drawn. Figure 7 shows the ensemble averages of the three models with and without no-lending constraint. Results on the right panel quantitatively confirm the Aiyagari, Marcet, Sargent, and Seppala (2002) result of asset accumulation in the long run both in the models with exogenous (*Exogenous* benchmark) and endogenous war probability (Baseline and *No Insurance*). When we impose the no-lending constraint in the left panel, the average long-run debt levels are significantly larger than in the *Exogenous* benchmark. These quantitative findings also confirm analytical results from Proposition 19 - long-run debt levels are higher when defense spending affects war probability. Both *deterrence* and *insurance* aspects contribute to higher long run debt levels as seen by the baseline model having the highest long-run debt.

Figure 7. Long-run dynamics in the model with and without lending constraints.



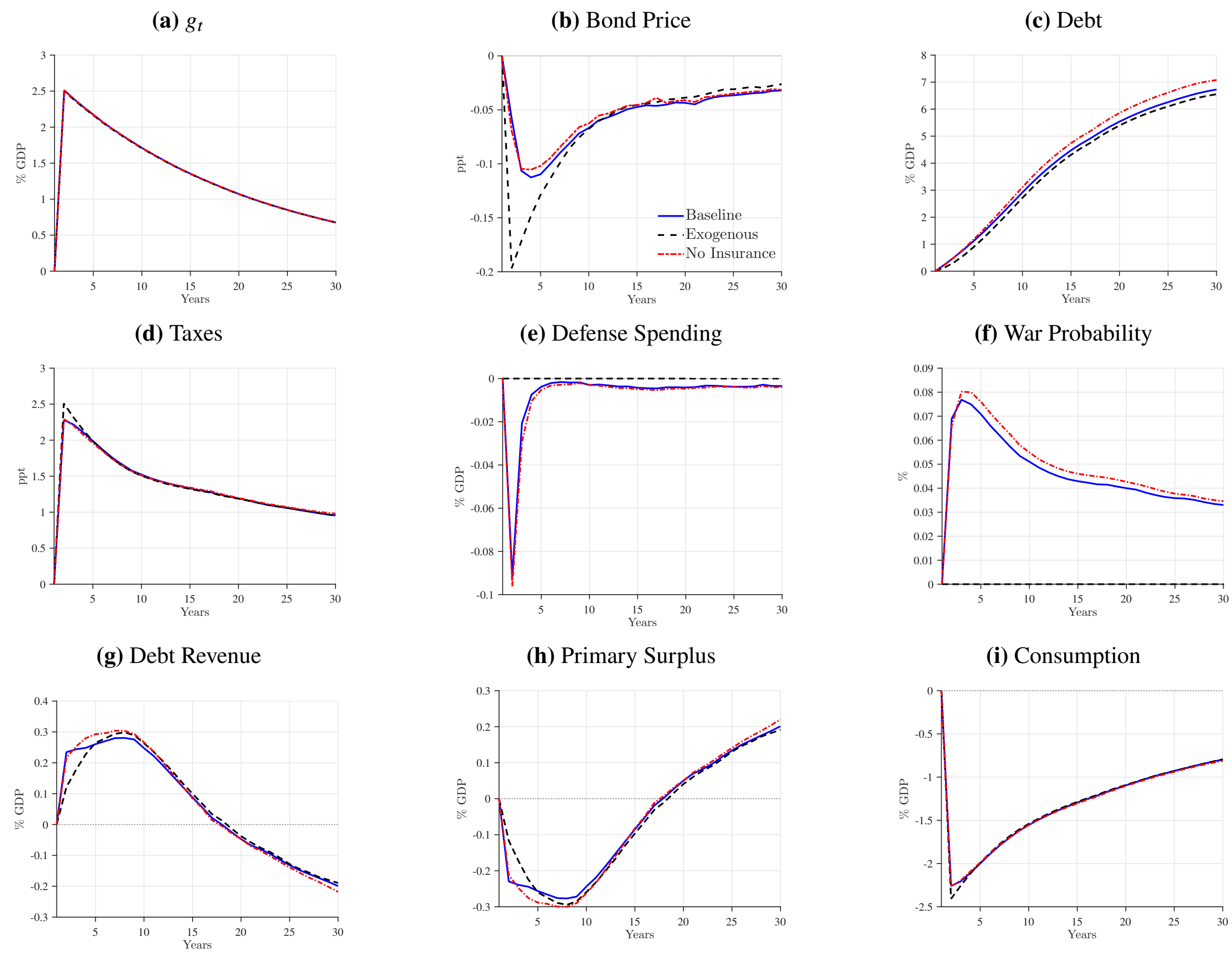
Notes: The figure shows ensemble averages from 5000 simulations of 1000 periods. The right panel shows average debt dynamics with the bond limits set at ±300% of GDP. The left panel shows the average debt dynamics in the model with a government no-lending constraint and an upper debt limit of 300% of GDP. The solid blue line is the baseline model, the red-dashed line is the counterfactual without the insurance channel, and the black-dotted line is the model where defense investment is ineffective and wars are exogenous.

5.2.2 Generalized Impulse-Responses

Government spending shocks We now investigate the model responses to unexpected government spending shocks. This exercise allows us to ask whether it is optimal to decrease investment in DS when other spending needs arise and what are the effects on prices and debt issuance. This allows for a neat comparison as the responses to g_t in the *Exogenous* benchmark are the same as in Aiyagari, Marcet, Sargent, and Seppala (2002). Figure 8 shows the generalized impulse-responses to a one-standard-deviation g_t shock. Response in the standard model (dot-dashed black line) highlights the typical tax-smoothing result: an increase in taxes is modest but very persistent while a large share of the shock is financed with debt. An initial increase in g_t by 2.5% of GDP leads to a $\sim$ 2.5 percentage point increase in taxes, and a subsequent increase in debt by roughly 7% of GDP. The shock is accompanied by an immediate drop in the bond's price as falling consumption in the shock period reduces household intertemporal motives to save.

The responses of the baseline model in the blue line show that it is optimal to cut defense spending. On impact, it falls by 0.1% of output or by 10% of the average peace-time D spending. Such a cut has two effects. First, it redirects labor tax income to financing g_t, easing the pressure to increase taxes. Second, a cut in defense spending allows the planner to manipulate bond prices. Falling defense spending increases the disaster probability. Such an endogenous rise in risk increases the household's precautionary saving motives and alleviates a fall in bond prices relative to the standard model. This price manipulation allows for cheaper borrowing than otherwise and enables further tax smoothing. A 0.1% drop in defense spending enables the planner to halve the fall in bond prices and achieve even better tax smoothing and higher borrowing, both in the baseline and in the *No Insurance* models. The responses in the baseline and in *No Insurance* are qualitatively similar, the main difference being that, due to the lack of insurance, the price-manipulation effect is stronger in the *No Insurance* case, resulting in larger defense cuts, smaller fall in prices, and more debt issuance. In appendix Section A.6, we show that similar price manipulation dynamics happen during war episodes. The depletion of defense capital due to *insurance* in the baseline model helps to mitigate a fall in bond prices, which helps to stabilize budget revenue and private consumption.

Figure 8. Generalized impulse-response to a government expenditure shock.

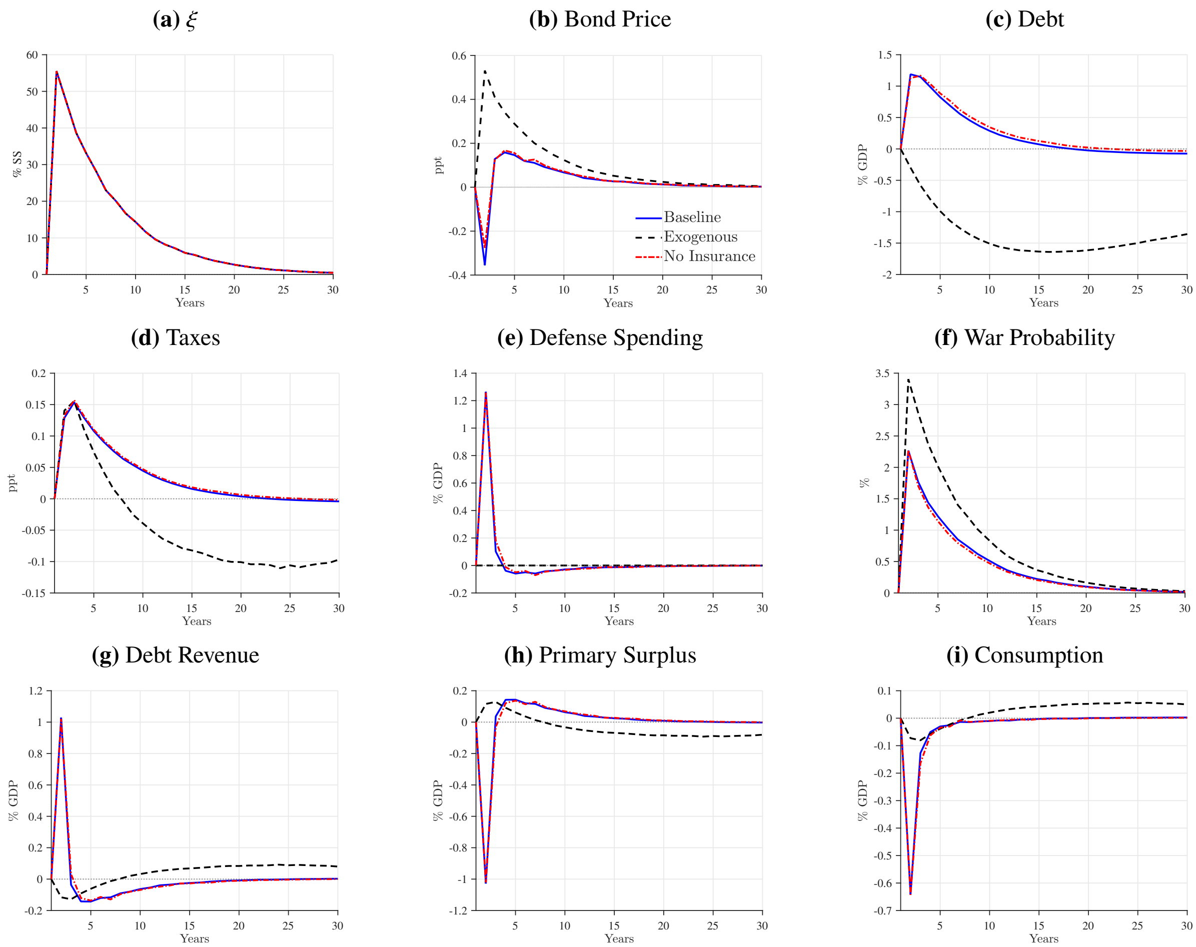


Notes: The figure shows the generalized impulse responses to a one-standard-deviation shock to ϵ_t^g. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line reports the benchmark where D is ineffective. Debt revenue is defined as $Revenue = Q_t b_{t+1} - b_t$. Primary surplus is defined as $s = \tau_t h_t - g_t - D_t$.

Geopolitical risk shocks Next, we consider optimal responses to an unexpected geopolitical risk shock. As this raises the disaster probability, it can be interpreted as an uncertainty shock. We start by looking at the responses in the standard model, where D is ineffective as optimal responses to such a shock have not been studied before. We consider a one-standard-deviation shock to ξ_t, or equivalently, a 3.5% increase in disaster probability as shown by the dot-dashed black line in panel (f).

In the standard model, where D is ineffective, the planner responds to increased risk by accumulating assets financed by an increase in taxes. Essentially, the planner uses non-contingent assets to create insurance against the disaster state. Such asset purchases are associated with a minor fall in consumption and an increase in the bond's price, as the household's precautionary motives also increase in the presence of higher uncertainty. In our baseline model, as well as in the *No insurance* benchmark, the *deterrence* motives are quantitatively strong and the planner forgoes insurance and responds by investing in DS, which is financed by a mix of taxes and debt. Comparing the baseline with the standard model, taxes increase by a similar amount but are used for different purposes. In the baseline, the planner responds by increasing defense investment by around 1.2% of GDP for deterrence motives as it mitigates the increase in the disaster probability to 2%. By reducing this probability, investment in DS effectively reduces the expected future tax distortions, which justifies financing it with debt. Such risk mitigation also lowers household precautionary savings motives, and bond prices fall in both the baseline and the *No Insurance* models. This price manipulation effect should potentially discourage DS investment. However, as we show in Section 6.3.1, the price manipulation term is quantitatively much less important than *deterrence*. Rather similar responses of the baseline and the *No Insurance* model suggest that the insurance aspect is also quantitatively small, at least to understand responses to unexpected shocks. Overall, in responding to uncertainty shocks, the planner weighs the benefits of building up reserves to meet the disaster state versus investing in DS to mitigate the disaster's risk, at the expense of a large fall in current consumption. The analysis shows that a fall in current consumption is optimal. Panels (g) and (h) are informative about the relative financing mix. Panel (h) plot shows that a 1.2% of GDP increase in defense investment is accompanied by a primary deficit of 1% of GDP, suggesting that it is optimal to finance defense investment in the wake of increasing geopolitical risk mainly with debt, despite the implied negative pressure on bond prices that defense investment creates.

Figure 9. Generalized impulse-response to a geopolitical risk shock



Notes: The figure shows the generalized impulse responses to a one-standard-deviation shock to ϵ_t^{ξ}. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line represents the benchmark model in which D is ineffective. Debt revenue is defined as $Revenue = Q_t b_{t+1} - b_t$. Primary surplus is defined as $s = \tau_t h_t - g_t - D_t$.

5.3 Smoothing taxes across states or over time?

In this exercise, we are interested in understanding how the planner achieves tax smoothing, whether by minimizing the variance of tax distortions over time or across states. The two-period model intuition states that the planner optimally issues more debt to finance defense spending in the presence of increased risks (Propositions 4 and 5). This implies that in the second period, the planner ends up with higher liabilities than in the standard model and consequently sacrifices tax smoothing across war and normal states while minimizing the war probability. We are interested in how this intuition carries through to the infinite horizon. In this setting, dynamics become more involved as the planner is able to borrow and cut defense spending in war states, helping to alleviate the tax distortions. Additionally, the recursive multiplier now captures not just the tax distortions arising from the current shocks but also the history of previous shocks as planners' policies become time-dependent.

We answer these questions by resolving the *No Insurance* benchmark with different values of the marginal effect of DS on war probability, as captured by $\frac{\partial P(DS,\xi)}{\partial DS}$. We do so by changing β_2, while simultaneously adjusting β_1 such that given the defense capital stock of 8.32% of output, the war probability is 7.93%, consistent with our calibration, reported in Table 3.

Table 4 shows selected moments from long simulations for several β_2 specifications, where more negative values are associated with the more negative marginal effect of DS and thus a stronger *deterrence* motive. As β_2 becomes more negative, the planner invests more in defense and reduces the war probability from 9.2% to 4.9% by increasing the average defense capital stock from around 5% to above 11% of GDP, which is qualitatively not surprising. As deterrence becomes stronger, the planner spends more on defense but incurs wartime spending less frequently. The average level of debt, reported in the first row is shaped by two effects. As the *deterrence* channel becomes stronger, the planner borrows more to finance defense spending. This first channel captures the ex-ante borrowing motive in the presence of geopolitical risks and is associated with higher levels of debt. The planner also borrows extensively during the war episodes, therefore, an economy with lower war probability should have less debt all else equal. Results show that ex-ante borrowing dominates and average debt increases as deterrence becomes stronger.

Next, we look at the implications for smoothing of tax distortions, which are summarized by the recursive multiplier. The debt optimality condition (11) states that planners' tax smoothing objectives imply that the recursive multiplier μ_t is a risk-adjusted martingale sequence. Hence, the closer its autocorrelation is to one, the better the planner is able to achieve the tax smoothing objectives. Results show that autocorrelation of the multiplier as well as debt monotonically increase towards one as the *deterrence* channel becomes quantitatively more important. Effectively, by investing in defense, the planner minimizes the frequency of disaster shocks, which allows the planner to smooth tax distortions over time. Such investments are mainly financed by debt, which becomes more volatile and more persistent. As the planner accumulates debt in the presence of high uncertainty, it optimally sacrifices the smoothing of distortions across states, as wars also tend to occur mostly in high uncertainty periods, when the planner has already borrowed extensively to finance deterrence. Consequently, the difference between the average multiplier values in the war and normal states increases as β_2 becomes more negative. In this sense, the *deterrence* motive shapes the policy towards smoothing distortions over time and away from smoothing across states.

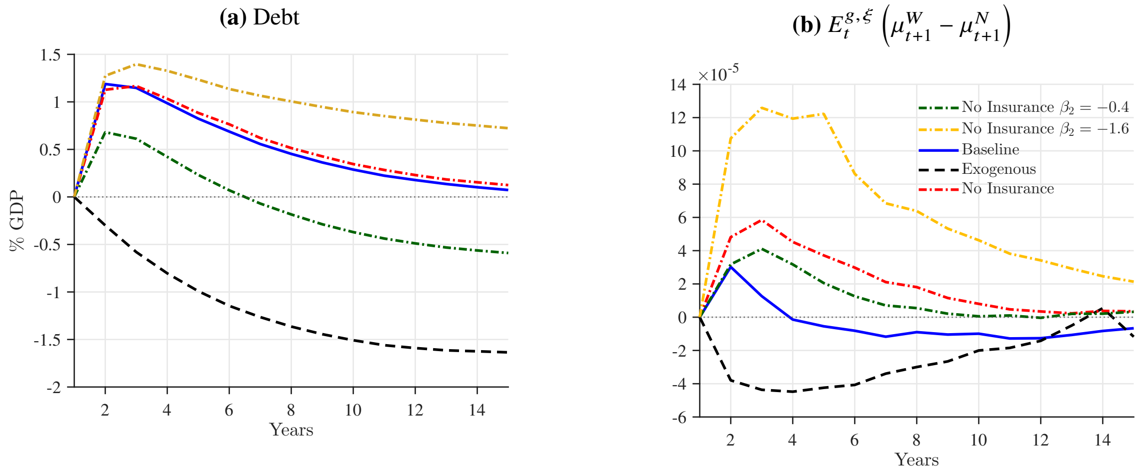
Table 4. Role of $\frac{\partial P(DS,\xi)}{\partial DS}$, selected moments.

Moments	$\beta_2 = -0.4$	$\beta_2 = -0.76$	$\beta_2 = -1.6$
Debt, % GDP			
$\mathbb{E}(b_t/Y_t)$	40.4275	51.8049	79.2647
$\sigma(b_t/Y_t)$	34.6243	40.6916	51.7344
$\rho(b_t/Y_t, b_{t-1}/Y_{t-1})$	0.9730	0.9800	0.9892
Taxes, ppt			
$\mathbb{E}(\tau_t)$	18.3880	18.9730	19.9157
$\sigma(\tau_t)$	3.0362	3.1805	3.4560
$\rho(\tau_t, \tau_{t-1})$	0.9654	0.9635	0.9594
Multipliers			
$\mathbb{E}(\mu_t)$	0.1138	0.1187	0.1272
$\sigma(\mu_t)$	0.0233	0.0249	0.0277
$\rho(\mu_t, \mu_{t-1})$	0.9641	0.9655	0.9771
$\mathbb{E}\left(\mathbb{E}_t^{g,\xi}(\mu_{t+1}^W - \mu_{t+1}^N)\right)$	0.0127	0.0131	0.0148
Other, %			
$\mathbb{E}((g_t + D_t)/Y_t)$	16.55	16.67	16.5
$\mathbb{E}(DS_t/Y_t)$	5.11	8.32	11.04
$\mathbb{E}(P(DS_t, \xi_t))$	9.18	7.93	4.89

Notes: The table shows selected moments from the *No Insurance* model solutions with different values of β_2. When changing β_2, we adjust β_1 so that the disaster probability is equal to 7.98% when defense stock DS is equal to 8.32% of GDP. These are the values in the *No Insurance* model with the estimated β_1 and β_2 parameters, reported in Table 3. Moments come from 5000 simulations for 1000 periods. We discard the first 200 periods in our calculations. Bond limits are set to 0 and 300% of GDP.

Figure 10 illustrates this state versus time smoothing point through the impulse response to the geopolitical risk shock. It shows responses of debt and expected multipliers to the same shock as in Figure 9 by adding the responses of model with strong and weak *deterrence*. The left panel shows that the response of debt becomes stronger and more persistent as *deterrence* becomes stronger, which confirms the two-period model intuition. Differently from the two-period logic, the initial increase in debt can now be rolled over into the future periods thus allowing for better cross-state smoothing of distortions. The right panel shows the conditional expectation of multipliers in wartime and in normal state. Here a stark difference emerges between the standard model with exogenous disasters and our model. In the standard model an accumulation of assets means that the planner builds a cushion that could potentially be used in the war state. This enables better cross-state smoothing as can be seen by the difference in the two multipliers going down, shown in the dashed black line. In our model the planner borrows and optimally accepts that future wars may occur when the debt is already large and the fiscal space is limited. Consequently, the difference between the two multipliers increases and the increase becomes larger with stronger *deterrence* and more borrowing. In this way the planner sacrifices cross-state smoothing in favor of time-smoothing. The response of the multipliers in the baseline model is more muted because increased defense investment strengthens the insurance channel and endogenously makes wars less severe. Hence higher debt sacrifices cross-state smoothing while the insurance channel enables it. Taken together, these findings confirm that proposition A.1.3 carries through to the infinite-horizon model.

Figure 10. Responses to a geopolitical risk shock with high and low β_2.



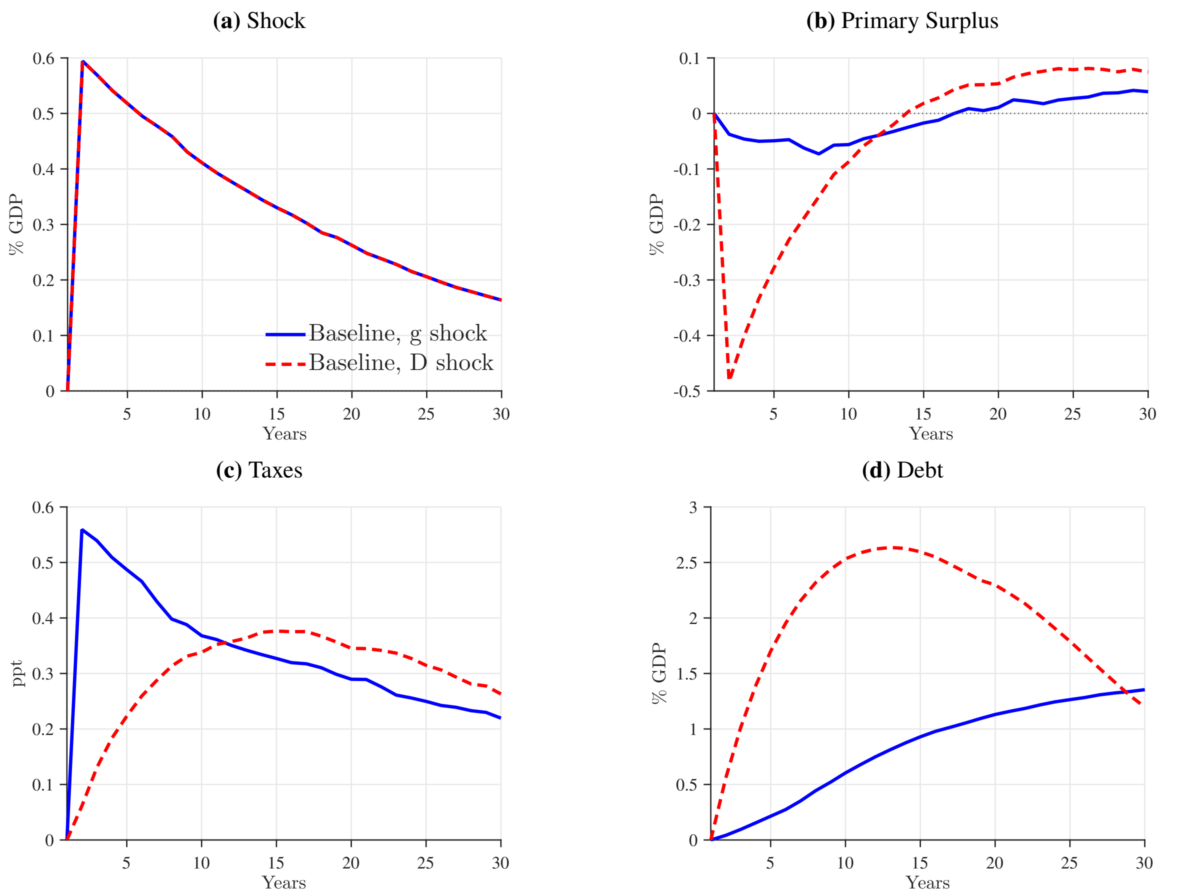
Notes: The figure shows generalized impulse responses to a one-standard-deviation shock to ξ_t. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line represents the benchmark model where D is ineffective. The dashed green line represents the No Insurance model with $\beta_2 = -0.4$ and the dashed yellow line is the No Insurance model with $\beta_2 = -1.6$. More negative values of β_2 are associated with stronger *deterrence* channel.

5.4 Policy Application: Optimal Mix of Defense Financing

So far, we have shown that in the presence of increasing geopolitical risk it is optimal to finance defense spending mainly through borrowing when such spending creates deterrence. In the last subsection, we study the role of primary deficit constraints in financing defense investment. In doing this, we aim to contribute to an ongoing discussion as to whether governments should lift the budget deficit rules when the borrowing is used to finance military spending. While historical examples show that such restrictions were typically be removed in times of heightened geopolitical risk (Marzian and Trebesch, 2015), theoretical guidance is still scarce. We contribute to this debate by conducting a policy exercise. Specifically, we ask how the optimal policy mix depends on the type of shock by considering the economy's responses to g_t and D_t shocks of the same size. In doing so, we aim to answer the question that policymakers typically ask: Is defense different from other types of spending? To answer this, we use our model to see how much more the planner should borrow to finance defense compared to the government expenditure shock of the same magnitude. Specifically, we compare the model responses to a $1/3$ standard deviation g_t shock and then look for the values of ξ_t shock such that the implied increase in defense investment exactly matches the government expenditure sequence.

Figure 11 presents the results. The solid blue line shows the baseline models' responses to a g_t shock and dashed-red line - same models' responses to an analogous increase in defense spending. The main observation is the striking difference in responses to these two shocks. Results show that most of the initial increase in defense spending is financed through borrowing and large primary deficits. On impact, the primary deficit falls by 0.5% of GDP, suggesting that approximately 80% of the initial outlay is financed through borrowing. This is in stark contrast to standard government spending shocks, where approximately only 10% of the initial increase is financed through borrowing. The timing of debt issuance and taxes also differs markedly. The response to g_t shock follows standard tax-smoothing prescription with an immediate increase in taxes with a gradual decline in the aftermath. The response to a defense spending shock entails backloading of tax distortions while most of the initial spending increase is financed with debt. This is how the planner achieves time-smoothing of tax distortions, while at the same time sacrificing the cross-state smoothing as such a large increase in debt implies that wars would occur when the outstanding debt is already large. Specifically, D_t spending helps to mitigate the disaster risk. Because disaster states are precisely those in which the planner's implementability constraint is likely to bind heavily, the deterrence affects the perceived disaster probability and therefore, lowers the value of the expected future multiplier on the implementability constraint. Tax smoothing condition then dictates that the current multiplier should also fall, which entails more borrowing. This is the mechanism identified in Proposition 4, which states that borrowing enables the planner to bring the future benefits of defense investment into the present.

Figure 11. Differential responses to D_t and g_t expenditure shocks.



Notes: The figure shows the generalized impulse-responses to a government expenditure shock (dashed lines) and to the ξ_t shock that induces the same path in D_t (solid lines). Blue lines represent the baseline model, and red lines represent the *No Insurance* benchmark.

6 Extensions

In this section, we extend the model from Section 3 to incorporate two salient features: (i) inflation, which allows the government to reduce the real value of nominal debt, and (ii) an option for the home country to launch a preemptive strike, which may arise when increasing military capabilities makes such attacks increasingly less costly. This second extension then captures the idea that higher military investment may in fact increase the war probability. Apart from these extensions, the model remains identical to that in Section 3.

6.1 Environment

Inflation. Although the United States did not default on its World War II debt, inflation substantially reduced its real value (Acalin and Ball, 2026; Hall and Sargent, 2011, 2022; Jiang, Lustig, Nieuwerburgh, and Xiaolan, 2026). For this reason, we add an option to use inflation by making bonds b_t nominal, as shown in the household budget constraint (21). On the one hand, nominal debt can be inflated away, allowing the planner to achieve some state contingency without having to raise taxes as much during war episodes.

$$q_t b_{t+1} + c_t = w_t h_t (1 - \tau_t) + \frac{b_t}{\pi_t}, \tag{21}$$

On the other hand, expected use of inflation feeds back into the bond prices as shown in equation 22. Hence, if the private sector expects the planner to inflate the debt away, this would immediately get reflected in the bond prices and may potentially lead to less borrowing when responding to the geopolitical risk shocks.

$$q_t = \beta \mathbb{E}_t \left[\frac{u_c(c_{t+1})}{u_c(c_t)} \frac{1}{\pi_{t+1}} \right], \tag{22}$$

We also add a New Keynesian block. Intermediate firms set prices $P_{i,t}$ and hire labor to maximize the expected discounted value of real dividends:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{u(c_t)}{u(c_0)} d_{i,t}, \quad \text{where} \quad d_{i,t} = \frac{P_{i,t}}{P_t} Y_{i,t} - w_t h_{i,t} - \Phi_t.$$

Demand for the intermediate good follows from static profit maximization by the final-good producer: $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{1}{\nu}} Y_t$, and price adjustment entails a Rotemberg cost (Rotemberg, 1982): $\Phi_t = \frac{\varphi}{2}(\pi_t - \pi)^2$. In a symmetric equilibrium ($P_{i,t} = P_t$), profit maximization yields the New Keynesian Phillips Curve (NKPC):

$$Y_t \left(\frac{\nu - 1}{\nu} + \frac{w_t}{z_t \nu} \right) - \Phi_\pi(\pi_t)\pi_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \Phi_\pi(\pi_{t+1})\pi_{t+1} \right] = 0. \tag{23}$$

Deterrence and Temptation. In the baseline model, it is never optimal for the home government to initiate a war. Conditional on conflict, the economic consequences are independent of who initiates it, so the home government weakly prefers to wait and see whether the foreign country attacks. Defense investment therefore affects the war probability only through the *deterrence* channel and always reduces it. There are at least two ways to overturn this implication. First, a larger military stock may increase the political influence of the military-industrial complex and thereby strengthen the government's incentive to initiate a conflict (Eisenhower, 1961; Mearsheimer, 2001). Second, the home government may have a first-mover advantage: a preemptive strike may shift military operations to foreign territory and thereby reduce the collateral damage incurred at home. We adopt the latter mechanism because it can be incorporated directly into the planner's problem while preserving the assumption that the government maximizes household welfare. In this case, higher defense spending may increase the equilibrium war probability through a *temptation* channel and potentially overturn our results regarding tax smoothing and borrowing to finance defense.

To introduce this channel, we allow the home government to initiate the war and assume that its economic cost depends on who starts the conflict. This assumption is motivated by the evidence in Federle, Meier, Müller, Mutschler, and Schularick (2026), who show that the costs of war differ substantially depending on whether military operations take place on domestic or foreign territory. Accordingly, war costs are lower when the home government initiates the conflict, because doing so allows military operations to be shifted abroad. Specifically, suppose $g^e = g^F$ if the war is initiated by the foreign government and $g^e = g^H(DS_t)$ if it is initiated by the home government. We further assume that $g^H(DS_t)$ is decreasing in the home country's defense capital stock, capturing the idea that greater military capability allows a war to be concluded more quickly and with less economic damage. At average values of DS_t, the damage from a home-initiated war is therefore lower than that from a foreign-initiated war. This creates an incentive for a preemptive strike and provides an additional reason for the home government to invest in DS_t, which we call the *temptation* channel. Wartime productivity is equal to z^W and does not depend on who initiates the conflict.

In period t, together with its other policy choices, the home government chooses the probability of attacking the foreign country in period $t+1$, denoted by P^H_{t+1}. The foreign government makes its decision at the end of period t, after observing the realization of the shocks and the home government's policies. We denote by $P^F(DS_t, \xi_t)$ the probability that the foreign government initiates a war. The probability of war in period $t+1$ is therefore

$$P(\mathcal{I}_{t+1} = 1) = P^H_{t+1} + (1 - P^H_{t+1})\, P^F(DS_t, \xi_t). \tag{24}$$

6.2 The Ramsey Problem under Incomplete Markets

In this subsection, we solve for the time-inconsistent Ramsey plan of the extension, following a similar approach to that adopted for the baseline model. Call the history of shocks $S^t = (g^t, \xi^t)$, as in the baseline. The Ramsey planner seeks to maximize household utility (1) by choosing the sequences of plans

$$\{c_t(S^t), l_t(S^t), b_{t+1}(S^t), D_t(S^t), DS_t(S^t), \pi_t(S^t), w_t(S^t), P^H_{t+1}(S^t)\}_{t=0}^T,$$

subject to the implementability constraint with multiplier μ_t, the law of motion for the defense stock (3) with multiplier μ^D_t, the New Keynesian Phillips Curve (23) with multiplier λ^π_t, the nominal bond bounds ($\overline{b}$, $\underline{b}$) with multipliers ($\overline{\Lambda}_t$, $\underline{\Lambda}_t$), and the defense stock bound with multiplier λ^D_t and the bounds on P^H_{t+1} with multipliers $\underline{\Lambda}^P_t, \bar{\Lambda}^P_t$. The planner takes into account that the defense stock affects the probability of war $P(DS, \xi)$ through equation (24), and given initial conditions $\mu_0 = b_0 = 0$ and DS_0 set such that the probability of war at time 0 is 10%. More formally, the recursive Lagrangian of the planner reads:

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t) + v(l_t) + \\
& + \mu_t \left(\Omega_t + \beta \mathbb{E}_t \left[u_c(c_{t+1}) \pi_{t+1}^{-1} \right] b_{t+1} - u_c(c_t) b_t \pi_t^{-1} \right) + \\
& + \mu^D_t \left(DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t \right) + \\
& + \lambda^\pi_t \left(u_c(c_t) Y_t \nu^{-1} \left(\nu - 1 + w_t z_t^{-1} \right) - u_c(c_t) \Phi_\pi(\pi_t) \pi_t + \mathbb{E}_t \left[\beta u_c(c_{t+1}) \Phi_\pi(\pi_{t+1}) \pi_{t+1} \right] \right) \\
& - \bar{\Lambda}_t b_{t+1} + \underline{\Lambda}_t b_{t+1} + \lambda^D_t D_t + \underline{\Lambda}^P_t P^H_{t+1} - \bar{\Lambda}^P_t P^H_{t+1} \Big\},
\end{aligned}$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t)(w_t h_t - g_t - D_t + \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)) - v_l(l_t) h_t$ is the government primary surplus in marginal utility terms. The first-order condition with respect to bonds is:

$$\mu_t = \mathbb{E}_t(n_{t+1}\mu_{t+1}) + \frac{1}{\beta \mathbb{E}_t \left[u_c(c_{t+1}) \pi_{t+1}^{-1} \right]} \left(\overline{\Lambda}_t - \underline{\Lambda}_t \right), \quad \text{where} \quad n_{t+1} \equiv \frac{u_c(c_{t+1}) \pi_{t+1}^{-1}}{\mathbb{E}_t u_c(c_{t+1}) \pi_{t+1}^{-1}}. \tag{25}$$

The intuition behind equation (25) is similar to the baseline case, equation (10), except that now the planner can use inflation to inflate away nominal bonds, creating state-contingent insurance. The optimal choice of P^H_{t+1} equates the household utility in the case of home-started war with the potential outcome in the case it does not start the war as shown in equation (26). In the case it does not start the war, the expected outcome worsens as the probability of the foreign-started war increases. This gives rise to a *temptation* channel as the home-started war is less costly. At the same time, the negative dependence of the cost through DS, $g^H(DS_t)$, captures the idea that countries with a large military capital stock may be more tempted to start wars.

$$
\begin{aligned}
&\beta E_t^{g,\xi}\left(U(c^H_{t+1}, l^H_{t+1}) - \left(P^F(DS_t,\xi_t)U(c^F_{t+1}, l^F_{t+1}) + (1-P^F(DS_t,\xi_t))U(c^N_{t+1}, l^N_{t+1})\right)\right) + \\
&\beta\mu_t b_{t+1} E_t^{g,\xi}\left(u_c(c^H_{t+1})/\pi^H_{t+1} - P^F(DS_t,\xi_t)u_c(c^F_{t+1})/\pi^F_{t+1} - (1-P^F(DS_t,\xi_t))u_c(c^N_{t+1})/\pi^N_{t+1}\right) + \underline{\Lambda}^P_t - \bar{\Lambda}^P_t + \\
&\beta\lambda^\pi_t E_t^{g,\xi}\left(u_c(c^H_{t+1})\Phi_\pi(\pi^H_{t+1})\pi^H_{t+1} - P^F(DS_t,\xi_t)u_c(c^F_{t+1})\Phi_\pi(\pi^F_{t+1})\pi^F_{t+1} - (1-P^F(DS_t,\xi_t))u_c(c^N_{t+1})\Phi_\pi(\pi^N_{t+1})\pi^N_{t+1}\right) = 0
\end{aligned} \tag{26}
$$

The optimality conditions with respect to D_t and DS_t retain the same structure as in the baseline model, except that (i) $P(DS_t, \xi_t)$ now follows Equation (24), and (ii) additional terms arise from nominal rigidities through the NKPC multipliers. As shown in the optimality condition for DS_t in Equation (27), additional terms related to the nominal rigidity show up in both *insurance* and *deterrence* terms. In war periods the planner may be more likely to resort to inflation to relax the implementability constraint through the use of inflation than in normal periods, while incurring real cost through nominal rigidities. By making wars less likely, the planner weights-in this expected reduction in real inflation costs. This shows up as the *deterrence NK* term. The planner also internalizes that higher defense investment will change the slope of the New Keynesian Phillips curve in the war period. This shows up as the *insurance NK* term. At the same time the active use of inflation during wars can help to smooth household wartime utility, making the original *deterrence* term quantitatively less important. All other optimality conditions are reported in Appendix A.4.

$$
\begin{aligned}
\mu^D_t = \;& \underbrace{\beta\mathbb{E}_t\left(\mu_{t+1}\frac{\partial(s_{t+1}u_c(c_{t+1}))}{\partial DS_t} + v'(l_{t+1})\frac{\partial l_{t+1}}{\partial DS_t}\right)}_{\text{Insurance}} + \underbrace{\beta\mathbb{E}_t\left(\lambda^\pi_{t+1}u_c(c_{t+1})\,\nu^{-1}(\nu - 1 + w_{t+1}z^{-1}_{t+1})\frac{\partial Y_{t+1}}{\partial DS_t}\right)}_{\text{Insurance NK}} \\
&\beta(1-P^H_{t+1})\left(\underbrace{\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}\mathbb{E}_t^{g,\xi}\left(U(c^F_{t+1}, l^F_{t+1}) - U(c^N_{t+1}, l^N_{t+1})\right)}_{\text{Deterrence}} + \underbrace{\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}\lambda^\pi_t\mathbb{E}_t^{g,\xi}\left(u_c(c^F_{t+1})\Phi_\pi(\pi^F_{t+1})\pi^F_{t+1} - u_c(c^N_{t+1})\Phi_\pi(\pi^N_{t+1})\pi^N_{t+1}\right)}_{\text{Deterrence NK}}\right) \\
&+ \underbrace{\beta(1-P^H_{t+1})\,b_{t+1}\mu_t\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}\mathbb{E}_t^{g,\xi}\left(\frac{u_c(c^F_{t+1})}{\pi^F_{t+1}} - \frac{u_c(c^N_{t+1})}{\pi^N_{t+1}}\right)}_{\text{Price Manipulation}} + \underbrace{\beta\mathbb{E}_t\left(\mu^D_{t+1}(1-\delta) - \mu^D_{t+1}\mathcal{I}_{t+1}\frac{\partial\mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e)}{\partial DS_t}\right)}_{\text{Future Terms}}
\end{aligned} \tag{27}
$$

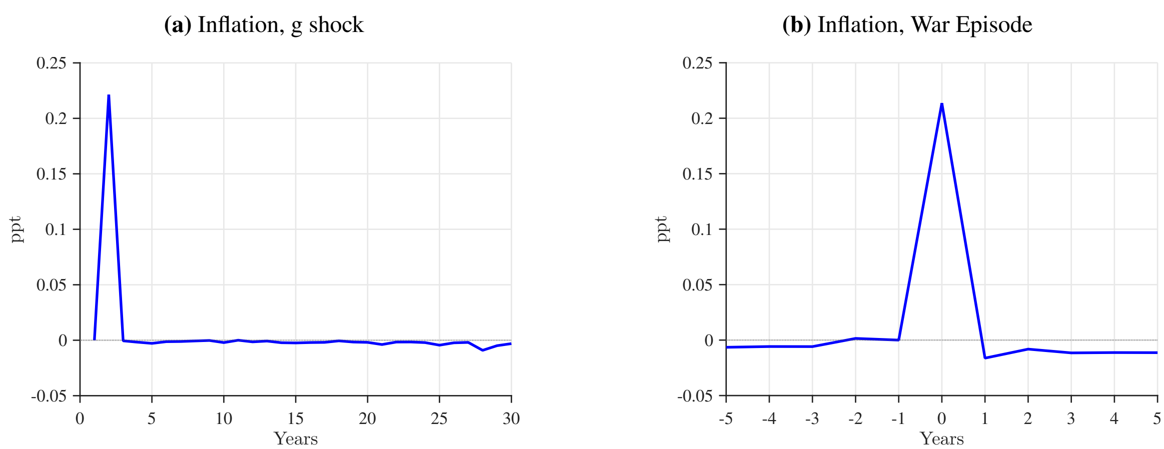
6.3 Results

This section presents the quantitative results by introducing the two extensions sequentially. We begin with the model featuring optimal inflation but without the incentive to attack first. We then incorporate the *temptation* motive into the model with inflation.

6.3.1 Inflation

To quantify the role of endogenous inflation, we set the price elasticity of demand $1/\nu$ to 10, which is a standard value used in the literature. Parameter φ, which governs the strength of the nominal rigidity, is set to 20, implying a slope of the New Keynesian Phillips curve of 0.0413, in line with the range of values in Clarida, Gali, and Gertler (1999). We start by looking at the inflation responses to spending shocks in Figure 12. The left panel shows the generalized impulse response to a peace-time government spending shock as in Figure 8, while the right panel shows the average inflation dynamics during war episodes. Results confirm the findings from the previous literature (Siu, 2004; Schmitt-Grohe and Uribe, 2004; Faraglia, Marcet, Oikonomou, and Scott, 2013) that the resource costs of nominal rigidities tend to dominate the state-contingency benefits of active inflation use. The planner uses inflation to relax the implementability constraint when the spending shocks arise, but its use is quantitatively modest - roughly a 0.2 percentage point increase during wars as well as responding to unexpected spending shocks equaling 2.5% of GDP.
The fact that inflation reacts very similarly to a war shock and to a much smaller but persistent spending shock may seem counterintuitive. The reason for the short-lived response to a spending shock in the left panel is that persistent inflation increase would negatively affect bond prices and the cost of borrowing. Therefore the planner concentrates all the inflation's response in the initial period.

Figure 12. Inflation responses.



Notes: The figure shows inflation's responses to shocks. The left panel shows the generalized impulse response to a one-standard-deviation government spending shock, shown in Figure 8. The right panel shows the mean inflation dynamics during war episodes. The war data come from 5000 simulations of 1000 periods, which gives 396995 war episodes in total.

To what extent does such a limited use of inflation affect the main result regarding borrowing for defense? The role of inflation in smoothing cross-state distortions can be illustrated most clearly through the bond optimality condition in (25). Under quasilinear preferences, in the absence of borrowing constraints and without the option of a preemptive strike, it simplifies to

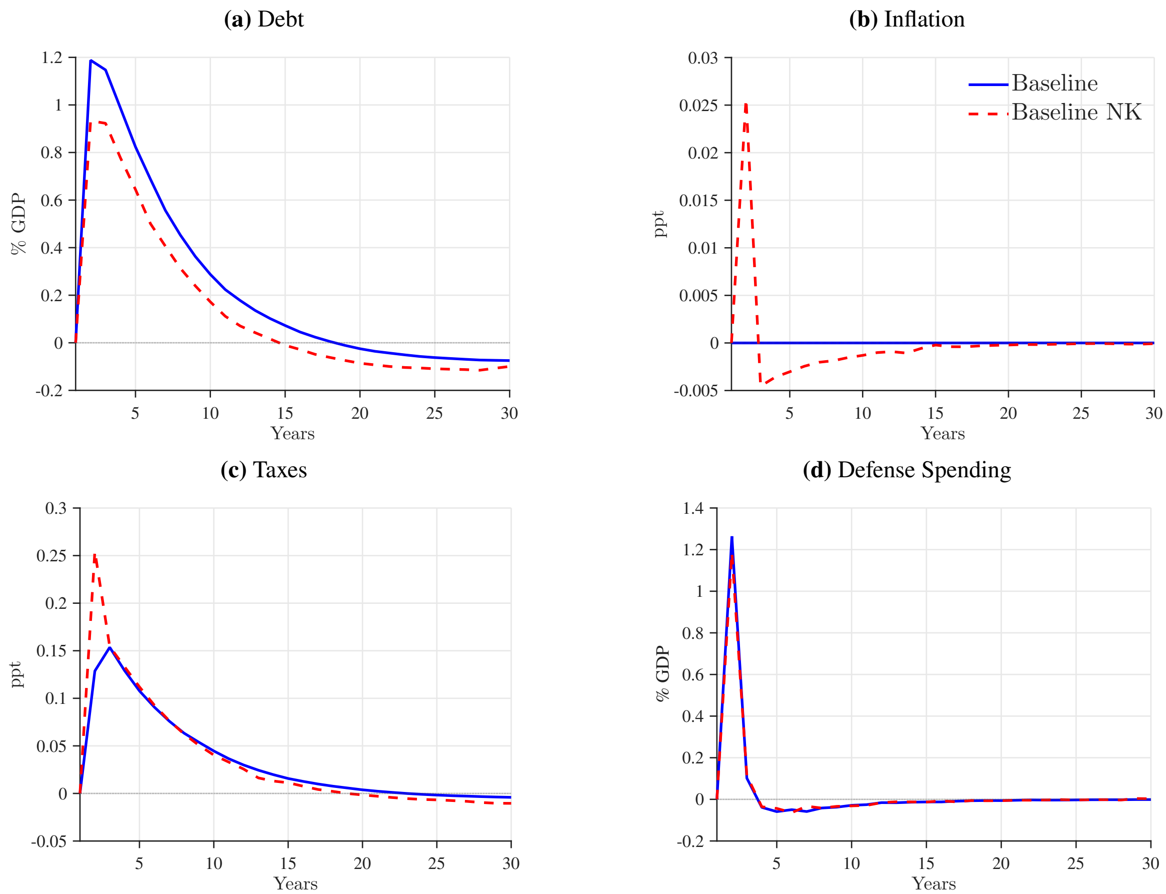
$$\mu_t/\mathbb{E}_t(\pi_{t+1}) = P(DS_t,\xi_t)\mathbb{E}_t^{\xi,g}\left(\mu_{t+1}^F/\pi_{t+1}^F\right) + (1-P(DS_t,\xi_t))\mathbb{E}_t^{\xi,g}\left(\mu_{t+1}^N/\pi_{t+1}^N\right).$$

It shows that a higher wartime inflation, (π^F), allows for greater smoothing of tax distortions across states. In other words, it makes wars less extreme events from the perspective of the planner's implementability constraint. As a consequence, the marginal effect of additional defense investment on the expected value of future distortions,

($\mathbb{E}(\mu_{t+1}/\pi_{t+1})$), decreases as inflation becomes more volatile, potentially shifting the optimal financing of defense toward current taxation and away from debt.

To investigate this mechanism quantitatively, Figure 13 compares the generalized impulse response functions to a geopolitical risk shock in the baseline model (solid blue line) with the extended model featuring the New Keynesian block (dashed red line). The results show that the presence of the New Keynesian block, together with optimal inflation, does indeed shift the optimal financing mix toward current taxes and away from debt, although the effects are modest. In the extended model, the planner responds to geopolitical risk shocks by investing less in defense and relying slightly less on borrowing. By introducing an additional source of state contingency and making wars less extreme from the perspective of the budget constraint, inflation effectively weakens the deterrence mechanism. However, as shown above, the use of inflation remains limited because the costs associated with nominal rigidities quantitatively outweigh the state-contingency benefits of inflation.

Figure 13. Inflation's Responses to the Geopolitical Risk Shock.

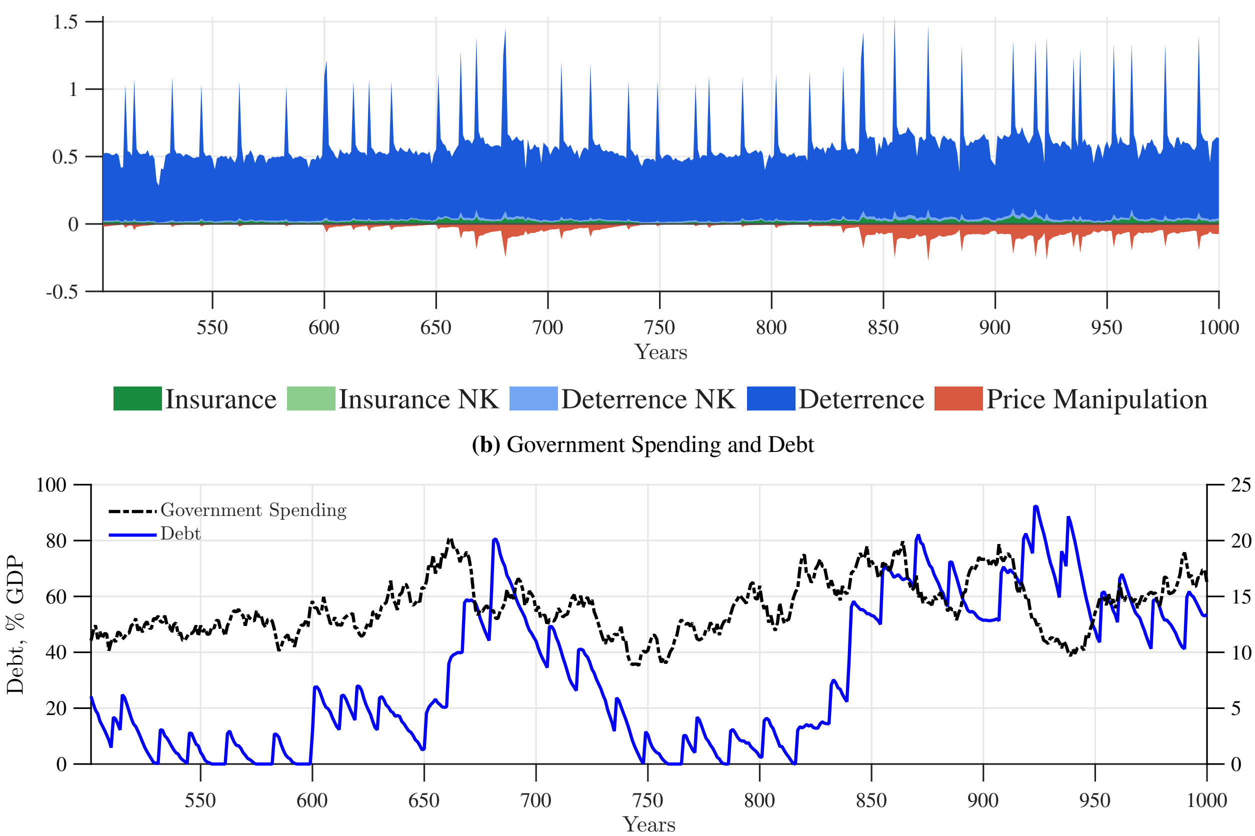


Notes: The figure shows generalized impulse response functions to a one-standard-deviation shock to ξ_t in the baseline model (solid blue) and in the extended model with the New Keynesian frictions (dashed red).

Lastly, to understand what drives defense investment, we decompose the individual terms in the optimality condition (27) for DS. Panel (a) of Figure 14 shows that the *deterrence* term is by far the most important, whereas the new *Deterrence NK* term is quantitatively small because the use of inflation is only modest. The results also show that both *Insurance* terms are relatively small. While this insurance channel allows the planner to meet almost half of the additional wartime spending needs by depleting the defense capital, as we show in appendix Section A.6, the planner nevertheless invests actively in defense during wars. This is because the planner recognizes that excessive depletion of defense capital would further increase the probability of future wars and therefore seeks to avoid it. Consequently, even when the *Insurance* channel is present, the planner spends almost as much during wars; the only difference is in the spending composition. The results also show that the *Price Manipulation* term contributes negatively to defense investment. An additional unit of defense capital lowers bond prices and causes the implementability constraint to bind more tightly when debt is positive. Indeed, panel (c) shows that the price manipulation term becomes increasingly important as debt levels rise. The role of the *Price Manipulation* term does not need to always be negative. In fact, if the planner was accumulating assets, an extra unit of defense investment would relax the implementability constraint as falling prices would increase the return on assets. This term would also be positive in alternative models where a decrease in geopolitical risk increases bond prices. For instance, this would occur if our model was extended to sovereign default (Arellano, 2008; Pouzo and Presno, 2016) as then an extra unit of defense spending would reduce the default risk by making wars less likely.

Figure 14. DS Decomposition.

(a) Decomposition of DS channels



(b) Government Spending and Debt

Notes: Panel (a) reports a term-by-term decomposition of the first-order condition for defense investment in equation (27). The green areas capture the insurance channel, the blue areas capture the deterrence channel, and the red area captures the incentive to manipulate bond prices. The lighter shades denote the components arising from the New Keynesian block. Components above zero raise the marginal value of defense investment, while components below zero lower it. Panel (b) shows the simulated paths of government debt, measured on the left axis, and peacetime government spending, measured on the right axis, over the same periods. The figure uses the stochastic simulation employed in the numerical solution of the model.

6.3.2 Temptation

In this last exercise, we are interested in understanding how results change if DS is not only used for defense, but could tempt the planner to attack first. As the main model's mechanism relies on the idea that by reducing the future war probability, higher D investment justifies more borrowing, it is interesting to see how results change when this mechanism is not built-in exogenously into the model. To accommodate this idea, we give the home planner the option to strike first where the costs of such a strike fall in accumulated defense capital.

We follow Federle, Meier, Müller, Mutschler, and Schularick (2026), who estimate that war damage in war-site economies is approximately 67% greater than in other belligerent countries where military operations do not take place on. Given this, we set the extra spending needs and the TFP drop in a war started by the home country to be 60% of the foreign-started war counterparts. We also assume that the home-started war spending decreases in DS, capturing the idea that wars should be less costly for more capable militaries.[11]

Implications of preemptive strike on tax smoothing can be best understood through the bond optimality condition, which under quasilinear preferences reads as

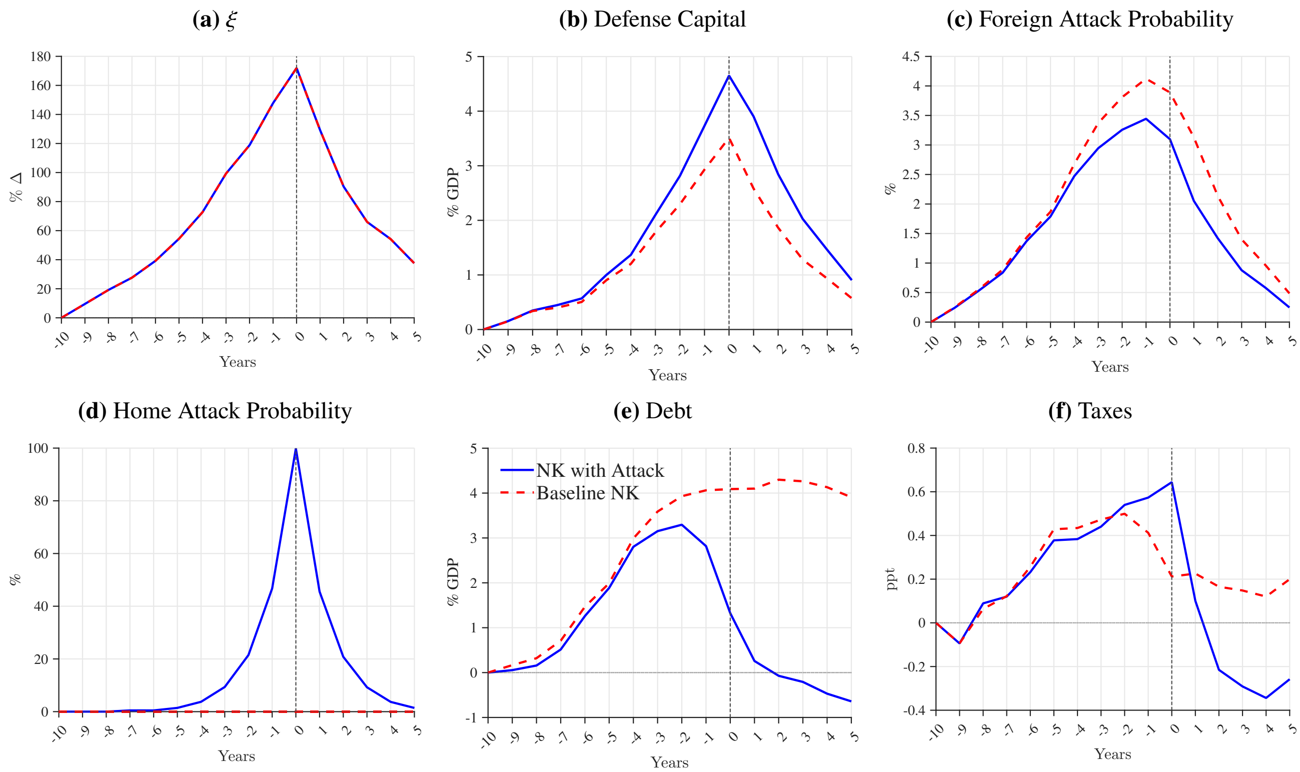
$$\mu_t/\mathbb{E}_t(\pi_{t+1}) = P^H_{t+1}\mathbb{E}^{\xi,g}_t\left(\mu^H_{t+1}/\pi^H_{t+1}\right) + (1-P^H_{t+1})\left(P(DS_t,\xi_t)\mathbb{E}^{\xi,g}_t\left(\mu^F_{t+1}/\pi^F_{t+1}\right) + (1-P(DS_t,\xi_t))\mathbb{E}^{\xi,g}_t\left(\mu^N_{t+1}/\pi^N_{t+1}\right)\right).$$

It suggests that the planner has the incentive to conduct a preemptive strike when the excess burden of taxation in that state, μ^H, becomes preferred to the wait-and-see option. Hence the planner only attacks when either the probability of foreign attack is high and/or the expected tax distortions in the case of preemptive strike are significantly lower than in the case of foreign attack. We find that preemptive strikes are rare events occurring with only 0.3% probability and as we show in Appendix A.6, model dynamics outside such episodes closely resemble the model without such attack option.

We instead focus on highlighting model differences around the preemptive strike episodes. Figure 15 shows the mean dynamics around preemptive strike episodes in the NK model with attack option and in the Baseline NK model shown in Section 6.3.1. As panel (a) shows, an increase in geopolitical risk needs to be large and abrupt to make the preemptive strike worthwhile as such attack episodes typically follow a 160% rise in ξ_t process. Panel (b) shows that the strike option provides the incentive to build up disproportionately more defense capital as this makes the preemptive strike cheaper. Such buildups result in endogenously lower foreign attack probability as shown in panel (c). That is, the option of preemptive strike provides the incentive to aggressive military build-up, which at the same time further deters foreign attacks. Panel (e) shows the debt path. An increase in geopolitical risk is related to rising debt levels in both models as debt is used to finance defense capital. At the same time, both debt and taxes remain elevated in the aftermath of the high geopolitical risk episodes in the Baseline NK model, in sharp contrast to the NK Attack model. The reason is twofold. As the average attack probability begins to increase early meaning that in some samples preemptive strikes occur already before the peak episode. The planner does not borrow in such events, hence the average debt in the NK with Attack model begins to go down already before the peak episode. Similarly, without the preemptive strike option the planner endogenously accumulates less military capital and faces higher foreign attack probability. Hence, on average, it faces more foreign attacks and accumulates more wartime debt and maintains higher taxes for longer. Nevertheless, as we show in Appendix A.6, outside of such episodes, optimal policies are very similar in both models.

[11] We assume that $g^H(DS)$ is a function of DS relative to the long-run average defense capital EDS, where $EDS = \mathbb{E}(DS)$, so that home-started war is less costly when the defense capital is higher relative to the average. More specifically, $g^H(DS) = \mathcal{S}(0.6g^F \cdot \frac{DS^{-\epsilon}}{E(DS)^{-\epsilon}}, g^F)$, where $\mathcal{S}(\cdot)$ is a smooth minimum function of the form $\mathcal{S}(x_1,x_2) = \frac{1}{\alpha}\ln(e^{\alpha x_1} + e^{\alpha x_2})$. We set $\epsilon = -5$.

Figure 15. Mean Dynamics around Attack Episodes.



Notes: The figure reports event-study averages around episodes in which the home planner's attack probability exceeds 90 percent. The dashed red line corresponds to the baseline New Keynesian model described in Section 6.3.1, while the solid blue line corresponds to the same model augmented with the option of a preemptive strike. Both economies are simulated using identical shock sequences. We first identify preemptive-strike episodes in the model with the attack option and then extract the corresponding dates from the baseline model. Each panel reports average dynamics relative to the mean ten periods preceding the attack. The statistics are computed from 500 simulations of 1,000 periods each, yielding 520 preemptive-strike episodes. Because the figure reports conditional event averages, the timing of policy adjustments differs across episodes; consequently, variables such as debt may begin to adjust before the average attack date.

7 Conclusion

This paper studies Ramsey policy when the planner manages an endogenous disaster risk, represented by the risk of war. The planner optimally invests in defense capital in response to time-varying geopolitical risk, besides using standard policy tools, such as distortionary taxes and debt. Defense capital serves a dual purpose. It drives down the disaster risk (*deterrence*), and higher existing defense capital also helps to meet the additional spending needs in the war state (*insurance*). We show that these aspects greatly alter optimal policies for taxes and debt. As additional defense investment drives down the war probability, it reduces the likelihood of having to raise taxes in the future war states and hence makes the future better from the fiscal point of view. As a consequence, we prove analytically and show quantitatively that it is optimal to finance defense spending by borrowing and giving up tax smoothing across states to favor tax smoothing over time. Our model with endogenous disasters features higher average debt levels and suggests active borrowing responding to uncertainty shocks. More broadly, we view this paper as also relevant and applicable to questions such as climate change or public equity investment.

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A Mathematical Appendix

The Mathematical Appendix is organized as follows. First, we report all derivations and proofs. Second, we report the full derivation of the optimal policy under full commitment and details about the solution algorithm.

A.1 Derivations and Proofs

This subsection contains the derivation of the Optimal Tax Rate and the five propositions reported in the main body of the paper.

A.1.1 Optimal Tax Rate

Start by laying out the optimality conditions for consumption and leisure:

$$\begin{aligned} & u_{c,t} + \mu_t \Omega_{c,t} - b_t u_{cc,t}(\mu_t - \mu_{t-1}) = \lambda_t, \\ & -v_{l_t} + \mu_t \Omega_{h,t} = -\lambda_t z_t. \end{aligned}$$

Divide through to eliminate the multiplier λ_t to get an expression for z_t that reads

$$z_t = \frac{v_{l,t}(1 - \mu_t \Omega_{h,t}/v_{l,t})}{u_{c,t}(1 + \mu_t \Omega_{c,t}/u_{c,t} - u_{cc,t} b_t(\mu_t - \mu_{t-1})/u_{c,t})}.$$

Note that $\tau_t = 1 - \frac{v_{l,t}}{z_t u_{c,t}}$. Also, define $\epsilon_{cc} \equiv -\frac{u_{cc} c}{u_c}$, $\epsilon_{hh} \equiv -\frac{v_{ll} h}{v_l}$, and $\epsilon_{ch} \equiv \frac{v_{cl} h}{u_c}$. This allows us to express primary surpluses in terms of elasticities:

$$\frac{\Omega_{c,t}}{u_{c,t}} = \frac{u_{cc,t}}{u_{c,t}} + u_{c,t}/u_{c,t} - \frac{v_{lc,t}}{u_{c,t}} = 1 - \epsilon_{cc} - \epsilon_{ch},$$

$$\frac{\Omega_h}{v_l} = -\frac{u_{cl} c}{v_l} - \frac{v_l}{v_l} + \frac{v_{ll} h}{v_l} = -\epsilon_{hc} - 1 - \epsilon_{hh}.$$

Substitute in taxes and elasticities to get the following expression for z_t

$$z_t = (1 - \tau_t) z_t \frac{1 + \mu_t(1 + \epsilon_{hh})}{1 + \mu_t(1 - \epsilon_{cc}) + \epsilon_{cc} b_t / c_t(\mu_t - \mu_{t-1})},$$

and rearrange the equation above for τ_t to finally get

$$\tau = \frac{\mu_t(\epsilon_{hh} + \epsilon_{cc}) - \epsilon_{cc} b_t / c_t(\mu_t - \mu_{t-1})}{1 + \mu_t(1 + \epsilon_{hh})}.$$

A.1.2 Proof of Proposition 2

Proof.
Start by substituting out labor supply in terms of c, D, and g using the three aggregate resource constraints:

$$\begin{aligned} & c_0 + g_0 + D_1 = h_0 \rightarrow h_0 = h(c_0, g_0, D_1), \\ & c^N + g^N = h^N \rightarrow h^N = h(c^N, g^N), \\ & c^W + g^W - \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)) = h^W \rightarrow h^W = h(c^W, g^W, g^N, D_1). \end{aligned}$$

Hence, substitute out labor supply from the household's intra-temporal optimality condition $\tau_t = 1 - \frac{v_l(1-h_t)}{c_t}$

and express consumption as a function of D, τ, and g:

$$\tau_0 = 1 - \frac{v_l(1-h(c_0, g_0, D_0))}{u_c(c_0)} \to c_0 = c_0(\tau_0, g_0, D_1),$$
$$\tau^N = 1 - \frac{v_l(1-h(c^N, g^N))}{u_c(c^N)} \to c^N = c^N(\tau^N, g^N),$$
$$\tau^W = 1 - \frac{v_l(1-h(c^W, g^W, D_1))}{u_c(c^W)} \to c^W = c^W(\tau^W, g^W, g^N, D_1).$$

The bond's price $Q_0 = P(D_1)\frac{u_c(c^W)}{u_c(c_0)} + (1-P(D_1))\frac{u_c(c^N)}{u_c(c_0)}$ is then a function of $(g_0, g^N, g^W, D_1, \tau_0, \tau^N, \tau^W)$. These substitutions also allow to express the government's revenue in marginal utility terms $\Omega \equiv h\tau u_c(c)$ as a function of $(g_0, g^N, g^W, D_1, \tau_0, \tau^N, \tau^W)$. Finally, the planner's implementability constraints define a system that relates $(\tau_0, \tau^N, \tau^W, b_1)$ to $(D_1, g_0, g^N, g^W, b_0)$ as follows:

$$(g_0 + b_0)u_c(F(\tau_0, g_0, D_1)) = \Omega_0 + b_1 P(D_1)u_c(c^W) + (1-P(D_1))u_c(c^N), \tag{28}$$
$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N, \tag{29}$$
$$(g^W + b_1 - D_1(1-\delta))u_c(F(\tau^W, g^W, D_1)) = \Omega^W. \tag{30}$$

This can be simplified further by substituting out b_1 using the period 0's implementability constraint $b_1 = \frac{(g_0+b_0)u_c(F(\tau_0,g_0,D_1))-\Omega_0}{Q_0}$, which yields $b_1 = b_1(g_0, b_0, \tau_0, D_1, g^N, \tau^N, g^W, \tau^W)$. This gives the following system of two equations in two endogenous variables τ^N and τ^W:

$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N,$$
$$(g^W + b_1 - \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)))u_c(F(\tau^W, g^W, D_1)) = \Omega^W.$$

Implicitly defining $f(\tau^W, \tau^N, D_1, g_0)$ and assuming that τ_0 is constant, one can use the implicit function theorem to calculate $\left(\frac{\partial \tau^W}{\partial D_1}, \frac{\partial \tau^N}{\partial D_1}\right)$ and $\left(\frac{\partial \tau^W}{\partial g_0}, \frac{\partial \tau^N}{\partial g_0}\right)$. These objects then allow to get $\frac{\partial Q_0}{\partial D_1}$ and $\frac{\partial Q_0}{\partial g_0}$. It remains to be shown that $\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)} < 0$ and $\beta P(D_1)u_{cc}(c^W)\frac{\partial c^W}{\partial D_1} < 0$.

For what regards $\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)}$, $P'(D_1) < 0$ by assumption and $c^W \leq c^N$ as long as $g^W \geq g^N$. For what regards $\beta P(D_1)u_{cc}(c^W)\frac{\partial c^W}{\partial D_1}$, $u_{cc}(c^W) < 0$, we need to show the sign of $\frac{\partial c^W}{\partial D_1}$ using the household's intra-temporal condition:

$$u_c(c^W)(\tau^W - 1) + v_l(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N))) = 0.$$

Finally, the implicit function theorem yields:

$$\frac{\partial c^W}{\partial D_1} = -\frac{(1-\delta)v_{ll}(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1\phi, \phi(g^W - g^N)))\frac{\partial \mathcal{S}}{\partial D_1}}{u_{cc}(c^W)(\tau^W - 1) - v_{ll}(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)))} < 0.$$

■

A.1.3 Proof of Proposition 3

Proof. Using the optimal tax formula and assuming quasilinear preferences, so that $\epsilon_{cc} = 0$ one can express the multiplier in terms of tax rates:

$$\mu^W = \frac{\tau^W}{\epsilon_{hh} - \tau^W(1+\epsilon_{hh})}. \tag{31}$$

Hence, the model can be characterized by the following system of equations:

$$\frac{\tau_0}{\epsilon_{hh} - \tau_0(1+\epsilon_{hh})} = P(D_0)\frac{\tau^W}{\epsilon_{hh} - \tau^W(1+\epsilon_{hh})} + (1-P(D_0))\frac{\tau^N}{\epsilon_{hh} - \tau^N(1+\epsilon_{hh})},$$
$$g^W + b_1 = h(\tau^W)\tau^W,$$
$$g^N + b_1 = h(\tau^N)\tau^N,$$
$$g_0 + b_0 + D_1 = h(\tau_0)\tau_0 + \beta b_1,$$

where we have used the household's optimality condition $\tau_t = 1 - v_l(1-h_t)$ to express h as functions of the tax rate.

To simplify the system further, first express τ in terms of the multiplier: $\tau^x = \frac{\mu^x \epsilon_{hh}}{1+\mu^x(1+\epsilon_{hh})}$. Substitute out μ_0 using $\mu_0 = P(D_1)\mu^W + (1-P(D_1))\mu^N$. Also substitute b_1 using $b_1 = 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0)$, where it is understood that h^x is a function of τ^x, and τ^x is a function of μ^x. μ_0 is a function of μ^W, μ^N and D_1. Then the model can be summarized by the following system:

$$h^W\tau^W - g^W - 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0) = 0,$$
$$h^N\tau^N - g^N - 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0) = 0.$$

This system can be thought as an implicit function $f(\mu^W, \mu^N, D_1) = 0$. Apply the implicit function theorem to compute:

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -f_\mu^{-1} f_D, \tag{32}$$

where, after defining $H^x \equiv \frac{\partial h^x \tau^x}{\partial \mu^x}$ for ease of notation,

$$f_\mu = \begin{pmatrix} H^W + 1/\beta H_0 P(D_1) & 1/\beta H_0(1-P(D_1)) \\ 1/\beta H_0 P(D_1) & H^N + 1/\beta H_0(1-P(D_1)) \end{pmatrix},$$

and

$$f_D = \begin{pmatrix} -1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) \\ -1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) \end{pmatrix}.$$

In order to compute f_u^{-1} in equation (32), we need to compute the determinant $det(f_\mu)$ and the adjugate $adj(f_\mu)$. Respectively, these are:

$$\begin{aligned} det(f_\mu) &= [H^W + 1/\beta H_0 P(D_1)][H^N + 1/\beta H_0(1-P(D_1))] - 1/\beta H_0 P(D_1) 1/\beta H_0(1-P(D_1)) = \\ &= H^W H^N + 1/\beta H^W H_0(1-P(D_1)) + 1/\beta H^N H_0 P(D_1), \end{aligned}$$

and

$$adj(f_\mu) = \begin{pmatrix} H^N + 1/\beta H_0(1-P(D_1)) & -H_0 1/\beta(1-P(D_1)) \\ -H_0 1/\beta P(D_1) & H^W + H_0 1/\beta P(D_1) \end{pmatrix}.$$

Finally, multiply $f_\mu^{-1} = adj(f_\mu)/det(f_\mu)$ by f_D to rewrite (32) as:

$$\frac{\partial \mu}{\partial D_1} = -\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)} \begin{pmatrix} H^N + 1/\beta H_0(1 - P(D_1)) - 1/\beta H_0(1 - P(D_1)) \\ -H_0 1/\beta P(D_1) + H^W + H_0 1/\beta P(D_1) \end{pmatrix},$$

which further simplifies to

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = \underbrace{-\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)}}_{\equiv \mathcal{Z}} \begin{pmatrix} H^N \\ H^W \end{pmatrix}, \tag{33}$$

and where H^x can be written as:

$$H^x = \frac{\partial h^x \tau^x}{\mu^x} = \frac{\partial h^x \tau^x}{\partial \tau^x} \frac{\partial \tau^x}{\partial \mu^x} = \underbrace{\frac{\partial h^x \tau^x}{\partial \tau^x}}_{\text{Laffer slope}} \frac{\epsilon_{hh}}{1 + \mu^x(1 + \epsilon_{hh})}.$$

Assuming the economy is on the left-hand side of the Laffer curve, then $H^W > 0$, $H^N > 0$, and $H_0 > 0$. Hence, $det(f_\mu) > 0$. On the left-hand side of the Laffer curve $\tau^W \geq \tau^N$. According to equation (31), this implies that $\mu^W > \mu^N$, hence $-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) < 0$ and $\frac{\epsilon_{hh}}{1+\mu^W(1+\epsilon_{hh})} \leq \frac{\epsilon_{hh}}{1+\mu^N(1+\epsilon_{hh})}$. The fact that $det(f_u) > 0$ and that $-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) < 0$ imply that $\mathcal{Z} > 0$. Hence, in order to establish whether $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$, we need to investigate whether or not $H^N > H^W$. For this purpose, we are left with the task to study the terms $\frac{\partial h^N \tau^N}{\partial \tau^N}$ and $\frac{\partial h^W \tau^W}{\partial \tau^W}$. Given that we assumed that the economy is on the left-hand side of the Laffer curve and that the Laffer curve is single peaked and, given differentiability, this also means that the Laffer curve is concave in τ. When the Laffer curve is concave in τ, we have $\frac{\partial h^N \tau^N}{\partial \tau^N} > \frac{\partial h^W \tau^W}{\partial \tau^W}$. Hence, $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$. ■

A.1.4 Proof of Proposition 4

Proof. Using the period 0's budget constraint, the response of debt to g_0 and D_1 is given by

$$\frac{\partial b_1}{\partial g_0} = 1/\beta\left(1 - \frac{\partial h_0 \tau_0}{\partial g_0}\right),$$
$$\frac{\partial b_1}{\partial D_1} = 1/\beta\left(1 - \frac{\partial h_0 \tau_0}{\partial D_1}\right).$$

Debt is more responsive to D_1 when $\frac{\partial h_0 \tau_0}{\partial D_1} < \frac{\partial h_0 \tau_0}{\partial g_0}$. Following the notation from the proof above

$$\frac{\partial h_0 \tau_0}{\partial g_0} = \frac{\partial h_0 \tau_0}{\partial \mu_0} \frac{\partial \mu_0}{\partial g_0} = H^0 \frac{\partial \mu_0}{\partial g_0},$$
$$\frac{\partial h_0 \tau_0}{\partial D_1} = \frac{\partial h_0 \tau_0}{\partial \mu_0} \frac{\partial \mu_0}{\partial D_1} = H^0 \frac{\partial \mu_0}{\partial D_1}.$$

One needs to compare $\frac{\partial \mu_0}{\partial g_0}$ and $\frac{\partial \mu_0}{\partial D_1}$

$$\frac{\partial \mu_0}{\partial g_0} = P(D_1)\frac{\partial \mu^W}{\partial g_0} + (1 - P(D_1))\frac{\partial \mu^N}{\partial g_0},$$
$$\frac{\partial \mu_0}{\partial D_1} = P(D_1)\frac{\partial \mu^W}{\partial D_1} + (1 - P(D_1))\frac{\partial \mu^N}{\partial D_1} + P'(D_1)(\mu^W - \mu^N),$$

where, through the bond's optimality condition, μ_0 is a function of D_1, μ^W, and μ^N. Use the implicit function theorem to get the effect on μ^W and μ^N. In the proof above we have shown that

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)} \begin{pmatrix} H^N \\ H^W \end{pmatrix},$$

which was equation (33). Similarly, the marginal effect of g_0 is

$$\frac{\partial \boldsymbol{\mu}}{\partial g_0} = -\frac{-1/\beta}{det(f_\mu)} \begin{pmatrix} H^N \\ H^W \end{pmatrix}. \tag{34}$$

Assuming the economy is on the left-hand side of the Laffer curve, $\frac{\partial \mu^x}{\partial D_1} > \frac{\partial \mu^x}{\partial g_0}$ for $x \in \{N, W\}$. The debt choice b_1 responds more to D_1 than to g_0, iff $\frac{\partial \mu_0}{\partial D_1} < \frac{\partial \mu_0}{\partial g_0}$. Equivalently, using the bond optimality condition (16), $\frac{\partial \mathbb{E}_0(\mu_1)}{\partial D_1} < \frac{\partial \mathbb{E}_0(\mu_1)}{\partial g_0}$

Expanding and rearranging terms gives

$$P(D_1)\left(\frac{\partial \mu^W}{\partial D_1} - \frac{\partial \mu^W}{\partial g_0}\right) + (1 - P(D_1))\left(\frac{\partial \mu^N}{\partial D_1} - \frac{\partial \mu^N}{\partial g_0}\right) < P'(D_1)(\mu^N - \mu^W). \tag{35}$$

Using (33) and (34) it is easy to show that

$$\begin{aligned} \frac{\partial \mu^W}{\partial D_1} - \frac{\partial \mu^W}{\partial g_0} &= \frac{1}{det(f_\mu)} \frac{1}{\beta} H^N H_0 P'(D_1)(\mu^N - \mu^W), \\ \frac{\partial \mu^N}{\partial D_1} - \frac{\partial \mu^N}{\partial g_0} &= \frac{1}{det(f_\mu)} \frac{1}{\beta} H^W H_0 P'(D_1)(\mu^N - \mu^W). \end{aligned}$$

Using these expressions, the left-hand side of (35) is

$$\begin{aligned} &P(D_1)\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0 P'(D_1)(\mu^N - \mu^W) + (1 - P(D_1))\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0 P'(D_1)(\mu^N - \mu^W) = \\ &\underbrace{\left(P(D_1)\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0 + (1 - P(D_1))\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0\right)}_{\mathcal{K}} P'(D_1)(\mu^N - \mu^W). \end{aligned}$$

Given this, $\frac{\partial \mu_0}{\partial D_1} < \frac{\partial \mu_0}{\partial g_0}$ is equivalent to

$$\underbrace{\left(P(D_1)\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0 + (1 - P(D_1))\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0\right)}_{\mathcal{K}} P'(D_1)(\mu^N - \mu^W) < \frac{\partial P^W(D_1)}{\partial D_1}(\mu^N - \mu^W)$$

It remains to show that the $\mathcal{K}$ is less than one:

$$\mathcal{K} = \frac{1/\beta H_0 H^N P(D_1) + (1 - P(D_1))H_0 H^W 1/\beta}{1/\beta H_0 H^N P(D_1) + (1 - P(D_1))H_0 H^W 1/\beta + H^W H^N} < 1,$$

since both the numerator and the denominator are positive and $H^W H^N > 0$. Hence, $\frac{\partial b_1}{\partial D_1} > \frac{\partial b_1}{\partial g_0}$. ■

A.1.5 Proof of Proposition 5

Proof.

As shown in A.1.2, one can express hours and consumption as functions of policy variables, such as

$$c_0 = c_0(\tau_0, D_1, g_0); \quad c^N = c^N(\tau^N, g^N); \quad c^W = c^W(\tau^W, D_1, g^W, g^N),$$

and

$$h_0 = h_0(\tau_0, D_1, g_0); \quad h^N = h^N(\tau^N, g^N); \quad h^W = h^W(\tau^W, D_1, g^W, g^N).$$

Using the notation from A.1.2, the Ramsey problem reads as:

$$\max_{b_1, D_1, \tau_0, \tau^N, \tau^W} U(c_0, h_0) + \beta(P(D_1)U(c^W, h^W) + (1 - P(D_1))U(c^N, h^N))$$

s.t.

$$(g_0 + b_0)u_c(F(\tau_0, g_0, D_1)) = \Omega_0 + b_1 P(D_1)u_c(c^W) + (1 - P(D_1))u_c(c^N),$$
$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N,$$
$$(g^W + b_1 - D_1(1 - \delta))u_c(F(\tau^W, g^W, D_1)) = \Omega^W.$$

The optimality condition for D_1 would read as:

$$\frac{\partial U}{\partial c_0}\frac{\partial c_0}{\partial D_1} + \frac{\partial U}{\partial h_0}\frac{\partial h_0}{\partial D_1} + \mu_0\left(\frac{\partial \Omega_0}{\partial D_1} - (g_0 + b_0)u_{cc}\frac{\partial c_0}{\partial D_1}\right) + \beta\frac{\partial P}{\partial D_1}\left(U(c^W, h^W) - U(c^N, h^N)\right) + \beta\mu^W\left(\frac{\partial \Omega^W}{\partial D_1}\right).$$

One can solve the model for an arbitrary sequence of b_1 by choosing $D_1, \tau_0, \tau^N, \tau^W$ as well as μ_0, μ^N, μ^W. This would give a system of four optimality conditions and three implementability constraints and seven variables, namely $D_1, \tau_0, \tau^N, \tau^W, \mu_0, \mu^N, \mu^W$, for any given value of b_1. This would define an implicit function between $D_1, \tau_0, \tau^N, \tau^W, \mu_0, \mu^N, \mu^W$ and b_1. As a last step, one could choose b_1 and let other variables adjust optimally. More formally, the Planner's objective at this last step would be written as

$$\max_{b_1} \quad U_0(\tau_0(b_1), D_1(b_1)) + \beta(P(D_1)U^W(\tau^W(b_1), D_1(b_1)) + (1 - P(D_1))U^N(\tau^N(b_1))).$$

At the optimum the following must be true:

$$\frac{\partial U_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial b_1} + \beta P(D_1)\frac{\partial U^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial b_1} + \beta(1 - P(D_1))\frac{\partial U^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial b_1} + \left(\frac{\partial U_0}{\partial D_1} + \underbrace{\beta\frac{\partial P}{\partial D_1}(U^W - U^N)}_{\text{Deterrence}} + \underbrace{\beta\frac{\partial U^W}{\partial D_1}}_{\text{Insurance}}\right)\frac{\partial D_1}{\partial b_1} = 0,$$

where derivatives $\frac{\tau_0}{\partial b_1}, \frac{\tau^W}{\partial b_1}, \frac{\tau^N}{\partial b_1}, \frac{D_1}{\partial b_1}$ capture the optimal responses consistent with the Ramsey plan.

The model with endogenous disasters will have higher b_1 than the standard model iff

$$\left(\frac{\partial U_0}{\partial D_1} + \beta\frac{\partial P}{\partial D_1}(U^W - U^N) + \beta\frac{\partial U^W}{\partial D_1}\right)\frac{\partial D_1}{\partial b_1} > 0.$$

This can be shown by showing that both terms are positive.

1. To show that $\frac{\partial D_1}{\partial b_1} > 0$, use Topkis (1978). Define $x = \{-\tau_0, \tau^W, \tau^N, D_1\}$. Denote $x \wedge x' = \{min(x_1, x_1'), ..., min(x_n, x_n')\}$ and $x \vee x' = \{max(x_1, x_1'), ..., max(x_n, x_n')\}$. And say that $x' \geq x$ if $x_i' \geq x_i \forall i$.

   Define planners objective function as:

$$L = U(c_0, h_0) + \beta P(D_1)U(c^W, h^W) + \beta(1 - P(D_1))U(c^N, h^N) + \mu_0(h_0\tau_0 + \beta b_1 - D_1 - b_0 - g_0) +$$
$$\beta P(D_1)\mu^W(h_W\tau^W - b_1 - g^W) + \beta(1 - P(D_1))\mu^N(h^N\tau^N - b_1 - g^N),$$

   where it is understood that c and h are functions of taxes and D_1.

   If $L(x, b_1)$ is supermodular in x and has increasing differences in x and b_1, then x is nondecreasing in b_1. Or if $L(x, b_1)$ is differentiable, $\frac{\partial^2 L(x,b_1)}{\partial x \partial b_1} \geq 0$ (Topkis, 1978).

   One can show that $\frac{\partial D_1}{\partial b_1} > 0$ by showing that the planner's lagrangian satisfies increasing differences in a sense that $L(x', b_1') - L(x, b_1') > L(x', b_1) - L(x, b_1)$ (Topkis, 1978), where $b_1' > b_1$ and $x' > x$, and $x \in \{\tau_0, \tau^W, \tau^N, D_1\}$. $x' > x$ then means that every element of x' is at least as high as the respective element of x.

   $L(x, b_1)$ exhibits increasing differences in x and b_1 iff $L(x', b_1') - L(x, b_1') - (L(x', b_1) - L(x, b_1)) > 0$. Calculating it gives

$$\beta(\mu^W - \mu^N)(b_1' - b_1)(P(D_1) - P(D_1')) > 0.$$

   Hence, $L(x, b_1)$ has increasing differences and, therefore, $\frac{\partial D_1}{\partial b_1} > 0$.

   $L(x, b_1)$ is supermodular in x iff $L(x \vee x', b_1) + L(x \wedge x', b_1) \geq L(x', b_1) + L(x, b_1) \geq$. When $x' > x$, this holds with equality and $L(x, b_1)$ is supermodular in x.

2. To show that $\frac{\partial U_0}{\partial D_1} + \beta \frac{\partial P}{\partial D_1}(U^W - U^N) > 0$, take the first order condition of L with respect to D_1 to get:

$$\frac{\partial U_0}{\partial D_1} + \beta \frac{\partial P}{\partial D_1}(U^W - U^N) + \mu_0 \left(\tau_0 \frac{\partial h_0}{\partial D_0} - 1\right) + \lambda^D = 0.$$

   Assuming $\lambda^D = 0$, $\tau_0 \frac{\partial h_0}{\partial D_0} = \frac{\partial c_0}{\partial D_1} - 1$, therefore $\tau_0 \frac{\partial h_0}{\partial D_0} - 1 < 0$ and $\frac{\partial U_0}{\partial D_1} + \beta \frac{\partial P}{\partial D_1}(U^W - U^N) > 0$.

■

A.2 Optimal Policy under Full Commitment

We consider a full commitment approach to optimal debt and disaster management with incomplete bond markets.

Incomplete Markets

In this subsection, we solve for the time-inconsistent Ramsey plan under incomplete debt markets. The Ramsey planner seeks to maximize the household's utility (1) subject to a series of implementability constraints

$$b_t = \mathbb{E}_0 \left[\sum_{j=t}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right],$$

with multiplier μ and the law of motion for the defense stock

$$DS_t = DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e), \tag{36}$$

with multiplier μ_t^D. The Ramsey planner also needs to take into account that defense stock affects the disaster probability, i.e. $P(DS, \xi)$, and needs to take into account the $D_t > 0$ constraint, to which we assign multiplier λ_t^D. Additionally, the planner needs to respect the aggregate resource constraint

$$c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = z_t h_t = z_t(1 - l_t).$$

From the resource constraint we obtain an expression for l_t and substitute it out. Hence, the recursive Lagrangian of the planner reads:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t) + v(l_t) + \mu_t(\Omega_t + \beta \mathbb{E}_t u_c(c_{t+1}) b_{t+1} - u_c(c_t) b_t) \\ & + \mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t) - \overline{\Lambda}_t b_{t+1} + \underline{\Lambda}_t b_{t+1} + \lambda_t^D D_t \Big\},\end{aligned}$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t) c_t - v_l(l_t) h_t$.

The first-order condition with respect to c_t is:

$$0 = u_c(c_t) + v_l(l_t) \frac{\partial l_t}{\partial c_t} + \mu_t \left(\frac{\partial (s_t u_c(c_t))}{\partial c_t} \right) - u_{cc}(c_t) b_t (\mu_t - \mu_{t-1}). \tag{37}$$

The first-order condition with respect to b_{t+1} is:

$$\mu_t = \frac{\mathbb{E}_t(u_c(c_{t+1})\mu_{t+1})}{\mathbb{E}_t(u_c(c_{t+1}))} + \frac{1}{\beta \mathbb{E}_t\left[u_c(c_{t+1})\right]} \left(\overline{\Lambda}_t - \underline{\Lambda}_t \right). \tag{38}$$

The first-order condition with respect to D_t is:

$$0 = \lambda_t^D + v_l(l_t) \frac{\partial l_t}{\partial D_t} + \mu_t \left(\frac{\partial s_t u_c(c_t)}{\partial D_t} \right) + \mu_t^D \left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t} \right). \tag{39}$$

The first-order condition with respect to DS_t is:

$$\mu_t^D = \beta\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}\mathbb{E}_t^{g,\xi}\left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N)\right) + \beta\mathbb{E}_t\left(\mu_{t+1}\frac{\partial s_{t+1}u_c(c_{t+1})}{\partial DS_t} + v'(l_{t+1})\frac{\partial l_{t+1}}{\partial DS_t}\right)$$
$$+ \beta\mu_t b_{t+1}\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}\mathbb{E}_t^{g,\xi}\left(u_c(c_{t+1}^W) - u_c(c_{t+1}^N)\right) + \beta\mathbb{E}_t\left(\mu_{t+1}^D(1-\delta) - \mu_{t+1}^D\frac{\mathcal{I}_{t+1}\partial\mathcal{S}(DS_t(1-\delta)+D_{t+1},\phi g^e))}{\partial DS_t}\right). \tag{40}$$

The implicit derivatives contained in these first-order conditions are:

$$\frac{\partial s_t u'(c_t)}{\partial D_t} = -\frac{v'(l_t)}{z_t} - v''(l_t)\frac{\partial l_t}{\partial D_t}h_t - v'(l_t)\frac{\partial h_t}{\partial D_t}$$

$$\frac{\partial s_t u'(c_t)}{\partial c_t} = u''(c_t)c_t + u'(c_t) - v'(l_t)\frac{\partial h_t}{\partial c_t} - v''(l_t)\frac{\partial l_t}{\partial c_t}h_t$$

$$\frac{\partial s_t u'(c_t)}{\partial DS_{t-1}} = -v'(l_t)\frac{\partial h_t}{\partial DS_{t-1}} - v''(l_t)h_t\frac{\partial l_t}{\partial DS_{t-1}}$$

$$\frac{\partial l_t}{\partial D_t} = -\frac{1}{z_t}\left(1 - \mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e)}{\partial D_t}\right)$$

$$\frac{\partial h_t}{\partial D_t} = \frac{1}{z_t}\left(1 - \mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e)}{\partial D_t}\right)$$

$$\frac{\partial l_t}{\partial c_t} = -\frac{1}{z_t}$$

$$\frac{\partial l_t}{\partial DS_{t-1}} = \frac{1}{z_t}\mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e))}{\partial DS_{t-1}}.$$

Note that we did not explicitly take the optimality condition with respect to leisure. Instead, we used the aggregate resource constraint to substitute out leisure in terms of consumption. Also note that $\mathbb{E}^x$ denotes the expectation operator over g_{t+1} and ξ_{t+1} after integrating out uncertainty over the disaster state. These four optimality conditions together with the implementability constraints

$$\Omega_t + \beta\mathbb{E}_t u_c(c_{t+1})b_{t+1} - u_c(c_t)b_t = 0, \tag{41}$$

and the law of motion for DS_t equation (36) characterize the model equilibrium dynamics.

A.3 First-best Optimal Policy

We consider a social planner who directly chooses optimal allocations, optimal tax, debt, and disaster management. The law of motion for the defense stock is the same as equation (36), with associated multiplier μ_t^D. The social planner still needs to take into account that defense stock affects the disaster probability, i.e. $P(DS,\xi)$, and needs to take into account the $D_t > 0$ constraint, to which we assign multiplier λ_t^D. Additionally, the planner needs to respect the aggregate resource constraint

$$c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = z_t(1-l_t),$$

with associated multiplier λ_t. More formally, the Lagrangian of the social planner reads:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) + v(l_t) + \mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t) +$$
$$+ \lambda_t (z_t(1-l_t) - c_t - D_t - g_t + \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)) + \lambda_t^D D_t \Big\}.$$

The first-order condition with respect to c_t is:

$$0 = u_c(c_t) - \lambda_t. \tag{42}$$

The first-order condition with respect to l_t is:

$$0 = v_l(l_t) - \lambda_t z_t. \tag{43}$$

The first-order condition with respect to D_t is:

$$0 = \lambda_t^D + (\mu_t^D - \lambda_t)\left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t}\right). \tag{44}$$

The first-order condition with respect to DS_t is:

$$\begin{aligned} \mu_t^D =& \beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^{g,\xi} \left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N) \right) + \beta \mathbb{E}_t \left(\lambda_{t+1} \mathcal{I}_{t+1} \frac{\mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e)}{\partial DS_t} \right) \\ &+ \beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_{t+1} \partial \mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e))}{\partial DS_t} \right). \end{aligned} \tag{45}$$

Substitute out terms using $\lambda_t = \frac{v'(l_t)}{z_t}$ to get:

$$\frac{\partial l_t}{\partial D_t} = -\frac{1}{z_t}\left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t)}{\partial D_t}\right)$$

$$\frac{\partial l_t}{\partial DS_{t-1}} = \frac{1}{z_t} \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e))}{\partial DS_{t-1}}.$$

This substitution gives expressions that can be directly compared to the analogous optimality conditions of the Ramsey problem

$$0 = \lambda_t^D + v_l(l_t) \frac{\partial l_t}{\partial D_t} + \mu_t \left(\frac{\partial s_t u_c(c_t)}{\partial D_t} \right) + \mu_t^D \left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t} \right). \tag{46}$$

$$\mu_t^D = \beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^{g,\xi} \left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N) \right) + \beta \mathbb{E}_t \left(\mu_{t+1} \frac{\partial s_{t+1} u_c(c_{t+1})}{\partial DS_t} + v'(l_{t+1}) \frac{\partial l_{t+1}}{\partial DS_t} \right)$$

$$+ \beta \mu_t b_{t+1} \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^{g,\xi} \left(u_c(c_{t+1}^W) - u_c(c_{t+1}^N) \right) + \beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_{t+1} \partial \mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e))}{\partial DS_t} \right). \tag{47}$$

Lemma 1. Proof.

Let period utility be

$$U(c, h) = u(c) - v(h),$$

where h denotes labor. Assume

$$u'(c) > 0, \qquad u''(c) \leq 0, \qquad v'(h) > 0, \qquad v''(h) > 0.$$

Define the welfare loss from distortionary taxation as

$$L = [u(c_{FB}) - v(h_{FB})] - [u(c_R) - v(h_R)],$$

The resource constraint is $c = zh - g - D + \mathcal{IS}(DS(1-\delta) + D, \phi g^e)$. Denote by A the total government spending including g and defense, such that $A \equiv g + D - \mathcal{IS}(DS(1-\delta) + D, \phi g^e)$. Then $c = zh - A$.

The first-best allocation satisfies

$$zu'(zh_{FB} - A) = v'(h_{FB}),$$

whereas the Ramsey allocation satisfies

$$(1-\tau)zu'(zh_R - A) = v'(h_R).$$

- Differentiating L with respect to τ gives

$$\frac{\partial L}{\partial \tau} = -\left[zu'(zh_R - A) - v'(h_R)\right] \frac{\partial h_R}{\partial \tau}. \tag{48}$$

Rearranging the Ramsey labor first-order condition $v'(h_R) = (1-\tau)zu'(zh_R - A)$ we obtain $zu'(zh_R - A) - v'(h_R) = \tau zu'(zh_R - A)$. Substituting this into 48 gives

$$\frac{\partial L}{\partial \tau} = -\tau zu'(zh_R - A) \frac{\partial h_R}{\partial \tau}.$$

It remains to sign $\partial h_R / \partial \tau$.

Differentiating the Ramsey labor first-order condition with respect to τ gives

$$-zu'(zh_R - A) + \left[(1-\tau)z^2 u''(zh_R - A) - v''(h_R)\right] \frac{\partial h_R}{\partial \tau} = 0.$$

Therefore,

$$\frac{\partial h_R}{\partial \tau} = \frac{zu'(zh_R - A)}{(1-\tau)z^2 u''(zh_R - A) - v''(h_R)}.$$

Since $u'(c) > 0$, $u''(c) \leq 0$, $v''(h) > 0$, and $z > 0$, the denominator is negative. Therefore,

$$\frac{\partial h_R}{\partial \tau} < 0.$$

Substituting this expression into the derivative of the welfare loss gives

$$\frac{\partial L}{\partial \tau} = \frac{\tau \left[zu'(zh_R - A)\right]^2}{v''(h_R) - (1-\tau)z^2 u''(zh_R - A)} > 0$$

for $\tau > 0$. Thus, other variables fixed, the welfare loss from distortionary taxation is increasing in the labor tax rate.

- This part shows that $L^W > L^N$ assuming that $\tau^W > \tau^N$. War is an event when z falls and A increases, therefore it is enough to show that L increases following a simultaneous increase in A and τ and a fall in z. Formally, one needs to show that

$$\frac{\partial L}{\partial \tau} + \frac{\partial L}{\partial A} - \frac{\partial L}{\partial z} > 0$$

$$\begin{aligned}
\frac{\partial L}{\partial A} &= \left(u'(c_{FB})z - v'(h_{FB})\right) \frac{\partial h_{FB}}{\partial A} - u'(c_{FB}) \\
&\quad - \left[u'(c_R)\left(z\frac{\partial h_R}{\partial A} - 1\right) - v'(h_R)\frac{\partial h_R}{\partial A}\right] \\
&= u'(c_R) - u'(c_{FB}) - \left[zu'(c_R) - v'(h_R)\right] \frac{\partial h_R}{\partial A} \\
&= u'(c_R) - u'(c_{FB}) - \tau z u'(c_R)\frac{\partial h_R}{\partial A}.
\end{aligned}$$

Use the implicit function theorem differentiating the intra-temporal optimality condition to obtain $\frac{\partial h_R}{\partial A}$. This gives

$$(1-\tau)zu''(c_R)\left(z\frac{\partial h_R}{\partial A} - 1\right) = v''(h_R)\frac{\partial h_R}{\partial A}.$$

$$\frac{\partial h_R}{\partial A} = \frac{-(1-\tau)zu''(c_R)}{v''(h_R) - (1-\tau)z^2u''(c_R)}.$$

Therefore,

$$\frac{\partial L}{\partial A} = -u'(c_{FB}) + u'(c_R) + \frac{\tau(1-\tau)z^2u'(c_R)u''(c_R)}{v''(h_R) - (1-\tau)z^2u''(c_R)}.$$

Assume constant Frisch labor supply elasticity, such that $v(h) = B\frac{h^{1+\eta}}{1+\eta}$. The period utility is then $U(c,h) = u(c) - \frac{B}{1+\eta}h^{1+\eta}$.

Define

$$\varepsilon_u(c_R) \equiv -\frac{c_R u''(c_R)}{u'(c_R)}, \qquad s_R \equiv \frac{zh_R}{c_R}, \qquad r \equiv \frac{u'(c_{FB})}{u'(c_R)}.$$

Using the Ramsey first-order condition, $Bh_R^\eta = (1-\tau)zu'(c_R)$, we can rewrite the derivative as

$$\frac{1}{u'(c_R)}\frac{\partial L}{\partial A} = 1 - r - \tau\frac{\varepsilon_u(c_R)s_R}{\eta + \varepsilon_u(c_R)s_R}.$$

Thus $\frac{\partial L}{\partial A} > 0$ iff $1 - r > \tau\frac{\varepsilon_u(c_R)s_R}{\eta+\varepsilon_u(c_R)s_R}$. Now define $x \equiv \frac{h_R}{h_{FB}}$ and $y \equiv \frac{c_R}{c_{FB}}$. For $\tau > 0$, the Ramsey labor wedge implies $h_R < h_{FB}; c_R < c_{FB}$, so $x \in (0,1)$ and $y \in (0,1)$.. From the first-order conditions,

$$(1-\tau)\frac{u'(c_R)}{u'(c_{FB})} = \left(\frac{h_R}{h_{FB}}\right)^\eta,$$

or equivalently, $\tau = 1 - rx^\eta$. Using $c_{FB} - c_R = z(h_{FB} - h_R)$, we can express s_R as $s_R = \frac{zh_R}{c_R} = \frac{x(1-y)}{y(1-x)}$. The condition $\partial L/\partial A > 0$ can then be written as $1 - r > (1 - rx^\eta)\frac{\varepsilon_u(c_R)s_R}{\eta+\varepsilon_u(c_R)s_R}$. After rearranging,

$$\eta(1-r) > \varepsilon_u(c_R)s_R r(1-x^\eta).$$

Substituting for s_R, this becomes

$$\frac{\eta}{\varepsilon_u(c_R)}\frac{y(1-r)}{r(1-y)} > x\frac{1-x^\eta}{1-x}. \tag{49}$$

First, using the mean value theorem one can show that for $x \in (0,1)$ and $\eta > 0$,

$$x\frac{1-x^\eta}{1-x} < \eta.$$

It remains to show that the left-hand side of (49) is weakly greater than η. We impose that the elasticity of marginal utility is constant,

$$\varepsilon_u(c) \equiv -\frac{cu''(c)}{u'(c)} = \varepsilon \geq 1.$$

Since

$$-\frac{u''(c)}{u'(c)} = \frac{\varepsilon}{c},$$

we have

$$\log u'(c_R) - \log u'(c_{FB}) = \int_{c_R}^{c_{FB}} \frac{\varepsilon}{c}\,dc = \varepsilon\log\left(\frac{c_{FB}}{c_R}\right).$$

Therefore,

$$\frac{1}{r} = \frac{u'(c_R)}{u'(c_{FB})} = \left(\frac{c_{FB}}{c_R}\right)^\varepsilon = y^{-\varepsilon}.$$

It follows that

$$\frac{1-r}{r} = \frac{1}{r} - 1 = y^{-\varepsilon} - 1.$$

Since $\varepsilon \geq 1$ and $y \in (0,1)$,

$$y^{-\varepsilon} - 1 \geq \varepsilon\left(\frac{1-y}{y}\right)$$

therefore

$$\frac{1}{\varepsilon}\frac{y(1-r)}{r(1-y)} \geq 1.$$

$$\frac{\eta}{\varepsilon}\frac{y(1-r)}{r(1-y)} \geq \eta.$$

Together with

$$x\frac{1-x^{\eta}}{1-x} < \eta,$$

this establishes (49). Therefore $\frac{\partial L}{\partial A} > 0$ in the usual interior case with $\tau > 0$.

Differentiating L with respect to z, holding A and τ fixed, gives

$$\frac{\partial L}{\partial z} = u'(c_{FB})h_{FB} - u'(c_R)h_R - \tau z u'(c_R)\frac{\partial h_R}{\partial z}.$$

Differentiating the Ramsey first-order condition with respect to z, we obtain

$$\frac{\partial h_R}{\partial z} = \frac{(1-\tau)\left[u'(c_R) + zh_R u''(c_R)\right]}{B\eta h_R^{\eta-1} - (1-\tau)z^2 u''(c_R)}.$$

Using the definitions for s_R and the constant elasticity of marginal utility ε, the expression for $h_{R,z}$ can be written as

$$\frac{\partial h_R}{\partial z} = \frac{h_R}{z}\frac{1-\varepsilon s_R}{\eta + \varepsilon s_R}.$$

Therefore,

$$\frac{1}{u'(c_R)h_R}\frac{\partial L}{\partial z} = \frac{u'(c_{FB})}{u'(c_R)}\frac{h_{FB}}{h_R} - 1 - \tau\frac{1-\varepsilon s_R}{\eta + \varepsilon s_R}.$$

For $\tau > 0$, the Ramsey labor wedge implies $0 < y < x < 1$. Thus

$$\frac{1}{u'(c_R)h_R}\frac{\partial L}{\partial z} = \frac{r}{x} - 1 - \tau\frac{1-\varepsilon s_R}{\eta + \varepsilon s_R}.$$

Following the same steps as above, we can write

$$\tau = 1 - rx^{\eta}, \qquad s_R = \frac{x(1-y)}{y(1-x)}.$$

Moreover, since $\varepsilon_u(c) = \varepsilon$ is constant,

$$\log u'(c_R) - \log u'(c_{FB}) = \int_{c_R}^{c_{FB}} \frac{\varepsilon}{c}\, dc = \varepsilon \log\left(\frac{c_{FB}}{c_R}\right).$$

Therefore,

$$\frac{u'(c_R)}{u'(c_{FB})} = \left(\frac{c_{FB}}{c_R}\right)^{\varepsilon} = y^{-\varepsilon},$$

and hence

$$r = \frac{u'(c_{FB})}{u'(c_R)} = y^{\varepsilon}.$$

Now define

$$\Phi(r) \equiv \frac{r}{x} - 1 - (1 - rx^{\eta})\frac{1 - \varepsilon s_R}{\eta + \varepsilon s_R}.$$

The normalized derivative equals $\Phi(r)$. Since $r = y^{\varepsilon}$, it remains to show that

$$\Phi\left(y^{\varepsilon}\right) \leq 0.$$

$$\Phi\left(y^{\varepsilon}\right) = \frac{y^{\varepsilon}}{x} - 1 - \left(1 - y^{\varepsilon}x^{\eta}\right)\frac{1 - \varepsilon s_R}{\eta + \varepsilon s_R}.$$

Since the denominator is positive, $\Phi(y^{\varepsilon}) \leq 0$ is equivalent to

$$\left(\frac{y^{\varepsilon}}{x} - 1\right)(\eta + \varepsilon s_R) + \left(1 - y^{\varepsilon}x^{\eta}\right)(\varepsilon s_R - 1) \leq 0.$$

Using

$$s_R = \frac{x(1-y)}{y(1-x)},$$

the left-hand side can be written as

$$F(y) - (1 + \eta),$$

where

$$F(y) \equiv y^{\varepsilon}\left[\frac{\eta}{x} + x^{\eta}\right] + \varepsilon y^{\varepsilon-1}(1-y)\frac{1 - x^{1+\eta}}{1-x}.$$

For fixed $x \in (0,1)$, $F(y)$ is weakly increasing in y on $(0, x]$ whenever $\varepsilon \geq 1$. Therefore, since $y < x$,

$$F(y) \leq F(x).$$

Evaluating at $y = x$ $F(x) = (\eta + \varepsilon)x^{\varepsilon-1} + (1-\varepsilon)x^{\varepsilon+\eta}$. Define

$$G(x) \equiv (\eta + \varepsilon)x^{\varepsilon-1} + (1-\varepsilon)x^{\varepsilon+\eta}.$$

Then

$$G'(x) = (\varepsilon - 1)(\eta + \varepsilon)x^{\varepsilon-2}\left(1 - x^{1+\eta}\right) \geq 0,$$

because $\varepsilon \geq 1$, $\eta > 0$, and $x \in (0,1)$. Hence $G(x) \leq G(1) = 1 + \eta$ and $F(y) \leq 1 + \eta$. This implies $\Phi\left(y^{\varepsilon}\right) \leq 0$. Since $r = y^{\varepsilon}$, we obtain

$$\frac{1}{u'(c_R)h_R}\frac{\partial L}{\partial z} \leq 0.$$

Finally, because $u'(c_R) > 0$ and $h_R > 0$, $\frac{\partial L}{\partial z} \leq 0$.

Taken together, this implies that

$$\frac{\partial L}{\partial \tau} + \frac{\partial L}{\partial A} - \frac{\partial L}{\partial z} > 0$$

and the welfare loss from distortionary taxation in war state is higher than in the normal state.

■

Proof of Proposition 1. **Proof.** Assume that optimality conditions for the first-best and for the Ramsey policy hold. We use superscripts R and FB for Ramsey and first-best allocations, respectively. For $s \in \{W, N\}$, define the state-specific welfare loss

$$L^s_{t+1} \equiv U(c^{s,FB}_{t+1}, l^{s,FB}_{t+1}) - U(c^{s,R}_{t+1}, l^{s,R}_{t+1}), \qquad U(c,l) \equiv u(c) + v(l). \tag{50}$$

Let $\mu^{D,R}_t$ and $\mu^{D,FB}_t$ denote the multipliers on the defense-stock law of motion, and define

$$\Delta_t \equiv \mu^{D,R}_t - \mu^{D,FB}_t. \tag{51}$$

To simplify derivations, assume that *insurance* is absent ($\phi = 0$). The Ramsey optimality condition for DS_t is then

$$\begin{aligned} \mu^{D,R}_t = \beta P_{DS,t}\, \mathbb{E}^{g,\xi}_t &\left(U(c^{W,R}_{t+1}, l^{W,R}_{t+1}) - U(c^{N,R}_{t+1}, l^{N,R}_{t+1})\right) \\ &+ \beta \mu_t b_{t+1} P_{DS,t}\, \mathbb{E}^{g,\xi}_t \left(u_c(c^{W,R}_{t+1}) - u_c(c^{N,R}_{t+1})\right) + \beta(1-\delta)\mathbb{E}_t \left(\mu^{D,R}_{t+1}\right), \end{aligned} \tag{52}$$

whereas the first-best condition is

$$\begin{aligned} \mu^{D,FB}_t = \beta P_{DS,t}\, \mathbb{E}^{g,\xi}_t &\left[U(c^{W,FB}_{t+1}, l^{W,FB}_{t+1}) - U(c^{N,FB}_{t+1}, l^{N,FB}_{t+1})\right] \\ &+ \beta(1-\delta)\mathbb{E}_t \left[\mu^{D,FB}_{t+1}\right], \end{aligned} \tag{53}$$

with $P_{DS,t} \equiv \partial P(DS_t, \xi_t)/\partial DS_t < 0$.

The multiplier gap then follows

$$\begin{aligned} \Delta_t = \beta P_{DS,t}\, \mathbb{E}^{g,\xi}_t &\left[L^N_{t+1} - L^W_{t+1}\right] \\ &+ \beta \mu_t b_{t+1} P_{DS,t}\, \mathbb{E}^{g,\xi}_t \left[u_c(c^{W,R}_{t+1}) - u_c(c^{N,R}_{t+1})\right] + \beta(1-\delta)\mathbb{E}_t \left[\Delta_{t+1}\right]. \end{aligned} \tag{54}$$

For compactness, define the one-period wedge

$$\begin{aligned} \mathcal{W}_{t+1} \equiv P_{DS,t} \Big\{ &\mathbb{E}^{g,\xi}_t \left[L^N_{t+1} - L^W_{t+1}\right] \\ &+ \mu_t b_{t+1} \mathbb{E}^{g,\xi}_t \left[u_c(c^{W,R}_{t+1}) - u_c(c^{N,R}_{t+1})\right] \Big\}. \end{aligned} \tag{55}$$

Iterating (54) forward for any $J \geq 1$ gives

$$\Delta_t = \sum_{j=1}^{J} \beta^j (1-\delta)^{j-1} \mathbb{E}_t \left[\mathcal{W}_{t+j}\right] + \beta^J (1-\delta)^J \mathbb{E}_t \left[\Delta_{t+J}\right]. \tag{56}$$

Under the transversality condition

$$\lim_{J\to\infty} \beta^J (1-\delta)^J \mathbb{E}_t \left[\Delta_{t+J}\right] = 0,$$

the present-value representation is

$$\Delta_t = \sum_{j=1}^{\infty} \beta^j (1-\delta)^{j-1} \mathbb{E}_t \left[\mathcal{W}_{t+j}\right]. \tag{57}$$

It follows that the Ramsey and first-best shadow values of defense stock coincide, $\Delta_t = 0$, if

$$\sum_{j=1}^{\infty} \beta^j (1-\delta)^{j-1} \mathbb{E}_t \left(P_{DS,t+j-1} \mathbb{E}^{g,\xi}_{t+j-1} \left(L^W_{t+j} - L^N_{t+j} \right) \right) = \sum_{j=1}^{\infty} \beta^j (1-\delta)^{j-1} \mathbb{E}_t \left(\mu_{t+j-1} b_{t+j} P_{DS,t+j-1} \mathbb{E}^{g,\xi}_{t+j-1} \left(u_c(c^{W,R}_{t+j}) - u_c(c^{N,R}_{t+j}) \right) \right) \tag{58}$$

1. Because $P_{DS} < 0$, if $L^W > L^N$ and the price-manipulation term is absent, every current allocation-gap term is positive: the Ramsey planner assigns a larger marginal value to deterrence than the first-best planner.

2. If the price manipulation term is present, in the sense that the implementability constraint multiplier is greater than zero or $u_c(c^{W,R}) \neq u_c(c^{N,R})$, then there exists a path for debt $\{b_t\}_t$ such the price manipulation term offsets the allocation-gap term. If, in addition, $u_c(c^{W,R}) > u_c(c^{N,R})$, the allocation-gap term is offset with positive debt.

■

A.4 Optimality Conditions of the Extended model

Here we present the optimality conditions of the extended model from Section 6. This is the baseline model extended to have a New Keynesian block and with the possibility for the home country to start a war.

Incomplete Markets with Nominal Rigidities

Hence, the recursive Lagrangian of the planner reads:

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t) + v(l_t) + \\
& + \mu_t \left(\Omega_t + \beta \mathbb{E}_t \left[u_c(c_{t+1}) \pi^{-1}_{t+1} \right] b_{t+1} - u_c(c_t) b_t \pi^{-1}_t \right) + \\
& + \mu^D_t \left(DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t \right) + \\
& + \lambda^\pi_t \left(u_c(c_t) Y_t \nu^{-1} \left(\nu - 1 + w_t z^{-1}_t \right) - u_c(c_t) \Phi_\pi(\pi_t) \pi_t + \mathbb{E}_t \left[\beta u_c(c_{t+1}) \Phi_\pi(\pi_{t+1}) \pi_{t+1} \right] \right) \\
& - \bar{\Lambda}_t b_{t+1} + \underline{\Lambda}_t b_{t+1} + \lambda^D_t D_t + \underline{\Lambda}^P_t P^H_{t+1} - \bar{\Lambda}^P_t P^H_{t+1} \Big\},
\end{aligned}$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t)(w_t h_t - g_t - D_t + \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)) - v_l(l_t) h_t$ is the government primary surplus in marginal utility terms. And we substitute out h_t and l_t from the resource constraint:

$$Y_t = z_t h_t = c_t + g_t + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)) + \Phi(\pi_t)$$

The first-order condition with respect to P^H_{t+1} is:

$$\begin{aligned}
& \beta E^{g,\xi}_t \left(U(c^H_{t+1}, l^H_{t+1}) - \left(P^F(DS_t, \xi_t) U(c^F_{t+1}, l^F_{t+1}) + (1 - P^F(DS_t, \xi_t)) U(c^N_{t+1}, l^N_{t+1}) \right) \right) + \\
& \beta \mu_t b_{t+1} E^{g,\xi}_t \left(u_c(c^H_{t+1})/\pi^H_{t+1} - P^F(DS_t, \xi_t) u_c(c^F_{t+1})/\pi^F_{t+1} - (1 - P^F(DS_t, \xi_t)) u_c(c^N_{t+1})/\pi^N_{t+1} \right) + \underline{\Lambda}^P_t - \bar{\Lambda}^P_t + \\
& \beta \lambda^\pi_t E^{g,\xi}_t \left(u_c(c^H_{t+1}) \Phi_\pi(\pi^H_{t+1}) \pi^H_{t+1} - P^F(DS_t, \xi_t) u_c(c^F_{t+1}) \Phi_\pi(\pi^F_{t+1}) \pi^F_{t+1} - (1 - P^F(DS_t, \xi_t)) u_c(c^N_{t+1}) \Phi_\pi(\pi^N_{t+1}) \pi^N_{t+1} \right) = 0
\end{aligned} \tag{59}$$

The first-order condition with respect to c_t is:

$$0 = u_c(c_t) + v_l(l_t)\frac{\partial l_t}{\partial c_t} + \mu_t\left(\frac{\partial(s_t u_c(c_t))}{\partial c_t}\right) - u_{cc}(c_t)b_t(\mu_t - \mu_{t-1})\pi_t^{-1}$$
$$+ \lambda_t^\pi\left[\left(u_{cc}(c_t)Y_t + u_c(c_t)\frac{\partial Y_t}{\partial c_t}\right)\nu^{-1}(\nu - 1 + w_t z_t^{-1}) - u_{cc}(c_t)\Phi_\pi(\pi_t)\pi_t\right] + \lambda_{t-1}^\pi u_{cc}(c_t)\Phi_\pi(\pi_t)\pi_t. \quad (60)$$

The first-order condition with respect to b_{t+1} is:

$$\mu_t = \mathbb{E}_t(n_{t+1}\mu_{t+1}) + \frac{1}{\beta\mathbb{E}_t\left[u_c(c_{t+1})\pi_{t+1}^{-1}\right]}\left(\overline{\Lambda}_t - \underline{\Lambda}_t\right), \quad \text{where} \quad n_{t+1} \equiv \frac{u_c(c_{t+1})\pi_{t+1}^{-1}}{\mathbb{E}_t[u_c(c_{t+1})\pi_{t+1}^{-1}]}. \quad (61)$$

The intuition behind equation (61) is similar to the baseline case, equation (10), except that now the planner can use inflation to inflate away nominal bonds, creating state-contingent insurance.

The first-order condition with respect to D_t is:

$$0 = \lambda_t^D + v_l(l_t)\frac{\partial l_t}{\partial D_t} + \mu_t\left(\frac{\partial(s_t u_c(c_t))}{\partial D_t}\right)$$
$$+ \mu_t^D\left(1 - \mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t}\right)$$
$$+ \lambda_t^\pi u_c(c_t)\,\nu^{-1}(\nu - 1 + w_t z_t^{-1})\left(1 - \mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t}\right). \quad (62)$$

The first-order condition with respect to DS_t is:

$$\mu_t^D = \beta(1 - P_{t+1}^H)\frac{\partial P(DS_t, \xi_t)}{\partial DS_t}\mathbb{E}_t^{g,\xi}\left(u(c_{t+1}^F) + v(l_{t+1}^F) - u(c_{t+1}^N) - v(l_{t+1}^N)\right)$$
$$+ \beta b_{t+1}\mu_t(1 - P_{t+1}^H)\frac{\partial P(DS_t, \xi_t)}{\partial DS_t}\mathbb{E}_t^{g,\xi}\left(u_c(c_{t+1}^F)/\pi_{t+1}^F - u_c(c_{t+1}^N)/\pi_{t+1}^N\right) + \beta\mathbb{E}_t\left(\mu_{t+1}\frac{\partial(s_{t+1}u_c(c_{t+1}))}{\partial DS_t} + v'(l_{t+1})\frac{\partial l_{t+1}}{\partial DS_t}\right)$$
$$+ \beta\mathbb{E}_t\left(\mu_{t+1}^D(1-\delta) - \mu_{t+1}^D\mathcal{I}_{t+1}\frac{\partial\mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e)}{\partial DS_t}\right) + \beta\mathbb{E}_t\left[\lambda_{t+1}^\pi u_c(c_{t+1})\,\nu^{-1}(\nu - 1 + w_{t+1}z_{t+1}^{-1})\frac{\partial Y_{t+1}}{\partial DS_t}\right]$$
$$+ \beta\lambda_t^\pi(1 - P_{t+1}^H)\frac{\partial P(DS_t, \xi_t)}{\partial DS_t}\mathbb{E}_t^{g,\xi}\left(u_c(c_{t+1}^F)\Phi_\pi(\pi_{t+1}^F)\pi_{t+1}^F - u_c(c_{t+1}^N)\Phi_\pi(\pi_{t+1}^N)\pi_{t+1}^N\right). \quad (63)$$

The first-order condition with respect to inflation π_t is:

$$0 = v_l(l_t)\frac{\partial l_t}{\partial \pi_t} + u_c(c_t)b_t\pi_t^{-2}(\mu_t - \mu_{t-1}) + \mu_t\frac{\partial u_{c,t}s_t}{\partial \pi_t}$$
$$+ \lambda_t^\pi u_c(c_t)\nu^{-1}(\nu - 1 + w_t z_t^{-1})\Phi_\pi(\pi_t)$$
$$+ u_c(c_t)\left[\Phi_{\pi\pi}(\pi_t)\pi_t + \Phi_\pi(\pi_t)\right](\lambda_{t-1}^\pi - \lambda_t^\pi). \quad (64)$$

The first-order condition with respect to the real wage w_t is:

$$0 = \lambda_t^\pi u_c(c_t)Y_t\nu^{-1}z_t^{-1} + \mu_t\frac{\partial s_t u_{c,t}}{\partial w_t}. \quad (65)$$

The implicit derivatives contained in these first-order conditions are as in the baseline model, with the addition of:

$$z_t h_t = Y_t = c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) + \Phi_t$$

The insurance term differs depending on who started the war:

home starts: $\mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^H(DS_{t-1}))$

foreign starts: $\mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^F)$

Derivative of the insurance term:

home starts:
$$\frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial DS_{t-1}} = S_1()(1-\delta) + S_2()\phi \frac{\partial g^H}{\partial DS_{t-1}}$$
$$\frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t} = S_1()$$

foreign starts:
$$\frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial DS_{t-1}} = S_1()(1-\delta)$$
$$\frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t} = S_1()$$

and

$$\frac{\partial s_t u'(c_t)}{\partial D_t} = u_{c,t}\left(w_t \frac{\partial h_t}{\partial D_t} - 1 + \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t}\right) - v''(l_t)\frac{\partial l_t}{\partial D_t} h_t - v'(l_t)\frac{\partial h_t}{\partial D_t}$$

$$\frac{\partial s_t u'(c_t)}{\partial c_t} = u''(c_t)(w_t h_t - g_t - D_t + \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)) + u'(c_t) w_t \frac{\partial h_t}{\partial c_t} - v'(l_t)\frac{\partial h_t}{\partial c_t} - v''(l_t)\frac{\partial l_t}{\partial c_t} h_t$$

$$\frac{\partial s_t u'(c_t)}{\partial DS_{t-1}} = u_{c,t}\left(w_t \frac{\partial h_t}{DS_{t-1}} + \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial DS_{t-1}}\right) - v'(l_t)\frac{\partial h_t}{\partial DS_{t-1}} - h_t v''(l_t)\frac{\partial l_t}{\partial DS_{t-1}}$$

$$\frac{\partial u_{c,t} s_t}{\partial \pi_t} = u_{c,t} w_t \frac{\partial h_t}{\partial \pi_t} - \frac{\partial h_t}{\partial \pi_t} v'(l_t) - h_t v''(l_t)\frac{\partial l_t}{\partial \pi_t}$$

$$\frac{\partial u_{c,t} s_t}{\partial w_t} = u_c(c_t) h_t$$

$$\frac{\partial h_t}{\partial D_t} = \frac{1}{z_t}\left(1 - \mathcal{I}_t \mathcal{S}_1()\right)$$

$$\frac{\partial l_t}{\partial D_t} = -\frac{1}{z_t}\left(1 - \mathcal{I}_t \mathcal{S}_1()\right)$$

$$\frac{\partial l_t}{\partial c_t} = -\frac{1}{z_t}$$

$$\frac{\partial h_t}{\partial DS_{t-1}} = \frac{1}{z_t}\left(-\mathcal{I}_t \partial \mathcal{S}_1()(1-\delta) - \mathcal{I}_t^H \mathcal{S}_2()\phi \frac{\partial g^H}{\partial DS_{t-1}} + \mathcal{I}_t^H \frac{\partial g^H}{\partial DS_{t-1}}\right)$$

$$\frac{\partial h_t}{\partial l_t} = -1$$

$$\frac{\partial h_t}{\partial \pi_t} = \frac{1}{z_t}\Phi_\pi(\pi_t)$$

Note that, as in the baseline, we did not explicitly take the optimality condition with respect to leisure. Instead, we used the aggregate resource constraint to substitute out leisure in terms of consumption.

The solution using PEA requires to approximate the following terms:

$$Y_{1,t} \equiv u'(c_t)\mu_t/\pi_t$$

$$Y_{2,t} \equiv u'(c_t)/\pi_t$$

$$Y_{3,t} \equiv \mu_t^D(1-\delta) - \mu_t^D \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial DS_{t-1}}$$

$$Y_{4,t} \equiv u(c_t) + v(l_t)$$

$$Y_{5,t} \equiv \mu_t \frac{\partial(s_t u_c(c_t))}{\partial DS_{t-1}} + v'(l_t)\frac{\partial l_t}{\partial DS_{t-1}}$$

$$Y_{6,t} \equiv \lambda_t^\pi \, u_c(c_t)\, \nu^{-1}(\nu - 1 + w_t z_t^{-1})\frac{\partial Y_t}{\partial DS_{t-1}}$$

$$Y_{7,t} \equiv u_c(c_t)\Phi_\pi(\pi_t)\pi_t$$

A.5 Solution Algorithm

Here we provide a brief summary of the algorithm. More implementation details along with the sample code can be found in Valaitis and Villa (2024). PEA algorithm requires making a projection of expected value terms on the state variables. We do this by projecting the integrands in the expected value terms in the system of equations (37),(38), (39), (40), (36), (41) onto the state variables using an artificial neural network. We then use Gaussian quadrature to approximate the expected value terms. Solution algorithm:

1. Generate a sequence of shocks $\{g_t, \xi_t\}_{t=1}^T$. Given an educated guess, initialize the neural network $\mathcal{ANN}(g_t, \xi_t, \mu_{t-1}, b_{t-1}, DS_{t-1}, \mathcal{I}_t)$, where $\mathcal{I}_t$ indicates whether economy is in the disaster state.
2. Given this guess, simulate the model by solving the system of equations (37),(38), (39), (40), (36), (41) at every t to obtain sequences of endogenous variables.
3. Given the simulated sequence train the neural network and update network weights.
4. Check if the $\mathcal{ANN}$ predictions are consistent with the simulated data and the network weights do not change and simulated sequences are stable across iterations. If not, go back to step 2 and simulate the model again using the updated neural network.

Implementation. To help reach convergence, we follow Maliar and Maliar (2003) and impose moving bounds on debt at each iteration that are gradually relaxed.[12] T is set to 30000. We use a single hidden-layer artificial neural network $\mathcal{ANN}(\mathcal{I}_t)$ with 30 neurons in the hidden layer. We use the hyperbolic tangent sigmoid activation function and train the network using Levenberg-Marquardt back-propagation. To avoid overfitting, in initial iterations, we put a penalty on the size of the network weights and then gradually decrease this penalty.

Lagrange multipliers. In the practical implementation of the algorithm, instead of checking the debt constraints and the complementary slackness conditions, we follow Valaitis and Villa (2024) and allow the debt constraints to be violated, subject to a utility penalty. We replace the multipliers $\bar{\Lambda}_t$, $\underline{\Lambda}_t$ and λ_t^D with smooth functions of b_{t+1} and D_t that correspond to the derivatives of the penalty function.

In particular, we set

$$\bar{\Lambda}_t = \begin{cases} \phi(b_{t+1} - \bar{b}) + \ln(1 + \phi(b_{t+1} - \bar{b})) & \text{if } b_{t+1} \geq \bar{b} \\ 0 & \text{if } b_{t+1} < \bar{b}. \end{cases} \tag{66}$$

and

$$\underline{\Lambda}_t = \begin{cases} \phi(\underline{b} - b_{t+1}) + \ln(1 + \phi(\underline{b} - b_{t+1})) & \text{if } b_{t+1} \leq \underline{b} \\ 0 & \text{if } b_{t+1} > \underline{b}. \end{cases} \tag{67}$$

Similarly, we set

$$\lambda_t^D = \begin{cases} \phi_d(\underline{D} - D_t) + \ln(1 + \phi_d(\underline{D} - D_t)) & \text{if } D_t \leq \underline{D} \\ 0 & \text{if } D_t > \underline{D}. \end{cases} \tag{68}$$

We set the penalty parameters $\phi = 1000$, $\phi_d = 5000$, $\underline{b} = 0$, $\underline{D} = 0$ and $\bar{b} = 0.99$. The upper bound of debt corresponds to 300% of the average output.

A.6 Additional Results

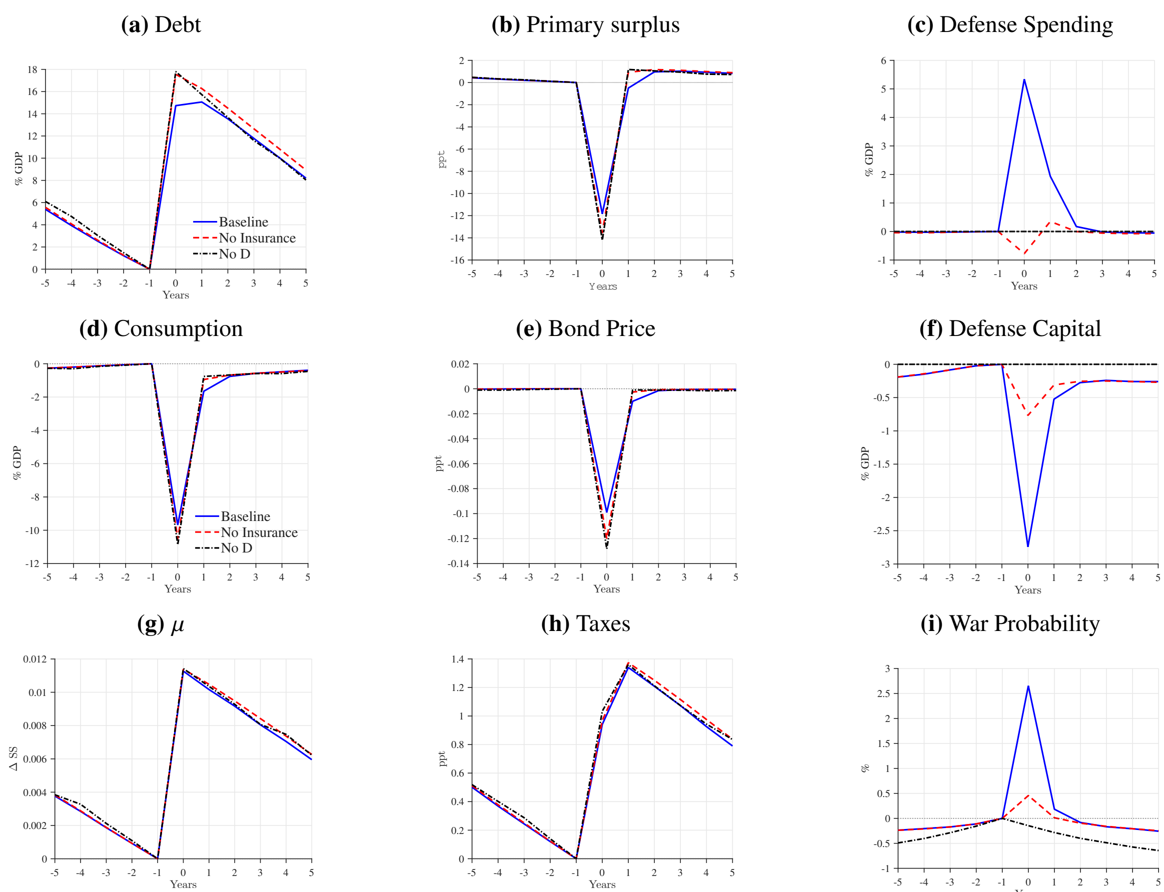
This section presents additional quantitative results.

[12] See Valaitis and Villa (2024) for details.

A.6.1 War Episodes

We look at how the *deterrence* and *insurance* channels shape the wartime dynamics. Figure A.1 shows the median dynamics around war episodes in the baseline, *No Insurance* and the model with exogenous war. Panel (a) reports debt dynamics, which are qualitatively similar in all three models. The peacetime dynamics are marked by the gradual decumulation of wartime debt, while during the war the planner borrows heavily. In the standard model, the planner borrows heavily and runs a primary deficit of 14% of GDP, accompanied by an 11% of GDP drop in consumption. Next, consider the *No Insurance* benchmark. A small cut in defense spending increases the probability of consecutive wars, which creates upward pressure on bond prices through household precautionary saving motives. Such policy allows the planner to run smaller deficits and is also associated with a smaller fall in consumption relative to the standard model. At the same time, the mechanism relies on a counterfactual finding that defense spending falls during wars. The dynamics of debt and surpluses are further dampened in the baseline model. While the planner now increases defense spending by 5% of GDP, this is still less than the total wartime spending needs and the defense capital stock falls significantly more than in the *No Insurance* model, creating further upward pressure on bond prices and allowing for cheaper borrowing. This, together with the insurance role of DS, allows the planner to run smaller deficits and experience smaller drops in consumption. Overall, the total primary deficits in the baseline are around 2% of GDP smaller than in the standard model.

Figure A.1. Median dynamics during war episodes.

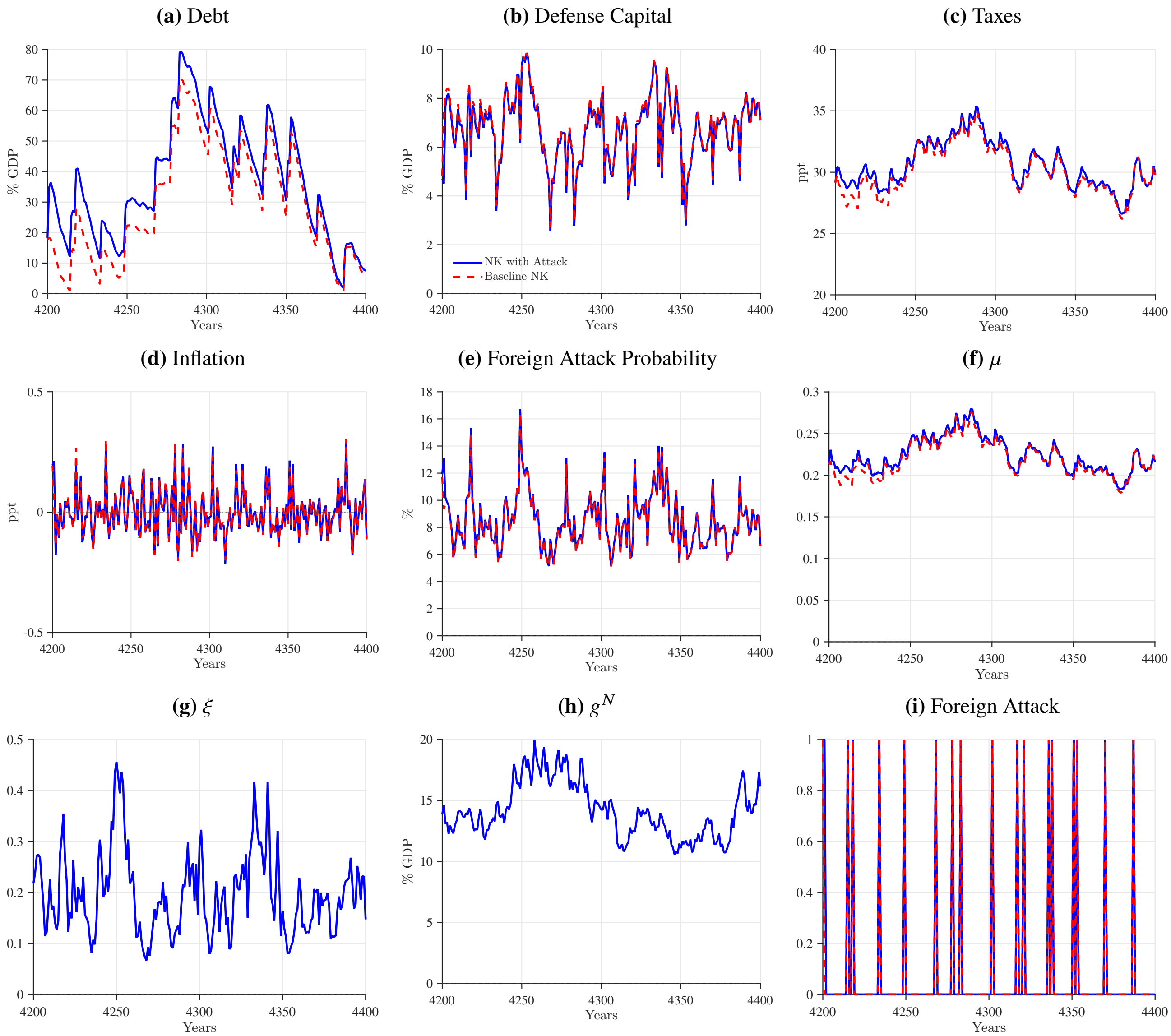


Notes: The figure shows the median model dynamics around war events. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line reports the benchmark where D is ineffective. Data come from 5000 model simulations of 1000 periods. In total, this gives 396995 war episodes in the baseline model.

A.6.2 Dynamics outside of preemptive strike episodes

A.2 shows an excerpt from the stochastic simulation of the Baseline NK model and the NK model with preemptive strike option. Results show that outside of preemptive strike episodes, model dynamics are nearly identical.

Figure A.2. Simulation Dynamics in Periods Without Preemptive Strikes



Notes: The figure shows an excerpt from a stochastic simulation in the Baseline NK model and in the NK model without an attack. It shows the selected subsample where the preemptive strike probability is zero.

B Data

In this section, we discuss our data sources and variable construction, used to calculate moments and the empirical impulse responses. We also provide a detailed description of the data used in the calibration.

B.1 Data used in the Calibration and in Section 2.1

Government Spending. Our measure of exogenous government spending is the total US government spending net of federal defense spending. Specifically, we use:

- GSP: government spending and investment (NIPA Table 3.9.5, line 1)
- ND: Federal defense spending (NIPA Table 3.9.5, line 17)
- $GSPF$: Government deflator (NIPA Table 3.9.4 line 1)
- NDF: Federal defense spending (NIPA Table 3.9.5, line 17)
- GDP: nominal GDP (NIPA Table 1.1.5, line 1)

In order to estimate ρ_g and σ_g, we construct the real government expenditure series as

$$g_t = GSP/GSPF - ND/NDF.$$

We then filter the linear trend before estimating ρ_g and σ_g. Our estimate of μ_g is:

$$\mu_g = \frac{GSP - ND}{GDP}.$$

B.2 Estimation of $P(DS, \xi)$ Parameters

Throughout the paper we use the historical series (GPRH, GPRHA, GPRHT). We measure ξ using the threats index (GPRHT) and we measure DSY_t by interpreting it as the US military capital stock (NIPA FA Table 7.1 line 22) as a fraction of GDP (NIPA Table 3.9.5, line 1). These data sources contain annual data for the period 1929 – 2023. We then convert the acts index (GPRHA) into a probability using the following transformation:

$$P_t = \frac{GPRHA_t - min(GPRHA)}{max(GPRHA) - min(GPRHA)}.$$

In order to harmonize our data into annual frequency, we take a 12-month moving average of P_t and ξ_t and pick the midyear value as the measure of P_t and ξ_t in period t. Our normalized series P_t rises above 0.5 twice: 1) during WWII and 2) during the September 11 episode. We label these as disaster episodes. Our objective is to estimate the role of defense investment on geopolitical risk in peace time. Hence, we add dummy variables to capture defense investment dynamics during our identified disaster episodes, which we denote as $D1_t$ and $D2_t$. We then estimate coefficients by minimizing the least squares of the nonlinear equation below:

$$P_t = \frac{1}{1 + e^{-\beta_1 - \beta_2 log(DSY_{t-1}) - \beta_3 log(\xi_{t-1}) - \beta_4 D1_{t-1} - \beta_5 D2_{t-1}}}.$$

Consistently with the model, our data for ξ_t is measured in the middle of period t, while data for DSY_t records the end of period quantity. For robustness, we consider the following four possibilities: 1) We use a single dummy variable to capture disaster episodes. 2) We fit absolute values instead of least squares. 3) We do not use disaster dummy variables. 4) We run a linear OLS in logs. All these cases always yield a negative coefficient for β_2 and a positive coefficient for β_3.

In our numerical work we then remain consistent with our estimation and use the defense stock DS scaled by the model GDP as an input in the $P(DS, \xi)$ function.

B.3 Construction of Tax Variables

To measure capital and labor taxes in the data we follow the approach of measuring average tax rates using national accounts data. This is a common approach initially introduced in Mendoza, Razin, and Tesar (1994). Since then, it has been repeatedly used in empirical work on taxation, such as Burnside, Eichenbaum, and Fisher (2004), Mertens and Ravn (2013), Leeper, Plante, and Traum (2010), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) and Clymo, Lanteri, and Villa (2023).
An alternative approach would be to account for non-linear tax schedules, with varying marginal tax rates Bhandari and McGrattan (2020). They construct a full marginal tax schedule for personal income taxes from US data, and model how business profits are taxed either as personal or corporate taxes. While this approach has the advantage of better capturing the specifics of a given country's tax system, it maps less directly into our representative agent model with linear tax schedule. In practice, however, Jones (2002) finds that the two approaches yield taxes of a similar magnitude and with a high correlation between the series.

Data for constructing tax variables For consistency with our model, we use annual data. We follow Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) and include both federal and state taxes when constructing our data, so that our taxes capture all taxes paid domestically by households in the US. The data come from the National Accounts (NIPA) tables provided by the Bureau of Economic Analysis (BEA). We name variables for use in the formulas below:

- Output and spending
  - DEF: output deflator (NIPA Table 1.1.4, line 1)
  - $NGDP$: nominal GDP (NIPA Table 1.1.5, line 1)
  - PCE: personal consumption expenditures (NIPA Table 1.1.5, line 2)
  - GSP: government spending and investment (NIPA Table 1.1.5, line 22)
- Incomes
  - CEM: compensation of employees (NIPA Table 1.12, line 2)
  - WSA: wage and salary accruals (NIPA Table 1.12, line 3)
  - PRI: proprietor's income (NIPA Table 1.12, line 9)
  - RI: rental income (NIPA Table 1.12, line 12),
  - CP: corporate profits (NIPA Table 1.12, line 13)
  - NI: interest income (NIPA Table 1.12, line 18)
- Taxes

– TPI: taxes on production and imports (NIPA Table 3.1, line 4)
– CT: taxes on corporate income (NIPA Table 3.1, line 5)
– CSI: contributions to Social Security (NIPA Table 3.1, line 7)
– PIT: federal, state, and local taxes on personal income (NIPA Table 3.2, line 3 plus NIPA Table 3.3, line 4)
– PRT: state and local property taxes (NIPA Table 3.3, line 9)

Data for, and construction of, other basic variables. Real GDP is nominal GDP over the price deflator (NGDP / DEF). Government spending to GDP is the GSP divided by $NGDP$ series. For government debt we start with data on nominal "Debt held by the public", which excludes debt held by other government departments. This is available from FRED (series FYGFDPUB) from 1939 onwards, and we extend back to earlier years using data from the CBO.[13] The data is annual year-end debt. We convert this to real debt by dividing by the GDP deflator.

For the local projection estimation, we use defense news shocks from Ramey and Zubairy (2018), which we convert from quarterly to annual by dividing the sum of news within the year by the total potential GDP of that year. The time spans of our raw data series are as follows. Our NIPA data run from 1929 to 2024, as does our debt data once extended with the CBO data. The news shock data runs from 1890 until 2016. We thus have complete coverage for 1929 to 2016 that we use to estimate the impulse-responses. Next, we turn to describing the construction of our tax series. The procedure closely follows Clymo, Lanteri, and Villa (2023). Nevertheless, we still explain it here in detail.

First step: Personal Income Tax. The personal income tax is a key tax in the US, which applies to income which will be classified as either labor or capital income in our model. Hence, a first step is to measure the average personal income tax in the data. In the data we directly measure before-tax personal income, PI_t, and personal income taxes paid, PIT_t, so measuring the personal income tax rate is simply done as $\tau_{p,t} = PIT_t/PI_t$. The personal income tax in the data is

$$\tau_{p,t} = \frac{PIT_t}{LI_t + CI_t} \tag{69}$$

where total taxable personal income, PI_t, is split into personal labor income, PLI_t, and personal capital income, PCI_t. These are defined as

$$PLI_t = WSA_t + PRI_t/2 \tag{70}$$

$$PCI_t = PRI_t/2 + RI_t + CP_t + NI_t \tag{71}$$

Labor income is wages and salaries plus half of proprietors income. The half split is arbitrary and from Jones (2002), who finds results are robust to how proprietors income is split. Capital income is made up of four components: half of proprietors income, and then rental income, corporate profits, and interest income.

[13] Specifically, there is historical CBO data which we choose not to use as our main data since it is measured as debt to GDP and only given to the first decimal place, and only up to the year 2000. The two data series are almost identical in their overlapping years, and we splice them together. The CBO data is as the Economic and Budget Issue Brief "Historical Data on Federal Debt Held by the Public", available at https://www.cbo.gov/publication/21728.

Capital Tax. In the data we measure capital taxes paid, KIT_t, and total taxable capital income, TCI_t. So measuring the tax is simply done as $\tau_{k,t} = KIT_t/TCI_t$:

$$\tau_{k,t} = \frac{\tau_{p,t} PCI_t + CT_t + PRT_t}{PCI_t + PRT_t} \tag{72}$$

The numerator is capital taxes paid. This is capital taxes paid out of personal income, plus taxes paid on corporate income and property taxes. The denominator measures total capital income which adds property taxes, PRT_t, back to personal capital income ($TCI_t = PCI_t + PRT_t$). Property taxes are subtracted from profits and hence missing from personal capital income, and so are added back to the denominator to properly measure total capital income.

Labor Tax. In the data we measure labor taxes paid, LIT_t, and total taxable labor income, TLI_t. So measuring the tax is simply done as $\tau_{l,t} = LIT_t/TLI_t$:

$$\tau_{l,t} = \frac{\tau_{p,t} PLI_t + CSI_t}{CEM_t + PRI_t/2} \tag{73}$$

The numerator is total income taxes. These come from two sources. Firstly, personal labor income is taxed at rate $\tau_{p,t}$. Secondly, there are additional contributions to social security, CSI_t, which are not taxed as personal income. The denominator is total labor income, which is total labor compensation, CEM_t, plus half of proprietor income ($TLI_t = CEM_t + PRI_t/2$).

Consumption Taxes. We do not use consumption taxes in our model or empirical exercises, but include details on how to construct consumption taxes using the Jones (2002) method here for reference. In a model, we would think of total consumption expenditure as $CS_t = (1 + \tau_{c,t})C_t$, where C_t is the amount of real good that is bought and CS_t is the total spending. The data gives CS_t and the tax bill, $CTAX_t = \tau_{c,t}C_t$, giving $\tau_{c,t} = CTAX_t/C_t = CTAX_t/(CS_t - CTAX_t)$:

$$\tau_{c,t} = \frac{TPI_t - PRT_t}{PCE_t - (TPI_t - PRT_t)} \tag{74}$$

The numerator is taxes on production and imports (TPI_t) less state and local property taxes (PRT_t). Production taxes are equivalent to consumption taxes in the standard model. Property taxes are included in production taxes in the data, but are better thought of as capital taxes, so are subtracted and counted in the capital tax instead. The denominator is consumption spending before taxes: PCE_t is personal consumption expenditure, which includes taxes, so the tax is subtracted.

B.4 Impulse Responses to Military Spending Shocks

In this section, we provide details of the impulse responses presented in Section 5.1. We are interested in the dynamics of taxes and debt to an exogenous increase in government spending, which we identify as the defense news shocks of Ramey and Zubairy (2018). They use a narrative approach to measure announced planned changes in government defense spending, as a fraction of potential GDP. In their work, they study the response of GDP to this shock to measure government spending multipliers, using local projections (Òscar Jordà, 2005). We adapt their approach to instead measure the response of fiscal instruments. Specifically, we use the actual values of the defense news shock as an instrument and apply local projection to estimate the impact on taxes and debt.

Our baseline specification, adapted from Ramey and Zubairy (2018), is as follows:

$$x_{t+h} - x_{t-1} = \alpha_h + A_h \cdot z_t + \beta_h \cdot Z_t + \phi \cdot \text{trend}_t + \epsilon_{t+h}, \qquad \text{for } h = 0, 1, 2, ..., H. \tag{75}$$

For any left-hand side variable x, we are regressing the forward difference h periods ahead, $x_{t+h} - x_{t-1}$, on the military spending shock, z_t, and a set of controls, Z_t. Our left-hand side variables include i) government spending to GDP, ii) federal debt to GDP iii) the level of labor taxes, iv) level of capital taxes, v) level of consumption taxes. Each is regressed separately, giving $5 \times H$ regressions with associated coefficients. We control for a trend which is a fourth-order polynomial of time. Z_t are the controls used in a typical local projection setup. In particular, Z_t consists of lags of all of the three left-hand-side variables and the shock z_t. Since we are using yearly data, we use two lags. We use robust standard errors.[14] Our impulse responses plot the coefficients A_h for each variable, multiplied by a scaling factor that is common to each variable. The scale is chosen to create an increase in government spending that is equivalent to an increase following a two-standard-deviation shock in the g_t AR(1) process estimated in Section 5.1.

Notice that the regression above is a simple OLS regression, but has an instrumental variables flavor since the independent variable of interest, z_t, is considered as an exogenous shock due to the narrative identification. Since government spending itself is one of the variables regressed on this variable we are automatically performing an effective "first stage" regression to check that total government spending does rise in response to the military spending shock. The remaining variables then investigate how the rest of the economy responds to this military-spending-induced rise in government spending.

Our results use all available data to maximize power. Accordingly, we use the full dataset from 1929 to 2016 for our regressions. The estimates using the full dataset are heavily influenced by the early military buildups in the twentieth century, as pointed out in Ramey and Zubairy (2018). Nevertheless, the use of the full sample allows us to obtain precise estimates of an exogenous increase in government spending.

[14] Specifically, the estimations are run in Stata using the ivreg2 package with options robust and bw(auto).