

Government Debt Management and Inflation with Real and Nominal Bonds*

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Abstract

Can governments use real bonds such as Treasury Inflation-Protected Securities (TIPS) to tame inflation? We propose a novel framework of optimal debt management with sticky prices and a government issuing nominal and real state-uncontingent bonds. A government debt portfolio with both nominal and real bonds helps completing markets unless the monetary policy stance renders them perfect substitutes. Under Full Commitment, the government borrows with nominal debt and accumulates real assets, to be able to use inflation to smooth taxes. With No Commitment, the government portfolio favors real bonds to strategically prevent future governments from monetizing debt ex-post. Quantitatively, our model with No Commitment is consistent with the small and persistent TIPS share in U.S. data. A higher TIPS share mitigates the commitment friction, and effectively curbs inflation.

Keywords: Government Debt Portfolio, TIPS, Incomplete Markets, Inflation Risk, Limited Commitment, Time-consistency, Markov-perfect Equilibria, Recursive Contracts, Fiscal-Monetary Interactions.

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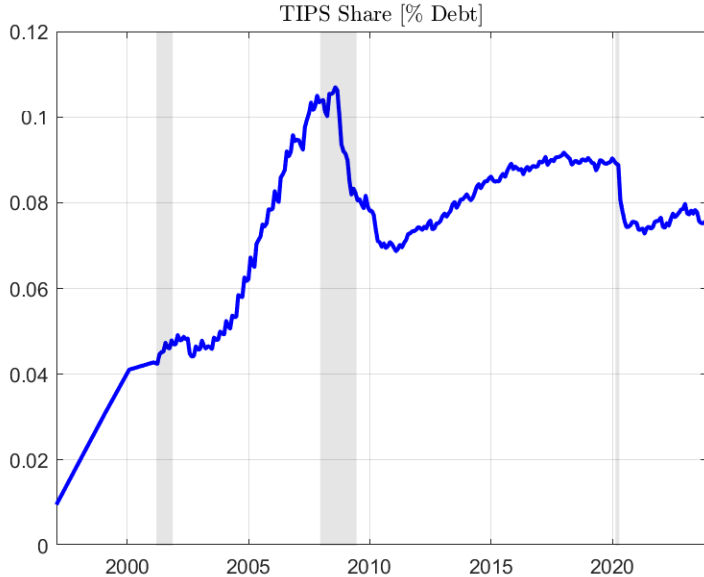
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1 Introduction

Economic policymakers in the U.S. face unprecedented challenges. For one, inflation concerns have returned. Indeed, after edging up to close to a 40-year high of around 8 percent in 2022, the current annual inflation rate in the U.S. remains persistently above the Federal Reserve’s target, with core inflation around 3 percent. At the same time, U.S. government debt has reached record levels and is projected to approach 175 percent of GDP by 2050, according to recent estimates from the Congressional Budget Office (CBO). In situations with such elevated debt levels, governments and central banks may be tempted to restore budget balance by monetizing debt, thereby exacerbating and creating persistent inflationary pressure.

In this paper, we examine government debt management in the presence of inflation concerns. In particular, we ask: Can governments effectively utilize inflation-indexed bonds to stabilize inflation? As shown in figure 1, the share of Treasury Inflation-Protected Securities (TIPS) in the U.S. government debt portfolio has been low and stable at around 8 percent following an initial expansion. Can observed debt portfolios and the TIPS share reflect governments’ debt management when inflationary pressure looms?

Figure 1: U.S. TIPS SHARE



Notes: The figure shows Treasury Inflation-Protected Securities (TIPS) at face value as a share of total government marketable debt. Source: U.S. Department of the Treasury (<https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt/summary-of-treasury-securities-outstanding>). Appendix A summarizes further stylized facts linked to our analysis.

Starting from the simple observation that real or indexed debt cannot be inflated away

ex-post, we examine the government’s optimal debt portfolio when it can issue both nominal and real state-uncontingent bonds in a novel framework of optimal debt management in the presence of sticky prices. Nominal bonds can be inflated away ex-post, but are more expensive as their prices reflect elevated inflation expectations. Real bonds, on the other hand, are cheaper but constitute a commitment ex-post for the government. As a benchmark, we first consider debt management of a government that can commit to future policies under Full Commitment (FC), and then characterize the policies of a government that responds strategically to the actions of future governments under No Commitment (NC). We thus solve for both the Ramsey equilibrium and the optimal time-consistent debt management policy. We provide both analytical and quantitative results that help us interpret the recent U.S. macroeconomic and financial experience.

Our main result is that our model with a government that cannot commit to fiscal policy describes the current U.S. macroeconomic experience qualitatively and quantitatively well, with a moderate but positive allocation to real bonds and stable inflation, consistent with the data. In sharp contrast, the benchmark economy under Full Commitment prescribes a large leveraged and volatile portfolio and volatile inflation in which the government sells nominal bonds to finance the purchase of real bonds. Our quantitative results thus suggest that debt management frameworks with No Commitment realistically capture the relevant constraints governments face and thus provide a natural starting point for policy analysis. In this quantitatively realistic specification with No Commitment, raising the real share in the government’s debt portfolio effectively helps curb inflation as it reduces future governments’ incentives to inflate away outstanding debt. Raising the real debt share thus helps governments to commit to low and stable inflation rates, so that indexed debt arises as an effective tool to mitigate the commitment friction.

Specifically, we consider a setting in which the government has to finance exogenous stochastic expenditures either by levying distortionary labor taxes or by issuing debt, much in the spirit of the literature started in the seminal work of [Lucas and Stokey \(1983\)](#) on optimal fiscal and monetary policy. By considering an optimal mix of fiscal and monetary policy under incomplete markets, we build on [Siu \(2004\)](#), [Schmitt-Grohe and Uribe \(2004\)](#), [Faraglia, Marcet, Oikonomou, and Scott \(2013\)](#), and [Lustig, Sleet, and Yeltekin \(2008\)](#). Our contribution is to allow the government to simultaneously issue real and nominal state-uncontingent debt. Inflation has real costs because of nominal rigidities through sticky prices and it affects the interest rates as the monetary authority sets the nominal short-term interest rate by responding to inflationary pressure following a Taylor rule. Under Full Commitment, government debt management reflects one key friction, namely limited insurance of fiscal shocks because of bond market incompleteness. We then relax the assumption of full commitment, and add, more realistically perhaps, another friction in that a succession

of governments cannot commit to future policies with No Commitment. More precisely, we assume that the government can commit to repaying debt but not to future taxes, inflation and debt policies.¹ In this setting with two frictions, government debt management naturally emerges as a trade-off between insurance and incentives.

Intuitively, the government can exploit a debt portfolio consisting of both nominal and real bonds to smooth fiscal shocks to the extent that inflation is sufficiently volatile, so that these securities are not perfect substitutes. Indeed, a key force that emerges in our environment is that reducing the substitutability between real and nominal bonds by means of inflation effectively helps the government complete the markets, and thus allows to hedge fiscal shocks better. Accordingly, a key trade-off arises in that the government needs to balance the costs of inflation with those of fiscal distortions. In an inflationary world, nominal bonds provide an effective hedging instrument, and the government uses debt to smooth fiscal shocks in such a way that the market value of outstanding government liabilities declines and the value of government assets rises when expenditures rise and inflation spikes up. Under Full Commitment, we find that the government finds it optimal to borrow by issuing nominal bonds and holding a negative position in real bonds, or in other words, saving by investing in real bonds, in sharp contrast to the recent U.S. macroeconomic experience. When we prevent the government from saving, or, in other words, from investing in bonds issued privately by households, the real debt share falls to zero. In this case, which is routinely entertained in the literature (Lustig, Sleet, and Yeltekin, 2008), there is no more role for real bonds and the hedging role of nominal bonds dominates. With No Commitment, in contrast, future governments have incentives to monetize previously accumulated nominal debt ex-post, to which households respond by raising the current government’s borrowing costs ex-ante. In this situation, the current government finds it optimal to borrow using real debt so as to lower borrowing costs and mitigate future governments’ incentive to inflate nominal debt away. Reducing future governments’ temptation to create inflation provides a natural novel role for real bonds in governments’ debt portfolio. Therefore, the realistic optimal portfolio emerges as a trade-off between insurance and incentives in a quantitatively relevant way.

Critically, we show that the commitment friction goes a long way towards explaining the difference between the observed debt portfolios in the data and the optimal allocations under Full Commitment. We show analytically that the optimal policy with No Commitment is strategically biased, designed not only to smooth fiscal policy but also to best respond to future governments in order to reduce borrowing costs. A hedging portfolio with levered positions constitutes an expensive financial choice ex-ante and exacerbates the tension

¹Effectively, we rule out the possibility of defaulting on debt since, as pointed in Gennaioli, Martin, and Rossi (2014), sovereign defaults entail disproportionately larger real costs than not committing to other policy tools and, therefore, they are infrequent in financially developed economies. Instead, we view inflation as a means to achieve “partial default” in such economies.

posed by the lack of commitment ex-post. Quantitatively, focusing on symmetric Markov-perfect equilibria, we find that with No Commitment, the inclusion of indexed bonds in the government’s debt portfolio robustly and significantly lowers inflation. By reducing future governments’ incentives to monetize debt, real debt thus acts as a commitment device to keep inflation stable, and leads to a realistic debt portfolio with moderate, but positive and stable real bond holdings, balancing insurance and incentives. Notably, mitigating future governments’ temptation to monetize debt renders inflation and debt endogenously sticky, thereby reducing debt portfolio rebalancing. A stable real debt share thus emerges naturally with a stable inflation rate.

Solving for the optimal policy under both Full and No Commitment and incomplete markets poses significant computational challenges. Full Commitment problems encompass sequences of forward-looking constraints (i.e., implementability constraints) that can be made recursive at the expense of introducing additional state variables. Echoing the approach of [Marcet and Marimon \(2019\)](#), we formulate the recursive Lagrangian to solve for the time-inconsistent optimal policy. In scenarios where markets are incomplete, the Ramsey planner is tasked with monitoring all commitments made in preceding periods. We tackle this computational challenge by exploiting a machine learning algorithm, based on artificial neural networks, as proposed in [Valaitis and Villa \(2024\)](#). More specifically, we build on a version of the parameterized expectations algorithm ([den Haan and Marcet, 1990](#)) that uses neural networks combined with stochastic simulation. A detailed description of the solution algorithm and its accuracy can be found in [appendix C.1](#). Solving for the optimal policy under No Commitment and incomplete markets is also challenging since, not only we need to take into account inter-temporal strategic interactions among governments and private sector given numerous policy tools (e.g., nominal debt, real debt, taxes, and inflation), but also account for the possibility that the policy functions may not be differentiable, as pointed out by e.g. [Krusell, Martin, and Rios-Rull \(2006\)](#). We solve for the Markov-perfect equilibrium by building on a methodology that does not require differentiability, introduced by [Clymo and Lanteri \(2020\)](#). A detailed description of the solution algorithm under No Commitment can be found in [appendix C.2](#).

1.1 Related Literature

Our paper is most closely related to the growing literature that examines monetary-fiscal interactions in the presence of realistic debt market specifications, such as. Much as we do, [Elenev, Landvoigt, Shultz, and Nieuwerburgh \(2022\)](#); [Elenev, Faria-e Castro, and Greenwald \(2019\)](#); [Corhay, Kind, Kung, and Morales \(2023\)](#) consider the interactions between fiscal policy, monetary policy and the government debt portfolio in computationally challenging New Keynesian settings. While they focus on the maturity composition of the government’s

debt portfolio, we focus on the nominal versus real debt composition. Similarly, [Diamond, Landvoigt, and Sanchez \(2022\)](#) additionally consider nominal mortgage debt. [Dittmar, Hsu, Roussellet, and Simasek \(2025\)](#) and [Hsu, Li, and Palomino \(2020\)](#) consider the interactions between nominal and real debt in quantitatively realistic settings. In contrast to them, we examine the optimal composition of the government debt portfolio. Relatedly, [Bretscher, Hsu, and Tamoni \(2020\)](#); [Li, Zha, Zhang, and Zhou \(2022\)](#); [Cieslak, Li, and Pflueger \(2023\)](#); [Bandi, Bretscher, and Tamoni \(2023\)](#); [Fang, Liu, and Roussanov \(2025\)](#) examine the interactions between fiscal policy, monetary policy, and bond pricing in quantitatively rich settings, but abstract from the optimal design of fiscal policy.

Our approach to understanding the optimal composition of the government debt portfolio builds on the literature that considers optimal fiscal policy with state-uncontingent government debt under Full Commitment ([Barro, 1979](#); [Aiyagari, Marcet, Sargent, and Sappala, 2002](#); [Angeletos, 2002](#); [Buera and Nicolini, 2004](#)) and optimal fiscal policy under No Commitment ([Klein, Krusell, and Rios-Rull, 2008](#); [Debortoli, Nunes, and Yared, 2017](#)). Optimal fiscal policy under Full Commitment dictates that the planner’s tax smoothing concerns pin down the value of state-uncontingent debt over time ([Barro, 1979](#)). When the planner can choose among multiple maturities, then the planner manipulates the maturity structure in order to achieve the same tax-smoothing objectives. In particular, [Angeletos \(2002\)](#) proves that the manipulation of maturity structure allows the planner to replicate the complete markets allocations. Such a policy prescribes issuing long-term debt and accumulate assets in short maturities. Quantitatively, [Buera and Nicolini \(2004\)](#) and [Faraglia, Marcet, Oikonomou, and Scott \(2019\)](#) find that such a portfolio would require highly leveraged and volatile bond positions. [de Lannoy, Bhandari, Evans, Golosov, and Sargent \(2024\)](#) develop a general framework that allows the characterization of the main forces that shape an optimal government portfolio in terms of statistics that are functions of macro and financial market data only. This literature studies the optimal portfolio allocation among real bonds of different maturity. We contribute by characterizing an optimal portfolio of real and nominal bonds of the same maturity, together with an endogenous choice for inflation and taxes both under Full Commitment and No Commitment. In our paper, monetary-fiscal policy interactions also highlight a new role for inflation, whose volatility is used to make nominal and real bonds imperfect substitutes, allowing to minimize tax distortions.

The paper also contributes to the literature on the optimal mix of monetary and fiscal policy with nominal state-uncontingent debt ([Bohn, 1988](#); [Siu, 2004](#); [Schmitt-Grohe and Uribe, 2004](#); [Lustig, Sleet, and Yeltekin, 2008](#); [Faraglia, Marcet, Oikonomou, and Scott, 2013](#); [Leeper and Zhou, 2021](#)). [Bohn \(1988\)](#) makes the case for the use of nominal debt in a Full Commitment setting, arguing that it can be inflated away when negative shocks realize, thus allowing the planner to relax the implementability constraint. These benefits

are weighted against the negative effect on nominal debt price and the real costs of inflation. When inflation has real costs, as is typically the case in New-Keynesian models, the cost of the active use of inflation coming from nominal rigidities outweighs the benefits described in [Bohn \(1988\)](#) and the planner makes only a limited use of inflation for tax-smoothing purposes ([Siu, 2004](#); [Schmitt-Grohe and Uribe, 2004](#); [Faraglia, Marcet, Oikonomou, and Scott, 2013](#)). Other papers elaborate on this point. [Lustig, Sleet, and Yeltekin \(2008\)](#) show that when the planner can choose among multiple maturities of nominal bonds, it is optimal to issue nominal debt of the longest maturity available as the long maturity insulates the bond prices from the contemporaneous increase in inflation and allows for better hedging of fiscal shocks. [Lustig, Berndt, and Yeltekin \(2012\)](#) show empirically that governments only engage in fiscal hedging to a limited degree. [Hilscher, Raviv, and Reis \(2022\)](#), on the other hand, show empirically that the scope for monetizing government debt is limited. Others papers study how optimal inflation depends on the starting level of government debt ([Leeper and Zhou, 2021](#)) or whether the monetary authority follows the Taylor rule ([Faraglia, Marcet, Oikonomou, and Scott, 2013](#)).

The paper also contributes to the literature on optimal fiscal policy under No Commitment. We characterize and solve for the Markov-Perfect equilibrium introduced in [Klein, Krusell, and Rios-Rull \(2008\)](#), who consider an economy without government debt and instead study the optimal capital and labor taxes. [Debortoli, Nunes, and Yared \(2017\)](#) show that commitment frictions change the implications for the optimal maturity structure of real debt. In our setting, commitment frictions create incentives for future governments to monetize debt, which increases borrowing costs for the current government. We study analytically and quantitatively how the optimal real and nominal portfolio and inflation are affected by the commitment friction.

The literature on the optimal choice between nominal and real bonds is scarce. [Alvarez, Kehoe, and Neumeyer \(2004\)](#) and [Persson, Persson, and Svensson \(2006\)](#) look for the maturity structure of the real and nominal bond portfolio that limits the use of surprise inflation in a setting with lack of commitment. [Alvarez, Kehoe, and Neumeyer \(2004\)](#) do it in an extension of the [Lucas and Stokey \(1983\)](#) setting without price rigidities and find that in order to minimize incentives to use surprise inflation, the planner should only issue real bonds. [Persson, Persson, and Svensson \(2006\)](#) characterize the optimal real and nominal bonds portfolio that makes the Ramsey policy time-consistent in a richer monetary setting. Both [Alvarez, Kehoe, and Neumeyer \(2004\)](#) and [Persson, Persson, and Svensson \(2006\)](#) do it analytically in models without uncertainty. This is a key difference from our paper, where the time-consistent Markov-perfect solution for the bond portfolio is a result of commitment friction and the hedging motive in the presence of uncertainty. [Fleckenstein, Lustig, and Longstaff \(2014\)](#) explore the relative pricing of nominal and real bonds, while [Pflueger and](#)

Viceira (2011) discuss the pricing of real bonds.

Lastly, the portfolio choice problem in our paper shares similarities with the sovereign debt literature, which examines the portfolio decisions between foreign and local currency debt (e.g., Bassetto and Galli, 2019; Ottonello and Perez, 2019; Bocola and Lorenzoni, 2020; Engel and Park, 2022). In these models, local currency debt serves as a hedge against shocks, as it can be eroded through inflation or currency devaluation, with its price reflecting these possibilities. This body of literature typically views hedging benefits as the ex-post use of inflation (or currency devaluation) in response to shocks. In contrast, our paper investigates the optimal use of real and nominal bonds, under both Full and No Commitment, within a closed New-Keynesian economy featuring risk-averse lenders, endogenous labor decisions, and distortionary taxation, with a focus on tax smoothing. While, the trade-offs in the No Commitment setting resemble those in Ottonello and Perez (2019); however, in our setting, the current planner must account for future planners' policies on inflation, taxes, and bonds, as all of these factors directly influence bond prices, labor, and output.

The paper is organized as follows. Section 2 describes the model environment. Section 3 presents how a benevolent government can finance its expenditures in a two-date setting under different assumptions on the government's commitment technology. Section 4 lays out the government's optimal debt management problems under Full Commitment and No Commitment in an infinite horizon setting. Section 5 presents the quantitative results from the infinite horizon model. Section 6 concludes.

2 Model

In this section, we describe our baseline environment with state-uncontingent nominal and real bonds. In this basic setting, the key friction is the lack of state-contingent bonds. That is, the value of outstanding debt at time t is independent of the realization of the shock at time t but, instead, measurable with respect to $t-1$. If state-contingent bonds were available, i.e. bond markets were complete, the trade-off between nominal and real bonds would not be meaningful. We embed this friction into a New Keynesian model with sticky prices à la Rotemberg (1982). A benevolent government needs to finance an exogenous stochastic expenditure stream and aims at doing so maximizing household welfare. As we describe in detail in sections 3 and 4, we start by assuming that the government operates under Full Commitment in the spirit of Lucas and Stokey (1983), and then relax that assumptions by introducing a second friction, namely the lack of commitment.

2.1 Environment

We consider a stochastic production economy populated by a continuum of identical households, a continuum of identical firms, a central bank, and a government. Time is discrete and infinite, $t = 0, 1, 2, \dots$

Preferences. Households rank streams of consumption c_t and leisure l_t according to the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(l_t)], \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor, and $u(\cdot)$ and $v(\cdot)$ are differentiable functions such that $u_c > 0$, $u_{cc} < 0$, $v_l > 0$, $v_{ll} < 0$.

Technology. A continuum of perfectly competitive intermediate firms, indexed by $i \in [0, 1]$, produces output through a linear production function $F(h_i)$, where hours worked are the only input. Intermediate goods are sold at a price $P_{i,t}$ to the final good producer. Aggregate output is given by $Y_t = A \cdot h_t$.

Resources. The resource constraint of the economy is given by

$$c_t + \Phi_t + g_t = Y_t, \quad (2)$$

where $h_t = 1 - l_t$ is labor, and g_t is an exogenous stochastic stream of government expenditures. Furthermore, we follow [Rotemberg \(1982\)](#) and assume each firm can set prices $P_{i,t}$ incurring the following convex quadratic reduced-form adjustment cost

$$\Phi_t = \frac{\varphi}{2} \cdot (\pi_t - \pi)^2,$$

where $\pi_t \equiv P_{i,t}/P_{i,t-1}$ denotes inflation, and π is the inflation target of the central bank.

Shocks. We assume that g_t follows an AR(1) process in logs. We denote by $g^t \equiv \{g_0, g_1, \dots, g_t\}$ a history of realizations of government spending. To simplify notation, we avoid explicitly denoting allocations as functions of histories g^t , but it is understood that c_t , and l_t are measurable with respect to g^t .

Households demand consumption goods, supply labor, and trade: (i) claims $S_{i,t}$ to the firm's i dividend $d_{i,t}$, (ii) nominal and (iii) real state-uncontingent government bonds denoted as B_t and b_t , respectively.² To simplify notation, we avoid explicitly denoting bonds as

²More precisely, B_{t+1} and b_{t+1} denote the amounts issued at time t divided by the price level at time t .

functions of histories g^{t-1} , but it is understood that B_t , and b_t are measurable with respect to g^{t-1} . The household budget constraint reads

$$c_t + Q_t B_{t+1} + q_t b_{t+1} + \int p_{i,t} S_{i,t+1} di = (1 - \tau_t) w_t h_t + \frac{B_t}{\pi_t} + b_t + \int (p_{i,t} + d_{i,t}) S_{i,t} di, \quad (3)$$

where Q_t is the price of nominal bonds, q_t is the price of real bonds, π_t denotes inflation, and $p_{i,t}$ is the price of the firm's claim to dividends.³ In equilibrium $S_{i,t} = 1$, since all households are identical.

2.2 Household and Firm Optimality

Households maximize utility (1) subject to their budget constraint (3). The intratemporal labor-consumption margin and the Euler equations for all savings instruments are

$$(1 - \tau_t) \cdot u_c(c_t) \cdot w_t = v_l(l_t), \quad (4)$$

$$u_c(c_t) \cdot Q_t = \beta \mathbb{E}_t u_c(c_{t+1}) \cdot \pi_{t+1}^{-1}, \quad (5)$$

$$u_c(c_t) \cdot q_t = \beta \mathbb{E}_t u_c(c_{t+1}) \cdot 1, \quad (6)$$

$$u_c(c_t) \cdot p_{i,t} = \beta \mathbb{E}_t u_c(c_{t+1}) \cdot [p_{i,t+1} + d_{i,t+1}]. \quad (7)$$

Intermediate firms set prices $P_{i,t}$ and hire labor to maximize the expected net present value of real dividends

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{u(c_t)}{u(c_0)} \cdot d_{i,t}, \quad \text{with} \quad d_{i,t} = \frac{P_{i,t}}{P_t} Y_{i,t} - w_t h_{i,t} - \Phi_t,$$

where the demand for the intermediate good is given by static profit maximization of the final good producer $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{1}{\nu}} Y_t$. In a symmetric equilibrium ($P_{i,t} = P_t$), the intermediate firm's profit maximization problem yields the New-Keynesian Phillips Curve (NKPC)

$$Y_t \cdot \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A\nu} \right) - \Phi_\pi(\pi_t) \pi_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \Phi_\pi(\pi_{t+1}) \pi_{t+1} \right] = 0. \quad (8)$$

³Notice that we did not allow households to trade risk-free bonds among themselves, since they are identical. In equilibrium these bonds would be in zero-net supply, rendering these bonds immaterial for equilibrium allocations.

2.3 Government

The government needs to finance spending g_t using labor income taxes and bonds, subject to the following budget constraint:

$$q_t b_{t+1} + Q_t B_{t+1} + \tau_t w_t h_t = g_t + b_t + \frac{B_t}{\pi_t}. \quad (9)$$

At date t , the government chooses current tax rate τ_t , and current bonds b_{t+1} and B_{t+1} , which are measurable with respect to g^t .

Given initial conditions b_{-1}, B_{-1} , the benevolent government chooses stochastic sequences of current tax rates τ_t and bonds B_t, b_t to maximize household utility (1).

2.4 Central Bank

We assume the central bank seeks to achieve an inflation target π by setting the nominal rate according to the following Taylor Rule:

$$Q_t^{-1} = \frac{1}{\beta} \pi \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi}. \quad (10)$$

In appendix B.1.1, we discuss the choice of having a separate monetary authority in detail.

2.5 Implementability Constraint

We now derive the implementability constraint of the government problem and follow [Lucas and Stokey \(1983\)](#) by taking the primal approach to the characterization of competitive equilibria, since this allows us to abstract away from bond prices and taxes.

The government budget constraint (9) can be combined with the private sector's first order conditions (4)-(6) to obtain a single implementability constraint for $t = 0, 1, \dots$ that reads:

$$\left(\frac{B_t}{\pi_t} + b_t \right) = s_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \left(\frac{B_{t+1}}{\pi_{t+1}} + b_{t+1} \right) \right], \quad (11)$$

where $s_t \equiv \left(1 - \frac{v_l(l_t)}{u_c(c_t)w_t} \right) \cdot w_t \cdot h_t - g_t$ denotes the government's surplus, and wage w_t can be obtained from the New-Keynesian Phillips Curve (8). Moreover, we substitute leisure and labor $l_t = 1 - h_t$ everywhere using the resource constraint (2). The implementability constraint (11) prices the government's liabilities $\frac{B_t}{\pi_t} + b_t$ as an expected net present value of surpluses. In line with the literature on fiscal policy design under market incompleteness (e.g., [Aiyagari, Marcet, Sargent, and Sappala, 2002](#); [Faraglia, Marcet, Oikonomou, and Scott,](#)

2019), we assume that there exist debts limits to prevent Ponzi schemes:

$$B_t \in [\underline{B}, \overline{B}], \quad b_t \in [\underline{b}, \overline{b}].$$

In our calibration, we let the bounds $(\underline{B}, \underline{b})$ be sufficiently low and $(\overline{B}, \overline{b})$ be sufficiently high so that they never bind in equilibrium. Note that we allow for $\underline{B} < 0$ and $\underline{b} < 0$, so that the government can lend to households, or, in other words, invest in private bonds. Such purchases can be ruled out by imposing $\underline{B} = 0$ or $\underline{b} = 0$. Forward substitution into equation (11) combined with a transversality condition then implies the following implementability condition:

$$\frac{B_t}{\pi_t} + b_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right].$$

We next turn to the government's optimal debt management problem which we will consider under different assumptions on its commitment technology. Before delving into the government's full dynamic optimization problem in section 4, we start by working out qualitatively the main economic trade-offs that governments face by means of simplified examples in the next section.

3 Debt Management: Inspecting the Mechanisms

In the following, we examine how a benevolent government can finance its expenditures optimally by a suitable choice of a government debt portfolio under different assumptions on the government's commitment technology. While most of our results are quantitative in nature and rely on simulations of our dynamic model that we present in sections 4 and 5, we find it instructive to work out the basic economic forces underlying these results by means of special one-period, two-date (0 and 1) cases of our full model. In these examples, we first consider debt management under Full Government Commitment, in the spirit of the seminal work of Lucas and Stokey (1983) to serve as a reference point. In such a setting, the actions of governments at $t = 0, 1$ can be captured by the choices of one benevolent, forward-looking planner in the spirit of Ramsey. We then relax that assumption to allow for a succession of governments, namely a government in power at $t = 0$ that responds strategically to the action of the government in power at $t = 1$ under No Commitment (NC).

3.1 Debt Management with Full Commitment

Under Full Commitment, the planner that represents governments seeks to find a government debt portfolio that maximizes intertemporal household welfare. Such a Ramsey problem can be thought of as a portfolio choice problem with incomplete markets in which the planner looks for the optimal government debt allocations to two securities, namely state-uncontingent nominal and real bonds together with inflation. Intuitively, optimal monetary-fiscal policy interactions highlight a new role for inflation to make nominal and real bonds imperfect substitutes, which then allows to create a portfolio mix that minimizes tax distortions.

To provide intuition about the economic forces that drive these allocations, we now examine stylized examples in which the objective of the planner is most transparent, namely specifications in which the economy is hit by a low and a high government expenditure shock. In such an environment, the planner's objective is to choose a portfolio of state-uncontingent bonds that replicates Arrow-Debreu securities. That is, the planner aims at implementing the complete markets allocation. We thus ask, how can the government use inflation fluctuations to replicate a portfolio of Arrow-Debreu securities?

Consider a one-period, two-date $t = 0, 1$ version of the model, where, for tractability, we specialize the utility over consumption to $u(c) = c$ and the disutility for labor to $v(h) = h^2/4$. Moreover, to begin with, we assume that at time 1 there are two realizations of the exogenous shocks, i.e. a low state (π_1^L, g_1^L) and a high state (π_1^H, g_1^H) , with $\pi_1^H \neq \pi_1^L$. Assume each realization realizes with a joint conditional probability $f(\pi_1, g_1 | \pi_0, g_0)$. Under the aforementioned conditions, the household optimality conditions imply $Q_0 = \beta \mathbb{E}_0 [\pi_1^{-1}]$, $q_0 = \beta$, and $h_t = 2(1 - \tau_t)w$. Firms take the exogenous sequence of prices as given and choose labor such that $w = A$, which we further assume is normalized to a unitary value.⁴ The resource constraint of the economy $c_t = h_t - g_t - \frac{\varphi}{2}(\pi_t - \pi)^2$ yields expressions for consumption. We follow the primal approach and substitute Q_0 , q_0 , and τ_t in the government budget constraints to get the following implementability constraints

$$\frac{B_0}{\pi_0} + b_0 + g_0 = h_0 \left(1 - \frac{h_0}{2}\right) + \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 + \beta b_1, \quad (12)$$

$$\frac{B_1}{\pi_1} + b_1 + g_1 = h_1 \left(1 - \frac{h_1}{2}\right). \quad (13)$$

The optimal policy under full commitment requires $\{h_0(g_0), h_1(g_0, g_1), B_1(g_0), b_1(g_0), \mu_0(g_0), \mu_1(g_0, g_1)\}$,

⁴In the spirit of the New-Keynesian Phillips Curve, this is equivalent to $w = A(1 - \nu)/\nu$ with $\nu = 1/2$.

such that welfare,

$$c_0 - \frac{h_0^2}{4} + \beta \mathbb{E}_0 \left[c_1 - \frac{h_1^2}{4} \right],$$

is maximized (where consumption is given by the resource constraints), subject to the implementability constraints (12) and (13). The first-order conditions with respect to nominal and real debt are

$$\mu_0 \mathbb{E}_0[1/\pi_1] = \mathbb{E}_0[\mu_1/\pi_1], \quad (14)$$

$$\mu_0 = \mathbb{E}_0[\mu_1]. \quad (15)$$

The first-order condition with respect to labor is

$$1 - h_t/2 - \mu_t(h_t - 1) = 0. \quad (16)$$

Proposition 1 formalizes the meaning behind equations (14) and (15), which are essentially tax smoothing conditions across states and time, as the shadow costs of tax distortions, μ , are “equalized” on average. In this simple example, the planner reaches complete markets as an equilibrium outcome.

Proposition 1 (*State-Smoothing with Nominal and Time-Smoothing with Real*). *Given initial conditions B_0, b_0, g_0, π_0 , optimal nominal and real debt management and tax management are such that smoothing of taxes and leisure is achieved across states with nominal debt*

$$\tau_1^H = \tau_1^L \text{ and } l_1^H = l_1^L, \quad (17)$$

where l_1^L and l_1^H denote leisure at time 1 in the low and high state, respectively. Moreover, smoothing of taxes and leisure is achieved across time with real debt

$$\tau_1^x = \tau_0 \text{ and } l_1^x = l_0, \quad (18)$$

where $x \in \{L, H\}$.

Proof. See appendix D.

Results (17) and (18) reveal that nominal debt is used for smoothing taxes across states, while real debt is used for smoothing taxes over time, to allow for full fiscal hedging with constant tax rates across states and time. In order to unpack this, we use the implementability

constraints (12) and (13) to express labor as a function of the portfolio choices:⁵

$$h_0 = 1 \pm \sqrt{1 - 2 \left(\frac{B_0}{\pi_0} + b_0 + g_0 - \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 - \beta b_1 \right)}, \quad (19)$$

$$h_1 = 1 \pm \sqrt{1 - 2 \left(\frac{B_1}{\pi_1} + b_1 + g_1 \right)}. \quad (20)$$

First, we substitute equation (20) in (17) to obtain the following cross-states smoothing condition

$$\frac{B_1}{\pi_1^L} + g_1^L = \frac{B_1}{\pi_1^H} + g_1^H, \quad (21)$$

which does not contain real debt. Second, we substitute equations (19)-(20) in equation (18) to obtain the following inter-temporal smoothing condition

$$\frac{B_0}{\pi_0} + b_0 + g_0 - \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 - \beta b_1 = \frac{B_1}{\pi_1^x} + b_1 + g_1^x, \quad (22)$$

where $x = \{L, H\}$. Note that since equation (22) needs to hold for both $x = \{L, H\}$, one can arbitrarily, and without loss of generality, choose the x that matches the realization of the shock at time 0, to further simplify

$$\frac{B_0 - B_1}{\pi_0} + b_0 - \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 = (1 + \beta)b_1. \quad (23)$$

These considerations lead us to formulate the following proposition, which pins down the optimal level of nominal and real debt.

Proposition 2 (*Optimal Nominal and Real Debt Management*). *Given initial conditions B_0, b_0, g_0, π_0 , optimal nominal debt management is such that*

$$B_1 = B_1^* \equiv \frac{g_1^H - g_1^L}{\pi_1^H - \pi_1^L} \cdot \pi_1^L \pi_1^H, \quad (24)$$

satisfies the intra-temporal (cross-states) smoothing condition (17). Given equation (24),

⁵Note that each implementability constraint yields two solutions for labor, one on each side of the Laffer Curve. Note that, although in principle one should check which one is optimal to determine the correct equilibrium allocations for consumption and leisure, it is immaterial for the bond optimal portfolio (i.e., both h_t solutions yield the exact same formulas for nominal and real bonds as in proposition 2).

optimal real debt management is such that

$$b_1 = b_1^* \equiv \frac{1}{1 + \beta} \left[\frac{B_0}{\pi_0} + b_0 - \left(\frac{1}{\pi_0} + \beta \mathbb{E}_0 \left[\frac{1}{\pi_1} \right] \right) B_1^* \right], \quad (25)$$

satisfies the inter-temporal smoothing condition (18).

Proof. See appendix D.

Note that if government expenditure shocks are *inflationary*, i.e. $\pi_1^H > \pi_1^L$ and $g_1^H > g_1^L$, then B_1^* is positive. Vice versa, if shocks are *deflationary*, i.e. $\pi_1^H < \pi_1^L$ and $g_1^H > g_1^L$, then B_1^* is negative. The sign for real debt depends on the initial amount of outstanding liabilities $B_0\pi_0^{-1} + b_0$. In order to gain intuition, we assume the government has zero net holdings of initial real liabilities, i.e. $B_0\pi_0^{-1} + b_0 = 0$. Under this condition, equation (25) reveals that b_1^* is negative with *inflationary* shocks and positive with *deflationary* shocks.

The intuition is simple. Holding a leveraged portfolio position enables the planner to achieve insurance without re-adjusting the debt structure. Under *inflationary* shocks, it is optimal to have positive nominal debt, as then inflation increases when a positive government expenditure shock hits, inducing the government liability to fall. This mechanism insures the government with an economic force that tends to relax the implementability constraint when it is most needed, by counterbalancing the high government expenditure shock with a falling liability. Moreover, with *inflationary* shocks, the position of real debt b_1^* should be negative, i.e. the government should accumulate real assets to smooth labor and taxes through time in high government expenditure inflationary times. The opposite applies with *deflationary* expenditure shocks. For example, in this case, the government chooses to optimally hold nominal assets that appreciate in periods of high government expenditures, helping to relax the implementability constraint. As empirically, government expenditures tend to be inflationary (Bohn, 1988), and thus the solution under Full Commitment suggests a counterfactual debt portfolio.

To conclude, in appendix B.1.1 we further enrich the example in this section by allowing the government to choose not only the debt portfolio, but also inflation. We find that some level of inflation volatility is optimal in this setting, while costly, because it allows the Ramsey planner to complete the market, and there are two equivalent welfare-optimal combinations of inflation with symmetric deflationary and inflationary shocks.

Next, we turn to a different, and perhaps more realistic, assumption on the commitment technology.

3.2 Debt Management with No Commitment

We now view the public sector as a succession of decision makers with *No Commitment* to future realized policies. Accordingly, we assume that there are two different governments in power at date 0 and 1. The government in power at $t = 0$ determines the initial level of the labor tax rate and inflation, together with the issuance of nominal and real non-contingent bonds that will be inherited by the government in power at $t = 1$. With that debt portfolio in place, the government in power at $t = 1$ can choose inflation optimally. In this setting, the government thus faces two frictions, namely an incentive friction with regards to future government policies on top of the incompleteness friction considered in the full commitment case. Naturally therefore, the optimal policy emerges from a trade-off between incentives and insurance.

Formally, we consider the same one-period model of as above, except that we assume the same utility as in the infinite-horizon model, i.e. utility for consumption $u(c) = \frac{c^{1-\eta_c}}{1-\eta_c}$, utility for leisure $v(l) = \chi l^{1-\eta_l}/(1-\eta_l)$, and a quadratic cost $\Phi(\pi_t) = \frac{\varphi}{2}(\pi_t - \pi)^2$. The government at $t = 1$ then takes as given the nominal, B_1 , and real, b_1 , bonds issued by the government in power at $t = 0$, as well as current expenditures g_1 . Formally, thus, the government at date 1 solves the following problem

$$\tilde{W}(B_1, b_1, g_1) = \max_{\pi_1, h_1} u(c_1) + v(1 - h_1)$$

subject to the implementability constraint (13) and the resource constraint $c_1 = h_1 - g_1 - \Phi_1$. Hence, the first-order condition with respect to π_1 of the government at date 1 is given by:

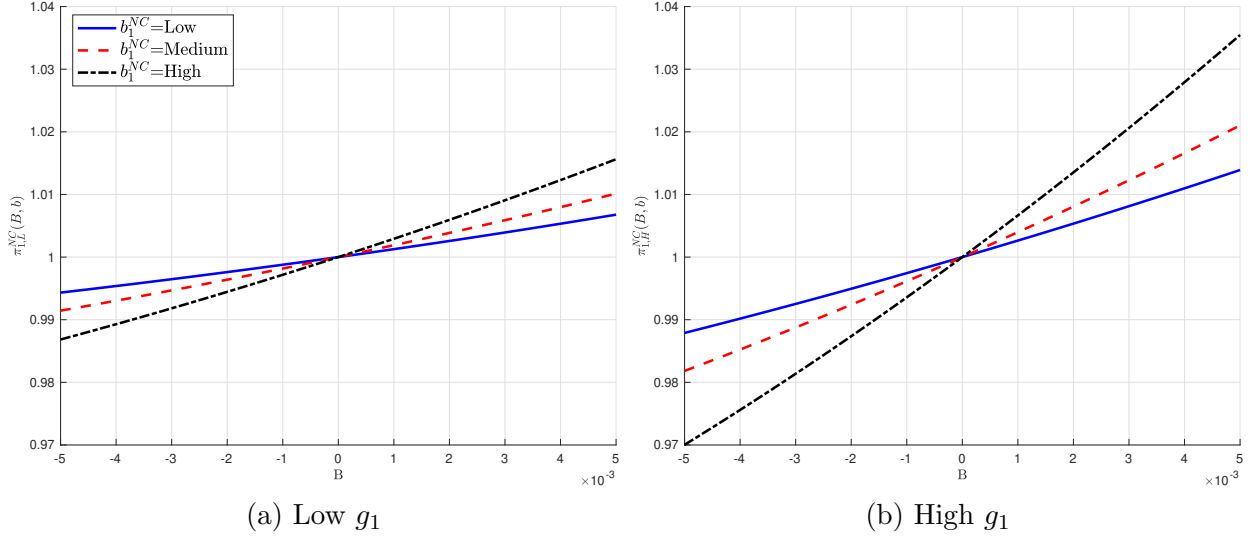
$$-u_c(c_1)\Phi_\pi(\pi_1) + \mu_1 \left(\frac{B_1}{\pi_1^2} + h_1 \frac{\partial \tau_1}{\partial \pi_1} \right) = 0. \quad (26)$$

This has an intuitive interpretation. On the one hand, the government at date 1 faces the marginal cost of nominal rigidities which, through the resource constraint, tends to lower consumption. On the other hand, the government internalizes the marginal benefit of inflating away nominal debt B_1 , which is inherited by the government in power at $t = 1$ and is a choice of the government in power at $t = 0$. Additionally, the government internalizes the effect of inflation on labor taxes. Figure 2 illustrates the inflation policy as a function of real and nominal debt for low and high realizations of government expenditure.⁶ The inflation policy is increasing in nominal debt and is more sensitive to the level of nominal debt when government expenditures at date 1 are high. Inflation also depends on the level of real debt.

⁶We solve the two-date No Commitment model using the following parameters: $\eta_c = 2, \eta_l = 1.8, \varphi = 0.1, \beta = 0.96, g \in \{0.1080, 0.1754\}$ with transition probability matrix $P = \begin{pmatrix} 0.55 & 0.45 \\ 0.45 & 0.55 \end{pmatrix}$ and $\pi = 1, \chi = 4.3276$.

Holding everything else fixed, a higher level of real debt means that the implementability constraint binds more, and the benefits of using inflation are larger. As a consequence, the sensitivity of inflation to nominal debt increases with the amount of real debt.

Figure 2: DATE TWO INFLATION POLICY FUNCTIONS



Notes: The figure shows the date two inflation policy function in terms of nominal and real debt. The left panel shows the policy at the low realization of g_1 and the right panel show the policy at the high realization of g_1 . X-axis denotes the nominal debt. Solid blue, dashed red and dot-dashed black lines show the policy function for different values of real debt.

Now we turn our attention to the government in power at date 0. This government chooses b_1 and B_1 in order to best respond to the government at date 1, taking the latter's policy as given. Formally, the government at date 0 solves the following problem

$$\max_{c_0, h_0, B_1, b_1} u(c_0) + v(1 - h_0) + \beta \mathbb{E}_0 \tilde{W}(B_1, b_1, g_1)$$

subject to the implementability constraint (12) and the resource constraint $c_0 = h_0 - g_0 - \Phi_0$.

In order to isolate the planner's trade-offs and to highlight the differences from the Full Commitment case, we derive the Generalized Euler Equations that neatly summarize the role of strategic interactions.⁷ The first-order conditions with respect to B_1 and b_1 , together

⁷Krusell, Martin, and Rios-Rull (2006) show that the value function under Markov-Perfect policy can contain jumps and consequently, may not be differentiable. We want to emphasize that we only use the first-order conditions to illustrate the economic trade-offs, but we do not use the Generalized Euler Equations to solve the problem numerically. Instead, we solve the model by directly maximizing the planner's value function.

with the envelope conditions, yield the following Generalized Euler Equations

$$\mu_0 \left(Q + \frac{\partial Q}{\partial B_1} B_1 + \frac{\partial q}{\partial B_1} b_1 \right) = \beta \mathbb{E}_0 \left[\frac{\mu_1}{\pi_1} \right], \quad (27)$$

$$\mu_0 \left(q + \frac{\partial Q}{\partial b_1} B_1 + \frac{\partial q}{\partial b_1} b_1 \right) = \beta \mathbb{E}_0[\mu_1], \quad (28)$$

which capture the intertemporal strategic interactions among governments and allow us to compare the policy trade-offs under No Commitment with the one-period model under Full Commitment in section 3.1. Note that each government's first-order conditions with respect to h_0 and h_1 yield expressions for μ_0 and μ_1 , equivalently to equation (16) in the Full Commitment case.

Expressions (27) and (28) reveal critical differences from their counterparts under Full Commitment (14) and (15), since they contain the strategic terms $\frac{\partial Q}{\partial B_1} B_1$ and $\frac{\partial Q}{\partial b_1} B_1$, $\frac{\partial q}{\partial B_1} b_1$ and $\frac{\partial q}{\partial b_1} b_1$, in addition to the “tax smoothing” terms involving μ . Through these additional terms, the government at date 0 internalizes the effects that its debt choice has on date 0 nominal and real rates. Consider, for example, the effect of nominal debt on the nominal bond price, $\frac{\partial Q}{\partial B_1}$. Expanding the $\frac{\partial Q}{\partial B_1}$ term, we have

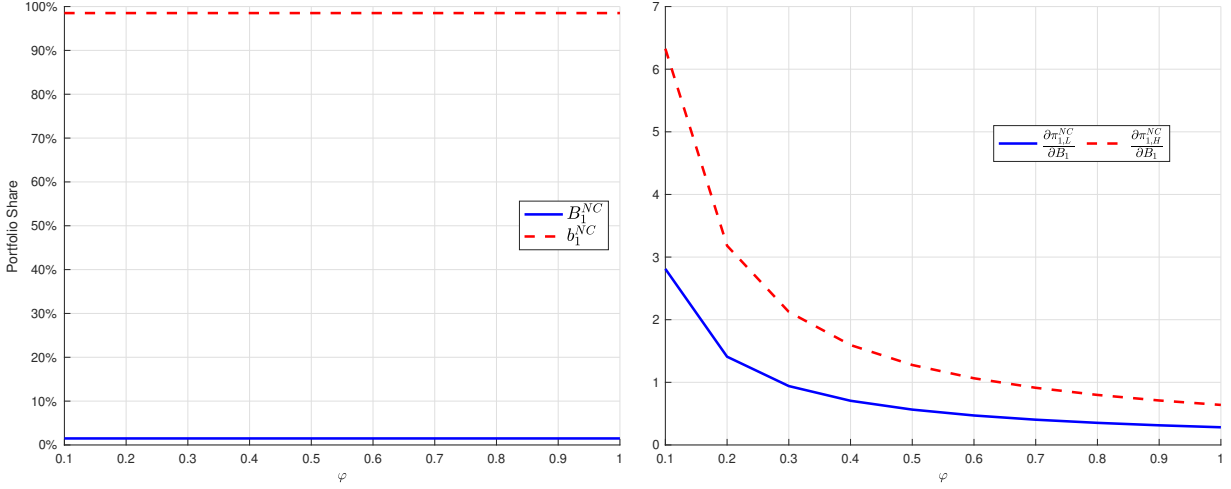
$$\frac{\partial Q}{\partial B_1} = \frac{\beta}{u_c(c_0)} \mathbb{E}_0 \left[\frac{u_{cc}(c_1)}{\pi_1} - \frac{u_c(c_1)}{\pi_1^2} \frac{\partial \pi_1}{\partial B_1} \right], \quad (29)$$

which shows that the sensitivity of nominal bond prices to the date-0 government's debt choice is, ceteris paribus, negatively related with the sensitivity of the date-1 government's inflation choice to the nominal debt outstanding. In other words, whenever households anticipate that the government at date 1 will resort to higher inflation, nominal bond prices tend to decline with rising nominal debt choices. Figure 2 shows that generally, $\frac{\partial \pi_1}{\partial B_1}$ is positive and higher nominal debt choice lowers nominal bond prices.

We now illustrate the trade-off between market incompleteness and the lack of commitment by solving the model by increasing the degree of nominal rigidity, controlled by the parameter φ . Intuitively, a higher degree of nominal rigidity should reduce the incentive to inflate away debt at date 1 and lowers the strategic bias terms in the Generalized Euler Equation, in this way altering the trade-off between the two frictions.

Figure 3 shows the optimal date 1 debt issuance and the associated bias terms for different values of φ . Note that the optimal policy under Full Commitment in section 3.1 prescribes issuing a leveraged portfolio position and, when the shocks are inflationary, the portfolio consists of nominal debt and real assets. The left panel of figure 3 shows that the optimal portfolio under No Commitment mainly consists of real bonds, while the nominal bond amount is positive, but close to zero. Essentially, the government at date 0 anticipates that

Figure 3: DATE ONE OPTIMAL PORTFOLIO AND CORRESPONDING INFLATIONARY BIAS



Notes: The figure shows equilibrium bond choices and strategic bias terms for different values of φ . The left panel reports the equilibrium bond choices B_1 and b_1 as a share of total debt portfolio. The right panel reports the strategic biases $\frac{\partial \pi_{1,L}^{NC}}{\partial B_1}$ and $\frac{\partial \pi_{1,H}^{NC}}{\partial b_1}$ at the equilibrium values of B_1 and b_1 .

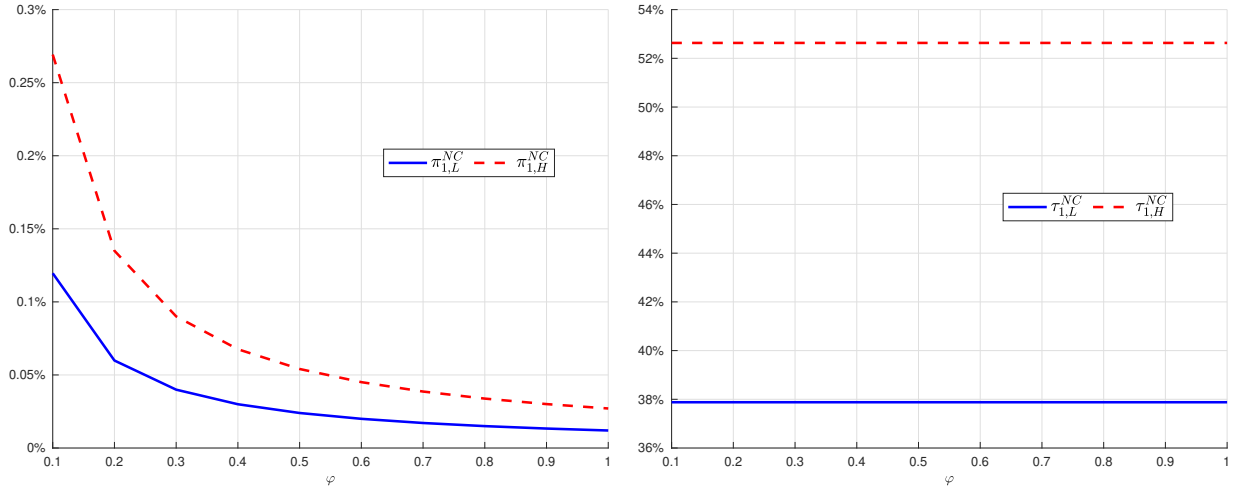
the government at date 1 will resort to inflation if the outstanding level of nominal debt is high, as illustrated in figure 2. The right panel of figure 3 shows the strategic bias terms at the equilibrium values of debt for high and low realizations of government expenditures at date 1. The figure suggests that the strategic bias terms in the Generalized Euler Equations are large and relevant, which is why the planner at date 0 optimally chooses a debt portfolio that minimizes the commitment rather than market incompleteness friction. The degree of nominal rigidity plays an important role in how tempting it is for the date 1 government to resort to inflation, which is seen from the fact that the strategic bias terms decrease in the value of φ .

These strategic interactions have implications for taxation. Indeed, equations (27) and (28) suggest that, under No Commitment, the government does not reach perfect fiscal hedging across states and time, even with just two realizations of shocks as in subsection 3.1. In fact, given the assumptions of subsection 3.1, equations (27) and (28) imply that $\mu_0 \neq \mu_1^L \neq \mu_1^H$ whereas, under Full Commitment, the planner was explicitly seeking to achieve $\mu_0 = \mu_1^H = \mu_1^L$ as implied by equation (17) of proposition 2. Indeed, the right panel of figure 4 shows two rather different values of taxes for the low and the high realization of g in date 1.

Again, this is a manifestation that the government at date 0 internalizes the effects that its nominal debt choice has on date 0 nominal rates, while internalizing the sensitivity of its choices on the future government's policy functions. In summary, the date 0 government is facing a trade-off between: (i) diminishing its nominal borrowing costs and (ii) smoothing fiscal policy. This tension drives the optimal portfolio allocations under the optimal

time-consistent policy: the hedging portfolio achievable with levered positions under Full Commitment is typically a sub-optimal choice under No Commitment. In fact, it would be an expensive financial choice ex-ante and would accentuate the tension posed by the lack of commitment ex-post. That is, it would give an incentive to the future government to use inflation excessively ex-post. This can be seen in the left panel of figure 4. The government at date 1 responds to shocks by resorting to inflation, the more so in high expenditure states and when nominal rigidities are low. However, in equilibrium, such use of inflation is limited. For example, inflation increases by 0.25% in high expenditure states for the value of $\varphi = 0.1$. Indeed, at time 0, the government issues minimal nominal debt as a result of internalizing such use of inflation. In this sense, the commitment friction provides a rationale for the results in Barro (2003) that governments should mostly issue real debt.

Figure 4: DATE TWO INFLATION, AND TAXES



Notes: The figure shows equilibrium inflation and taxes, solved for different values of φ . The solid blue line corresponds to the NC equilibrium value in the low realization of the g shock. The dashed red line corresponds to the NC equilibrium value in the high realization of the g shock.

3.2.1 Comparison with Single-Bond Economies

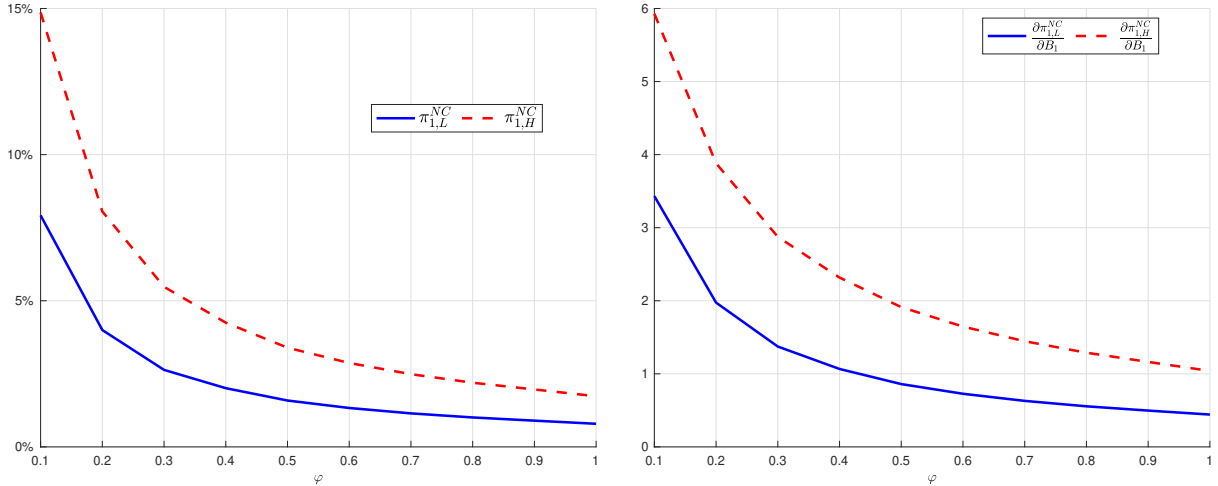
We now compare the benchmark two-bond economy to two restricted environments in which only one instrument is available: (i) TIPS only, and (ii) nominal bonds only.

Real Bonds Only. When only TIPS are available, the equilibrium is straightforward. Inflation is identically anchored at its steady state regardless of the degree of nominal rigidity. Since real bonds are indexed, inflation provides no fiscal relief; the only role of inflation in this environment would be to generate resource losses through Rotemberg adjustment costs. Hence, the government at date 1 optimally sets inflation at its target. Fiscal adjustment at date 1 is therefore implemented entirely through labor taxation.

Nominal Bonds Only. In contrast, restricting the government at date 0 to issue only nominal bonds fundamentally changes the equilibrium dynamics. The date-0 government must then rely exclusively on nominal debt to finance expenditures (or to roll over any initial liabilities), which creates a strong incentive for the date-1 government to use inflation ex post to erode its real value. As the planner at date 0 issues significant amounts of nominal debt, inflation at date 1 thus becomes an active policy instrument for relaxing the implementability constraint.

Figure 5 documents two implications. First, the derivatives of inflation with respect to nominal debt—both in low and high expenditure states—are substantially larger than in the benchmark economy with both instruments (see figure 3). This indicates that the inflationary bias is magnified when the portfolio cannot be tilted toward real bonds. Second, inflation itself rises substantially relative to the two-bond case: the left panel of figure 5 shows systematically higher equilibrium inflation rates than those in figure 4.

Figure 5: DATE TWO INFLATION AND INFLATIONARY BIAS WITH ONLY NOMINAL DEBT



Notes: The figure shows the equilibrium inflations and their corresponding strategic bias terms for different values of φ when only nominal debt is available to the date-0 government. The solid blue line corresponds to the NC equilibrium value in the low realization of the g shock. The dashed red line corresponds to the NC equilibrium value in the high realization of the g shock.

Taken together, these comparisons underscore the role of portfolio composition in mitigating the lack of commitment. With real bonds only, inflation is fully stabilized but tax smoothing is limited. With nominal bonds only, inflation helps smooth taxes across states but exacerbates the lack of commitment problem. In principle, the two-bond model nests these two cases: it allows the date-0 government to tilt the portfolio toward real bonds, thereby reducing commitment frictions while preserving some capacity for intertemporal fiscal insurance. In practice, as shown by the left panel of figure 3, the government at date 0 endogenously chooses policies quantitatively close to those in the real-bonds-only case.

4 Dynamic Debt Management

While the examples in the previous section provide qualitative guidance regarding optimal policies in the Full Commitment and the No Commitment case, we now return to the full dynamic model. In this section, we lay out governments' optimal debt management problems in the Full Commitment (in the spirit of [Lucas and Stokey 1983](#)) and No Commitment cases in the general case. The latter setting gives rise to time-consistent policies in that future governments will have no incentives to renegotiate policies ex-post. We turn to a detailed quantitative evaluation of these policies in section 5.

4.1 Full Commitment

Under Full Commitment, the government chooses stochastic sequences of allocations and prices $\{c(g^t), w(g^t), \pi(g^t)\}_{t=0}^{\infty}$ and stochastic sequences of nominal and real state-uncontingent bonds $\{B(g^{t-1}), b(g^{t-1})\}_{t=0}^{\infty}$ to maximize the household's utility (1), subject to the implementability constraint (11), with multiplier μ_t , the New-Keynesian Phillips Curve (8), with multiplier λ_t^π , the Taylor Rule (10), with multiplier λ_t^T , and the bounds $(\bar{B}, \underline{B}, \bar{b}, \underline{b})$, with multipliers $(\bar{\Lambda}_t, \underline{\Lambda}_t, \bar{\lambda}_t, \underline{\lambda}_t)$, and given $\mu_{-1} = \lambda_{-1}^T = \lambda_{-1}^\pi = 0$.⁸

The first order conditions with respect to nominal bonds B_t and real bonds b_t are

$$\mu_t \cdot \mathbb{E}_t [\pi_{t+1}^{-1} \cdot u_c(c_{t+1})] = \mathbb{E}_t [\mu_{t+1} \cdot u_c(c_{t+1}) \cdot \pi_{t+1}^{-1}] + \beta^{-1} (\bar{\Lambda}_t - \underline{\Lambda}_t), \quad (30)$$

$$\mu_t \cdot \mathbb{E}_t [u_c(c_{t+1})] = \mathbb{E}_t [\mu_{t+1} \cdot u_c(c_{t+1})] + \beta^{-1} (\bar{\lambda}_t - \underline{\lambda}_t). \quad (31)$$

Note that equations (30) and (31) pin down dynamics for the recursive multiplier μ_t on the implementability constraint, similar in spirit to those in [Aiyagari, Marcet, Sargent, and Sappala \(2002\)](#), reflecting market incompleteness. The multipliers μ_t capture the shadow value of relaxing the implementability constraint, as the government may have to resort to distortionary taxation to balance the budget. The first-order condition with respect to the wage w_t is

$$A\nu \cdot \mu_t \cdot u_c(c_t) + \lambda_t^\pi = 0. \quad (32)$$

This condition captures the trade-off between the marginal effect that wage has on the implementability constraint (11) through the government's surplus and the New-Keynesian

⁸Specifically, the planner maximizes (1) subject to constraints (11), (8), (10) multiplied by the marginal utility of consumption $u_c(c_t)$.

Phillips Curve (8). The first-order condition with respect to inflation π_t is⁹

$$\frac{v_l(l_t)}{u_c(c_t)} \frac{\Phi_\pi(\pi_t)}{A} - \lambda_t^\pi \mathcal{H}_t - \frac{\lambda_{t-1}^\pi \mathcal{K}_t}{u_c(c_{t-1})} = \mu_t \frac{\partial s_t}{\partial \pi_t} + B_t \frac{\mu_t - \mu_{t-1}}{\pi_t^2} + \left(\frac{\pi_t}{\pi}\right)^{-\phi_\pi - 1} \frac{\lambda_t^T \phi_\pi}{\pi^2} - \frac{\lambda_{t-1}^T}{\beta \pi_t^2}. \quad (33)$$

The terms on the left-hand side of equation (33) capture the marginal effects of the nominal rigidities. In particular, the first term captures the marginal increase in labor due to a marginal increase in inflation through the resource constraint. The additional two terms that contain λ_t^π and λ_{t-1}^π captures the inter-temporal effects of the nominal rigidities on the NKPC. On the right-hand side, the first term captures the marginal effect of inflation on primary surplus and the second term of relaxing the implementability constraint inter-temporally. Finally, the remaining two terms on the right-hand side, that contain λ_t^T and λ_{t-1}^T , capture the constraints imposed by the Taylor rule.

The remaining first-order condition with respect to consumption c_t is reported in appendix C.1, and is given by equation (75).

Special Case. We consider a special case with risk-neutral households $u(c_t) = c_t$ and no lending limits, $\underline{\lambda}_t = \underline{\Lambda}_t = 0$. In this case, equation (31) becomes

$$\mu_t = \mathbb{E}_t[\mu_{t+1}] + \beta^{-1} \bar{\lambda}_t.$$

Since the Lagrange multiplier on the borrowing limit is non-negative $\bar{\lambda}_t \geq 0$, then $\mu_t \geq \mathbb{E}_t[\mu_{t+1}]$. We can use the submartingale convergence theorem: μ_t converges almost surely.¹⁰ This last condition and result are equivalent to Aiyagari, Marcet, Sargent, and Sappala (2002): in the long-run the government accumulates enough *real assets* that it never needs to tax again. Differently from Aiyagari, Marcet, Sargent, and Sappala (2002), the simultaneous presence of both nominal and real bonds requires an extra condition to be satisfied. This is given by the optimal policy for nominal government debt (30). If, for illustration purposes, we further assume there are no lending and borrowing limits - i.e., $\underline{\lambda}_t = \underline{\Lambda}_t = \bar{\lambda}_t = \bar{\Lambda}_t = 0$ - we can combine (30) and (31) to get

$$\text{Cov}_t(\pi_{t+1}^{-1}, \mu_{t+1}) = 0. \quad (34)$$

Formally, in this special case, this condition states that under risk neutrality and in the absence of lending and borrowing limits, it is ex-ante optimal for the government to create policies such that, averaging over all future states, it does not have to resort to in-

⁹Define $\mathcal{H}_t \equiv \left(\frac{\nu-1}{\nu} + \frac{w_t}{A\nu}\right) \Phi_\pi(\pi_t) - \mathcal{K}_t$ and $\mathcal{K}_t \equiv \varphi(2\pi_t - \pi)$.

¹⁰To be precise, the proof in Aiyagari, Marcet, Sargent, and Sappala (2002) for the convergence of μ_t requires introducing lump-sum rebates that guarantee that $\mu_t \geq 0$.

flation to relax the implementability constraint. In contrast, in an environment such that $\text{Cov}_t(\pi_{t+1}^{-1}, \mu_{t+1}) > 0$, the marginal cost paid tomorrow of issuing one unit of nominal debt today would exceed the marginal benefit of relaxing the current implementability constraint by borrowing with that same unit of nominal debt, as suggested by (30). In our environment, the government has the outside option to use real debt. This outside option provides the government with the opportunity to relax the implementability constraint today, with marginal benefit μ_t , by just paying a marginal cost tomorrow that does not depend on inflation $\mathbb{E}_t[\mu_{t+1}]$, so the government would rather relax the implementability constraint today by using real debt. The government thus finds it optimal to relax the current implementability constraint by issuing real debt until it is left indifferent between using either real or nominal.

Condition (34) then suggests a new role for inflation due to the fact that both real and nominal bonds are simultaneously available to the government. This implies that in such a case, inflation is optimally used to reduce the substitutability between real and nominal bonds as this effectively helps the government complete the markets, which in turn helps smoothing fiscal shocks. In our model, a key trade-off thus emerges in that the government needs to balance the costs of inflation with those of fiscal distortions. This can be contrasted to the idea that the benefit of issuing nominal debt arises from the possibility of inflating it away, which was first proposed in Bohn (1988). In Bohn (1988), inflation is used to relax the government's budget constraint in times when it binds more. Such use of inflation is optimal because the covariance term, analogous to (34) is typically positive.¹¹ Our framework thus identifies a novel force driving inflation and another reason why governments should issue nominal debt. We note that in a calibrated version of our full model, the government may also want to relax the fiscal burden by issuing nominal bonds in bad times, due to risk aversion, for example.

4.2 No Commitment

In the setting with No Commitment, we focus on a symmetric Markov-perfect equilibrium and denote the state of the economy at time t by $x_t \equiv (B_t, b_t, g_t)$. In this environment, let all future governments set their policy according to functions $\tilde{c}(x)$, $\tilde{h}(x)$, $\tilde{w}(x)$, $\tilde{B}(x)$, $\tilde{b}(x)$, and $\tilde{\pi}(x)$. The current government takes these policies of future governments as given, and understands that it can only influence future governments' actions through its current policies. As these functions are unknown, we are looking for the solutions of a fixed-point problem. By construction then, as the policies are time-invariant functions of the current

¹¹Unlike in our paper, in Bohn (1988) nominal and real bonds are not chosen simultaneously and inflation is not chosen optimally. These differences result in a positive covariance between inflation and the shadow value of reducing government debt (see equation (15) in Bohn 1988). In this sense, in our framework the planner issues nominal debt and actively uses inflation, but for different purposes.

state x_t , these policies are time-consistent.

In this context, let $\tilde{W}(x)$ be the present discounted value of government utility (1) as associated with the policy functions introduced above, given the state of the economy x . With this notation at hand, the government in power at time t chooses allocations and wage (c, h, w) , with $h = 1 - l$, as well as policies (B', b', π) to maximize

$$u(c) + v(l) + \beta \mathbb{E} \left[\tilde{W}(x') | x \right], \quad (35)$$

subject to the resource constraint

$$Ah - c - g - \Phi(\pi) = 0, \quad (36)$$

with associated multiplier λ , and the implementability constraint

$$u_c(c) \cdot s + \mathbb{E} \left[\beta u_c(\tilde{c}(x')) \cdot \left(\frac{B'}{\tilde{\pi}(x')} + b(x') \right) | x \right] - u_c(c) \left(\frac{B}{\pi} + b \right) = 0, \quad (37)$$

with multiplier μ and where s denotes the government's surplus as specified in equation (11), the New-Keynesian Phillips Curve (8)

$$\mathcal{N}(x) \equiv u_c(c) \left(Y \cdot \left(\frac{\nu - 1}{\nu} + \frac{w}{A\nu} \right) - \Phi_\pi(\pi)\pi \right) + \mathbb{E} [\beta u_c(\tilde{c}(x')) \cdot \Phi_\pi(\tilde{\pi}(x'))\tilde{\pi}(x') | x] = 0, \quad (38)$$

with multiplier λ^π and the Taylor Rule (10)

$$\mathcal{T}(x) \equiv \bar{\pi}^{1-\phi_\pi} \pi^{\phi_\pi} \mathbb{E} [u_c(\tilde{c}(x')) \cdot \tilde{\pi}(x')^{-1} | x] - u_c(c) = 0, \quad (39)$$

with multiplier λ^T .¹² In this setting, we assume that the planner can commit to repaying debt but not to future taxes, inflation and debt policies, as is common in the literature on fiscal policy under no commitment (e.g., Klein, Krusell, and Ríos-Rull, 2008; Debortoli and Nunes, 2013; Debortoli, Nunes, and Yared, 2017; Martin, 2009). Effectively, we rule out the possibility of defaulting on debt since, as pointed in Gennaioli, Martin, and Rossi (2014), sovereign defaults entail disproportionately larger real costs than not committing to other policy tools and, therefore, they are infrequent in financially developed economies. Instead, we view inflation as means to achieve “partial default” in such economies.

Next, for illustrative purposes only, we derive and interpret Generalized Euler Equations (GEEs) that characterize the optimal time-consistent policy as they reveal key distinctive

¹²Note that since we dropped the subscript t from inflation and in order to avoid confusion we denote the inflation target as $\bar{\pi}$, instead of π .

features of this problem with respect to the Full Commitment case. We do so under the assumption that the policies under consideration exhibit sufficient differentiability properties. Note that in our numerical work we do not assume differentiability and use a global solution method that tackles problem (35) directly. A detailed description of the solution algorithm can be found in appendix C.2. Indeed, as pointed out, e.g. in Klein, Krusell, and Ríos-Rull (2008), the associated policy functions may contain jumps and be nondifferentiable. The associated optimality conditions are Generalized Euler Equations as they feature derivatives of future policies, that the current government takes as given, with respect to the current policy. Here, to illustrate the key economic trade-offs, we follow the literature on Markov-perfect fiscal policy (e.g., Klein, Krusell, and Ríos-Rull, 2008; Debortoli and Nunes, 2013; Debortoli, Nunes, and Yared, 2017) and focus our attention on policies that are differentiable functions of the “natural” state space x . Under the assumption of differentiability, it is possible to derive and interpret Generalized Euler Equations that characterize the optimal time-consistent policy.

The first-order conditions with respect to consumption, labor, and wage are

$$\lambda = u_c(c) - \mu u_{cc}(c) \left(\frac{B}{\pi} + b \right) + \mu u_{cc}(c)s + \mu u_c(c) \frac{\partial s}{\partial c} + \lambda^\pi \mathcal{N}_c + \lambda^T \mathcal{T}_c, \quad (40)$$

$$v_l(l) = \lambda A + \mu u_c(c) \cdot \frac{\partial s}{\partial h} + \lambda^\pi u_c(c) A \cdot \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A} \right), \quad (41)$$

$$0 = \mu u_c(c) \frac{\partial s}{\partial w} + \lambda^\pi u_c(c) h. \quad (42)$$

In equation (40), the government equalizes the marginal effect on the resource constraint today (λ) with the marginal utility gain of an additional unit of consumption today plus the impact of that additional unit of consumption through s in the implementability constraint today, plus the marginal impacts on the Phillips Curve and Taylor Rule, plus the second order effects of an additional unit of consumption on the future government’s implementability constraint. In equation (41), the government offsets the marginal disutility of labor with the marginal increase in production, the marginal effects on the implementability constraint through s , plus the marginal impact of h on the Phillips Curve. Finally, in equation (42) the government sets the wage by equating the marginal effect of the wage on the implementability constraint (through government surplus s) with the marginal effect on the New-Keynesian Phillips Curve.

The first-order conditions with respect to nominal bonds, real bonds, and inflation are

$$0 = \beta \mathbb{E} [\tilde{W}_B(x')|x] + \mu \beta \mathbb{E} [S_{B'}(x')|x] + \lambda^\pi \mathcal{N}_{B'} + \lambda^T \mathcal{T}_{B'}, \quad (43)$$

$$0 = \beta \mathbb{E} [\tilde{W}_b(x')|x] + \mu \beta \mathbb{E} [S_{b'}(x')|x] + \lambda^\pi \mathcal{N}_{b'} + \lambda^T \mathcal{T}_{b'}, \quad (44)$$

$$0 = -\lambda \Phi_\pi + \mu u_c(c) \frac{B}{\pi^2} + \lambda^\pi \mathcal{N}_\pi + \lambda^T \mathcal{T}_\pi, \quad (45)$$

where $S(x') \equiv u_c(\tilde{c}(x')) \cdot (B' \tilde{\pi}(x')^{-1} + b(x'))$. To set the nominal and real bonds, the social planner balances the expected present discounted value of an additional unit of B or b on the future government's continuation value with the marginal impacts on the Taylor Rule and New-Keynesian Phillips Curve plus the expected marginal effect on the consumer's Euler equation (S). Inflation is optimally chosen by equating the marginal effects on the Taylor Rule and New-Keynesian Phillips Curve with the marginal effect on the implementability constraint (through the marginal utility of consumption and the amount of nominal bonds).

An important difference between these optimality conditions and their counterparts in the Full Commitment problem of the previous subsection is that past multipliers on the implementability constraint (11) are absent here, because the government disregards the effects of current policy on past decisions of the private sector, and in particular past bonds. Moreover, the derivatives of the future policy functions appear within the terms $\mathbb{E} [S_{B'}(x')|x]$, $\mathbb{E} [S_{b'}(x')|x]$, $\mathcal{N}_{b'}(x)$, and $\mathcal{T}_{b'}(x)$, rendering these optimality conditions GEEs.

The envelope conditions on the government's continuation function $\tilde{W}$ for B and b , $\tilde{W}_B(x) = \mu \cdot u_{cc}(c) \cdot \pi^{-1}$ and $\tilde{W}_b(x) = \mu \cdot u_{cc}(c)$, synthesize the second order effects on consumption $u_{cc}(c)$ expressed in real terms by dividing by inflation (in the case of nominal bonds). Imposing a symmetric Markov-perfect equilibrium, we can use the envelope conditions to back out the GEEs

$$0 = \beta \mathbb{E} \mu(x') u_{cc}(\tilde{c}(x')) \pi'^{-1} + \mu \beta \mathbb{E}_t S_{B'} + \lambda^\pi \mathcal{N}_{B'} + \lambda^T \mathcal{T}_{B'}, \quad (46)$$

$$0 = \beta \mathbb{E} \mu' u_{cc}(\tilde{c}(x')) + \mu \beta \mathbb{E}_t S_{b'} + \lambda^\pi \mathcal{N}_{b'} + \lambda^T \mathcal{T}_{b'}. \quad (47)$$

Markov-Perfect Equilibrium. We focus on a symmetric Markov-perfect equilibrium (MPE). More formally, a symmetric MPE consists of a set of policy functions for allocations and wage $c(x) = \tilde{c}(x)$, $h(x) = \tilde{h}(x)$, $w(x) = \tilde{w}(x)$, for bonds and inflation $B'(x) = \tilde{B}(x)$, $b'(x) = \tilde{b}(x)$, $\pi(x) = \tilde{\pi}(x)$ that maximize (35) taking equations (36)-(42) as constraints. Note that our definition of equilibrium does not use the GEEs and does not assume differentiability of policy functions. Indeed, as pointed out, e.g. in Klein, Krusell, and Ríos-Rull (2008), the associated policy functions may contain jumps and be nondifferentiable. We want to emphasize that the analysis above with GEEs is done for illustrative purposes only. In our

numerical work we do not assume differentiability and use a global solution method that tackles problem (35) directly. In particular, we follow a computational methodology similar in spirit to the one introduced by Clymo and Lanteri (2020).¹³

5 Quantitative Results

In this section, we provide a detailed quantitative analysis of our calibrated full dynamic models.

5.1 Baseline Calibration

We start by describing the calibration strategy and discuss parameter choices that are common across the Full Commitment and the No Commitment Case. We then specialize the discussion of parameter values to the respective cases, and discuss the relevant solution algorithms. All parameter values are summarized in table 1.

We parameterize the utility function as follows: $u(c) \equiv \frac{c^{1-\eta_c}}{1-\eta_c}$ and $v(l) \equiv \chi \frac{l^{1-\eta_l}}{1-\eta_l}$, with $\eta_l = 1.8$ to match a unitary Frisch elasticity of labor supply and $\chi = 4.3276$ to normalize average labor to 1/3 of the time endowment in the stochastic simulation of the Full Commitment model. The production function is linear $F(h) \equiv Ah$, where A is normalized to a unit value.

We calibrate fiscal parameters using data from Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015). In particular, we use measures of government expenditures and labor tax rates for the period 1971-2013.¹⁴ We also use this data to compute the average ratio of government spending to GDP, which is around 20 percent. We calibrate the exogenous process for g_t as an AR(1) in logs, formally $\log g_{t+1} = (1 - \rho_g) \log \mu_g + \rho_g \log g_t + \epsilon_t$, with ϵ_t normally distributed with mean zero and standard deviation σ_g . We then match the average ratio of government spending to GDP, as well as the standard deviation and autocorrelation of linearly detrended (log) government spending, using the data described above.

We set the price elasticity of demand $1/\nu$ to 10, which is a standard value used in the literature. The Taylor rule responds only to deviations from the steady state inflation rate. We set the steady state inflation rate to 2 percent, which is the Fed target level. The parameter φ , which governs the strength of the nominal rigidity, is set to 20 to render the slope of the New-Keynesian Phillips Curve 0.0413 in line with the range of values in Clarida, Gali, and Gertler (1999).

¹³A detailed description of the solution algorithm can be found in appendix C.2. While we cannot prove that the Markov-Perfect Equilibrium is unique, we verify that our solution method always converges to the same policy from a large grid of initial guesses.

¹⁴We convert the data from a quarterly to an annual frequency, obtained as average values in each year. The data can be found at the following link:
<https://www.openicpsr.org/openicpsr/project/112890/version/V1/view>.

Table 1: PARAMETER VALUES

		Parameter	Value
Preferences	Discount factor	β	0.96
	Risk aversion	η_c	2
	Labor disutility	χ	4.3276
	Labor elasticity	η_l	1.8
Firm	Price elasticity	$1/\nu$	10
	Adjustment cost	φ	20
Monetary Policy	Response to inflation	ϕ_π	1.2
Government	Average g	μ_g	0.068
	Volatility of $\log(g)$	σ_g	0.016
	Autocorr. of $\log(g)$	ρ_g	0.977

Notes: The table reports the parameter values used in the quantitative part of the paper. The same calibration is used in the Full Commitment model without TIPS bonds and in all extensions in presented in the appendix section B.1.2 and in the No Commitment model in section 5.3.

5.2 Quantitative Results with Full Commitment

We solve the optimal policy under Full Commitment using an algorithm similar in spirit to the Parameterized Expectations Algorithm (den Haan and Marcet, 1990). We provide extensive details on the solution method in appendix C, which has been proposed by Valaitis and Villa (2024). This method relies on a neural network to approximate forward-looking terms in the optimality conditions as functions of the state vector.

We begin by comparing our calibrated model to a counterfactual scenario where the government can only issue nominal bonds. When the government cannot issue TIPS, the Ramsey planner faces a trade-off between responding to shocks using distortionary taxes versus inflation. On the one hand, by inflating away the nominal liability, the government can relax its budget constraint without increasing labor taxes. In a setting where the government has access to state-uncontingent bonds only, it can thus take advantage of inflation to render these bonds' real payoffs state-contingent ex-post. On the other hand, elevated inflation expectations lower nominal bond prices as well. The addition of inflation-protected securities in the government debt portfolio qualitatively changes the use of inflation, as shown in equation (34). As we show below, this force is highly relevant in the quantitative model with risk-averse agents.

Table 2 compares the unconditional moments of the main policy variables in the two models. In the two bonds model, inflation is volatile, allowing the planner to approximate the complete markets allocation. In line with the results from a one-period model, optimal policy prescribes borrowing in nominal bonds and accumulating real assets. This occurs because, as in the one-period model, such a leveraged position allows hedging spending shocks, thus allowing to reduce the tax distortions as long as the expenditure shocks are inflationary.

This can be seen from the baseline model having less volatile taxes than the No TIPS model. Importantly, and differently from the one-period setting, in the full, dynamic model with TIPS, government expenditure shocks are endogenously inflationary, as determined by the Taylor rule. Indeed, intuitively, government expenditure shocks raise taxes and depress consumption, thereby raising interest rates. By the Taylor rule, then, this creates upward pressure on inflation. At the same time, expenditure shocks are expansionary, as households increase their labor supply in response, raising output.

The result on the composition of the government debt portfolio thus resembles the policy prescription from the optimal maturity literature to borrow in the maturity with the most volatile price and to accumulate assets in the maturity with the least volatile price ([Angeletos, 2002](#)). In our case, nominal bond prices are more volatile because they include an inflation premium. Inflation plays a key role in hedging. The planner finds it optimal to have volatile and procyclical inflation because such a policy means that nominal and real bond prices become less and less correlated, allowing the planner to achieve better hedging. In fact, the more volatile and procyclical the inflation, the lower the leverage that is needed to hedge shocks. Indeed, once TIPS bonds become available, inflation volatility increases five times in this Full-Commitment setting. In the end, the planner chooses inflation by balancing the benefits of hedging against the costs incurred due to nominal rigidities. While most of the literature has found that the cost of nominal rigidities typically outweighs the benefits of inflating away nominal debt in models without TIPS bonds ([Siu, 2004](#); [Faraglia, Marcet, Oikonomou, and Scott, 2013](#)), inflation is used actively for hedging purposes in our full model. We note that high inflation volatility goes in hand with a very volatile real debt share with minimal persistence, in sharp contrast to the empirical evidence. As we illustrate in [appendix B](#) figure [A.3](#) by means of a simulated path of the key variables in the full model, the government finds it optimal to rebalance the composition of its debt portfolio actively and frequently to take advantage of movements in inflation. While in the FC baseline case volatile inflation plays the role of the shock absorber, in the FC No TIPS case the total amount of debt outstanding does so when TIPS are not available. Indeed, total debt volatility goes up by a factor of four relative to the benchmark in the specification without TIPS. This is in sharp contrast to the setting with No Commitment that we examine below, in which total debt is endogenously stable.

Overall, the baseline results from the dynamic model under Full Commitment confirm the intuition obtained from the one-period setting, in that with inflationary government expenditure shocks, the government finds it optimal to borrow with nominal bonds, and to save through real bonds. Moreover, the dynamic model shows that the government finds it optimal to effectively use inflation to relax its budget constraint, thereby actively rebalancing its debt portfolio. Importantly, in the dynamic model, government expenditure shocks are

Table 2: SUMMARY OF MOMENTS: POLICY VARIABLES

Description	Moments	FC No TIPS	FC Baseline
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	2.0	1.8
Avg. Tax [%]	$\mathbb{E}(\tau)$	22.0	23.2
Avg. Real to GDP	$\mathbb{E}(b/Y)$	-	-0.2
Avg. Nominal to GDP	$\mathbb{E}(B/Y)$	-0.2	0.3
Autocorr. Real Share	$\rho_1(b/(b+B))$	-	0.046
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.587	0.822
Corr. Tax and GDP	$\rho(\tau, Y)$	-0.434	0.441
Corr. Inflation and GDP	$\rho(\pi, Y)$	-0.245	0.667
Corr. Inflation and Real	$\rho(\pi, b)$	-	0.455
Corr. Inflation and Nominal	$\rho(\pi, B)$	0.74	-0.588
Std. Inflation	$100\sigma(\log(\pi))$	0.033	0.164
Autocorr Inflation	$\rho_1(\pi)$	0.6268	0.8981
Std. Debt	$100\sigma(\log(b+B))$	16.3840	4.2275
Autocorr Debt	$\rho_1(b+B)$	0.9999	0.9984

Notes: The table reports sample moments from simulating the equilibrium dynamics of the Full Commitment model with and without TIPS bonds using the same realization of government expenditure shocks. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments.

endogenously inflationary, whereby inflation is determined through the Taylor rule. In the appendix, we examine a number of extensions of the model to show that the baseline result of a positive allocation to nominal bonds and a negative allocation to real bonds is remarkably robust across specifications.

5.3 Quantitative Results with No Commitment

We now turn to a calibration of the full dynamic model with No Commitment to examine the quantitative implications of the lack of commitment. To start and to benchmark our results against the Ramsey equilibrium, we use the identical parameterization as in that case, as summarized in table 1. The results for the main policy variables are reported in table 3. We report results for the baseline case with real and nominal bonds, as well as the case with nominal bonds only.

As the table shows, the government finds it optimal to issue a significant amount of real debt, in line with the example examined in the one-period case. Naturally, this is because with No Commitment, real debt emerges as an effective commitment device to prevent future governments from monetizing debt ex-post. At the same time, to provide insurance, the government finds it optimal to use nominal bonds. In fact, the optimal allocation is negative, in that the government saves using nominal bonds. In equilibrium, thus, households borrow in nominal terms, but less so when inflation is high. Therefore, governments save less when the value of their assets is eroded more by inflation. When we restrict our policy

Table 3: SUMMARY OF MOMENTS: POLICY VARIABLES

Description	Moments	NC No TIPS	NC Baseline
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.67	1.93
Avg. Tax [%]	$\mathbb{E}(\tau)$	26.0	25.81
Avg. Real to GDP	$\mathbb{E}(b/Y)$	-	0.65
Avg. Nominal to GDP	$\mathbb{E}(B/Y)$	0.30	-1.06
Autocorr. Real Portfolio Weight	$\rho_1(b/(b+B))$	-	0.899
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.99	1.00
Corr. Tax and GDP	$\rho(\tau, Y)$	0.953	0.998
Corr. Inflation and GDP	$\rho(\pi, Y)$	-0.719	0.932
Corr. Inflation and Real	$\rho(\pi, b)$	-	0.946
Corr. Inflation and Nominal	$\rho(\pi, B)$	0.975	-0.808
Std. Inflation	$100\sigma(\log(\pi))$	0.141	0.056
Autocorr Inflation	$\rho_1(\pi)$	0.954	0.944
Std. Debt	$100\sigma(\log(b+B))$	0.481	0.121
Autocorr Debt	$\rho_1(b+B)$	0.978	0.950

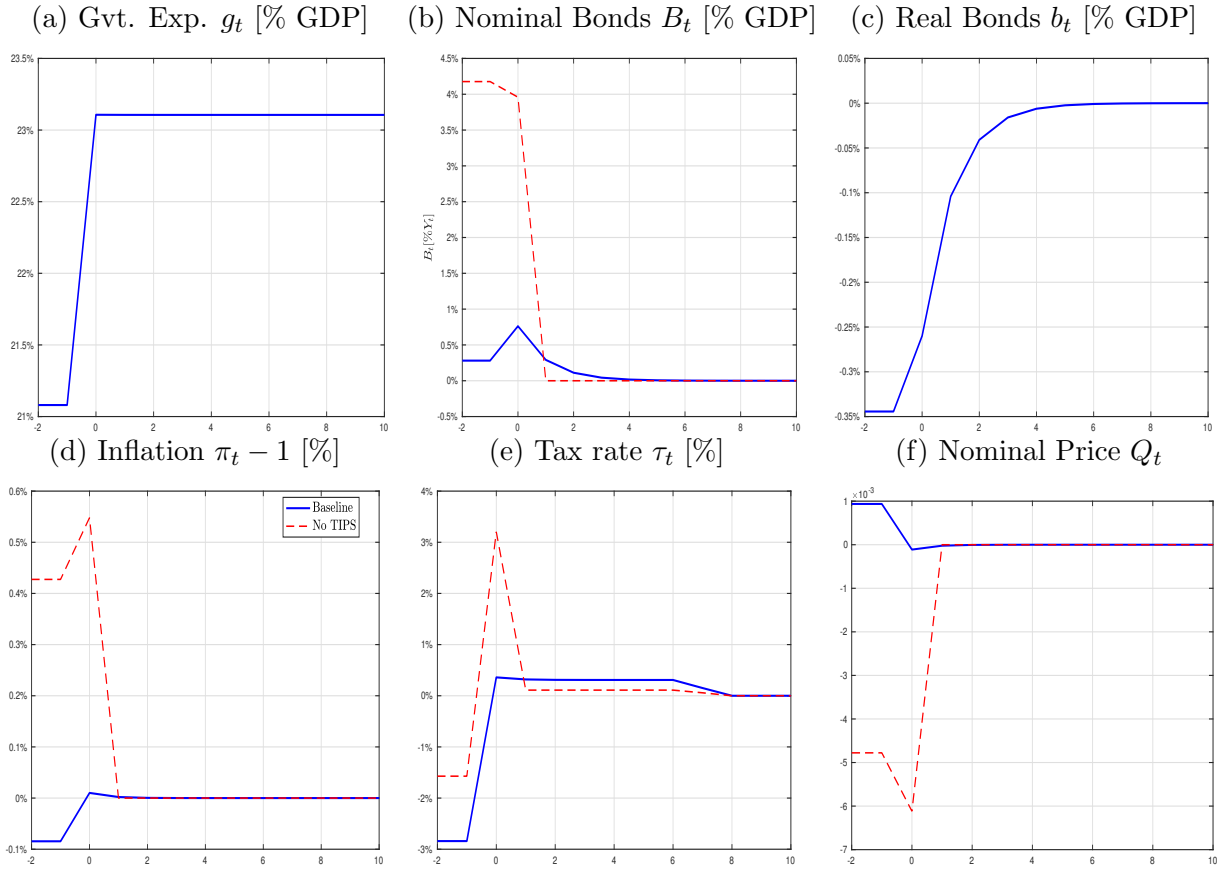
Notes: The table reports sample moments of the No Commitment model with and without TIPS bonds from simulating model equilibrium dynamics using the same realization of government expenditure shocks. The simulation is initialized at $b = B = 0$, and we drop the first 100 periods before calculating moments.

instrument to nominal bonds, we find a small positive nominal bond position, since holding a large nominal debt position would be costly given the future government temptation to use inflation. Intriguingly, average inflation is lower in the absence of real bonds. In this case, in equilibrium, large nominal bond positions would be costly with No Commitment, so that smaller positions reduce the temptation to resort to high inflation. In contrast, once real bonds are available as a commitment device, the government can sustain somewhat higher inflation, as a negative allocation to nominal bonds curbs its temptation to use inflation excessively ex-post. Hence, negative nominal debt serves as a commitment device, as any future government relying on inflation would effectively diminish the value of its own inherited nominal assets by inflating them away. In view of this, inflation is also substantially less volatile than in the benchmark with Full Commitment. We note that total debt as well as the real debt share are much more stable than in the Full Commitment case, so that they no longer take up the important role of shock absorbers that they played in the latter case. Intuitively, this is because large movements in debt would raise the temptation to monetize it, reflected in movements in inflation expectations and nominal bond prices. The availability of TIPS in the baseline specification with No Commitment alleviates that temptation as we discuss next.

Figure 6 illustrates these dynamics by plotting the responses of the model's endogenous variables to an exogenous increase in government spending in the Baseline and in the No TIPS model. Responses to an expenditure shock in period 0 highlight that real bonds act

as a commitment device. In the Baseline model, the government holds the portfolio of real debt and nominal assets, which limits its incentives to resort to inflation when the shock hits. As the second panel shows, the inflation's response is dampened compared to the No TIPS model, where the shock is absorbed by inflating away the nominal debt.¹⁵ As a consequence, the government in the Baseline model can mitigate the effect of the shock on nominal prices (bottom right panel) and does not need to increase labor taxation by as much as in the No TIPS model. In this sense, the presence of TIPS bonds limits the use of inflation in response to expenditure shocks. This is the exact opposite of what happens in the Full Commitment setting of section 5.2, where the hedging motives lead the Ramsey planner to engineer more inflation volatility compared to the No TIPS benchmark. In this sense, reducing future governments' temptation to monetize debt endogenously stabilizes inflation.

Figure 6: NO COMMITMENT: CONDITIONAL DYNAMICS AFTER AN EXPENDITURE SHOCK



Notes: The figure displays the dynamics of fiscal variables around a shock that increases government spending, at $t = 0$. X-axes report time t . Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds.

Under the baseline calibration, the optimal policies under Full Commitment and No

¹⁵Unconditional inflation volatility, measured as $100\sigma(\log(\pi))$, also decreases in the No Commitment model to 0.1 compared to 0.164 in the Full Commitment.

Commitment both exhibit rather stark implications for the composition of government debt portfolios that seem to contrast rather sharply with the U.S. macroeconomic experience, in that they both feature significant negative allocations to one type of bonds. In the case of Full Commitment, the government chooses to save by means of real bonds, whereas in the case with No Commitment, the government chooses to save by lending through nominal bonds. Realistically, perhaps, governments do not lend through the bond market, neither by means of nominal or real bonds, and neither do households borrow from the government at a large scale. To capture this added element of realism, we next solve a version of our models in which we impose a no-lending constraint, by restricting the government’s bond allocations to be non-negative and preventing the government from investing in private bonds, while we do not model and are agnostic about the exact source of this friction. We implement these constraints by setting the lower bounds on debt $\underline{B} = \underline{b} = 0$. We report the results in table 4 and present the baseline results without no-lending constraints alongside to facilitate comparison.

Table 4: SUMMARY OF MOMENTS: POLICY VARIABLES

Description	Moments	FC	FC No Lend	NC	NC No Lend
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.8	1.97	1.93	1.67
Avg. Tax [%]	$\mathbb{E}(\tau)$	23.2	25.3	25.8	25.4
Avg. Real Share	$\mathbb{E}(b/(B + b))$	-0.5	0	-1.6	0.07
Avg. Nominal Share	$\mathbb{E}(B/(B + b))$	1.463	0.998	2.6	0.93
Autocorr. Real Share	$\rho_1(b/(b + B))$	0.046	0.957	0.899	0.935
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.822	0.777	1.00	0.997
Corr. Tax and GDP	$\rho(\tau, Y)$	0.441	0.366	0.998	0.988
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.667	0.545	0.932	0.505
Corr. Inflation and Real	$\rho(\pi, b)$	0.455	-0.071	0.946	-0.719
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.588	0.644	-0.808	0.679

Notes: The table reports sample moments from simulating model equilibrium dynamics using the same realization of government expenditure shocks. The columns report moments from the baseline Full Commitment and No Commitment models and alternative models with no lending constraints for both Full and No Commitment. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments.

The relevant results are presented in the columns labeled “No Lend.” With such no-lending constraints, the Full Commitment case now prescribes a government debt portfolio consisting exclusively of nominal bonds, with no role for real bonds. This finding highlights the quantitative relevance of nominal bonds for fiscal hedging in that the state-contingent flavor of nominal bonds plays the dominant role in the government’s debt policy. On the other hand, notably, in the case of No Commitment with a no-lending constraint, the government’s debt portfolio now features a sizable share of real bonds (7%), reflecting the relevance of the commitment friction in this setting. Importantly, in the case with No Commitment, the real debt share now qualitatively resembles that in the data, although quantitatively the

commitment friction is overstated in our benchmark calibration. We note, however, that for robustness purposes our benchmark calibration is based on the long data sample ranging from 1929 to 2021, substantially longer than the TIPS period.

5.3.1 Interactions Between TIPS, Monetary, and Fiscal Policies

In this section, we take the No Commitment model with no-lending constraint as a starting point and examine quantitatively how the slope of the New Keynesian Phillips curve and the strength of the monetary policy’s response to inflation affect the relevance of the commitment friction and the real debt portfolio share. In this analysis, we investigate the TIPS share with parameter choices that are less precisely measured, but perhaps more relevant to the recent U.S. macroeconomic experience in which the government issued TIPS. Both cases reveal key mechanisms to understand the TIPS share observed in the U.S. data.

In this section, we ask two questions aimed at exploring the interaction between fiscal and monetary policy. We note that we examine these interactions in a setting in which the fiscal authority operates under no commitment, while the monetary authority commits to a Taylor rule. As discussed in appendix section B.1.1 and shown in table 4, commitment to a Taylor rule is not sufficient to eliminate the tendency of future governments to inflate away debt ex-post, as demonstrated by a significant portfolio share of TIPS bonds. With that in view, we first examine how changing the real costs of inflation affects the degree to which the commitment friction matters under No Commitment and thereby impacts the real debt portfolio share. We vary the strength of the monetary policy’s response to inflation, controlled by ϕ_π . Second, we vary the real costs of inflation by changing the slope of the New Keynesian Phillips curve controlled by parameter φ . We think that these are relevant cases in view of ongoing discussions revolving around the changing slope of the Phillips curve and Federal Reserve’s response to inflation in recent years.

Table 5 shows the results. Column (a) shows the no-lending specification with the baseline parameters, and columns (d) and (e) show the specification where monetary policy reacts to inflation more aggressively. Intuitively, the more dovish the central bank is in responding to inflation, the less costly it becomes to manipulate inflation ex-post, as this has a smaller effect on the nominal rates. We see that the portfolio share in column (a) is 7%, suggesting that the commitment friction is relevant in the baseline case and is in line with the average TIPS portfolio share observed in the data. As the monetary policy becomes more dovish, the portfolio share of real bonds goes down to 23% and 53% in columns (b) and (c), respectively. Next, in columns (d) and (e) we make the slope of the New Keynesian Phillips Curve steeper by decreasing parameter φ . Intuitively, this also makes it less costly to use inflation ex-post as prices are less sticky and inflation less costly, making the commitment friction more relevant. Columns (b) and (c) show that quantitatively, this is relevant as the real debt

portfolio shares increase to 22% and 55%, respectively. Together, these results lead to two important conclusions. First, the state of the economy, as summarized by the slope of the New Keynesian Phillips curve, and the conduct of monetary policy shape the relevance of the commitment friction and thus the optimal level of real debt. Second, within the reasonable range of parameter values, the real debt share takes values close to that in the U.S. data.

Table 5: NO COMMITMENT AND PORTFOLIO SHARES

Description	Moments	Model				
		$\phi_\pi = 1.2$ $\varphi = 20$ (a)	$\phi_\pi = 1.2$ $\varphi = 15$ (b)	$\phi_\pi = 1.2$ $\varphi = 10$ (c)	$\phi_\pi = 1.17$ $\varphi = 20$ (d)	$\phi_\pi = 1.15$ $\varphi = 20$ (e)
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	1.67	1.57	1.11	1.58	1.53
Avg. Tax [%]	$\mathbb{E}(\tau)$	25.45	25.81	25.82	25.41	25.40
Avg. Real Share	$\mathbb{E}(b/(b+B))$	0.07	0.22	0.55	0.23	0.53
Avg. Nominal Share	$\mathbb{E}(B/(b+B))$	0.93	0.78	0.45	0.77	0.47
Autocorr. Real Share	$\rho_1(b/(b+B))$	0.935	0.906	0.886	0.958	0.958
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.997	0.995	1.00	1.00	1.00
Corr. Tax and GDP	$\rho(\tau, Y)$	0.988	0.974	0.998	0.999	0.999
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.505	-0.710	0.793	0.985	0.973
Corr. Inflation and Real	$\rho(\pi, b)$	-0.719	-0.726	-0.241	-0.716	-0.822
Corr. Inflation and Nominal	$\rho(\pi, B)$	0.679	0.820	0.245	0.716	0.822

Notes: The table reports the average inflation, taxes, portfolio weights of real and nominal bonds, and salient correlations among monetary and fiscal policy instruments. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments. All moments are calculated in a simulation with $T = 10000$ periods.

The TIPS Share and Inflation. Intuitively, real bonds in the No Commitment framework emerge as a commitment device that curbs future governments' incentives to inflate debt ex-post. We now examine to what extent real bonds are effective in taming inflation by exploring the systematic relation between the share of TIPS bonds in the portfolio and inflation. Doing so, we ask, does issuing more real debt help reduce current inflation?

To start, in figure 7 we illustrate the impact of tilting the portfolio towards real debt on the inflation level, holding the amount of total debt fixed, starting from a model in column (a) in table 5, where the real debt portfolio share is the closest to the data.

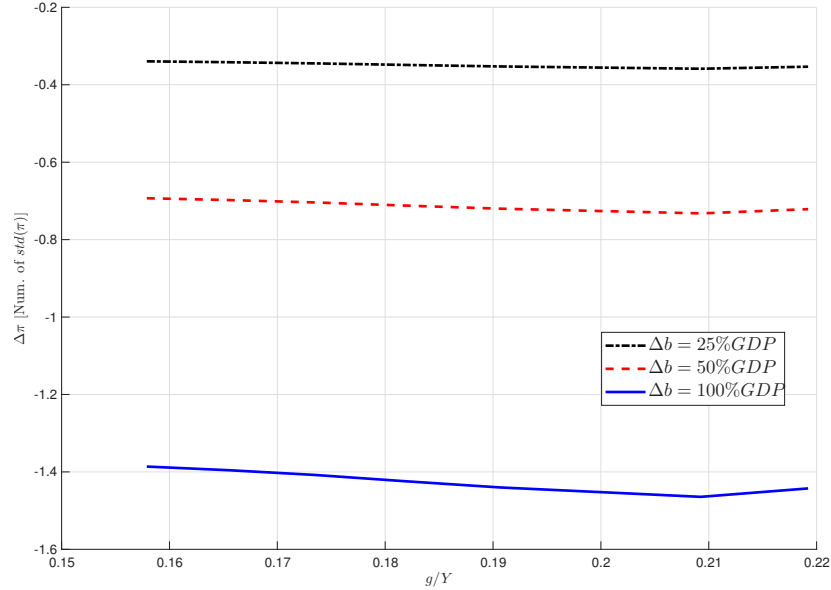
We achieve this by calculating the derivative of the equilibrium policy functions of inflation with respect to real and nominal debt, denoted as $\frac{\partial \pi}{\partial b}$ and $\frac{\partial \pi}{\partial B}$, respectively.¹⁶ Specifically, we examine the derivatives of inflation with respect to current debt choices. Intuitively, in a symmetric Markov-perfect equilibrium, these derivatives measure the cross-sensitivity between contemporaneous policy functions. We assess the impact of debt composition on inflation through the following expression:

$$\Delta \pi_t = \frac{\partial \pi_t}{\partial b_{t+1}} \cdot \Delta b_{t+1} + \frac{\partial \pi_t}{\partial B_{t+1}} \cdot \Delta B_{t+1}, \quad (48)$$

¹⁶Note that these same terms appear inside the Generalized Euler equations and are internalized by each government's best response to the future governments.

where we alter the bond positions, keeping the total amount of debt constant, i.e. $\Delta b_{t+1} = -\Delta B_{t+1}$.

Figure 7: INFLATION AND NOMINAL-REAL PORTFOLIO REBALANCING



Notes: The figure shows the change in inflation derived from rebalancing the portfolio towards real bonds, starting from the portfolio shares that replicate the U.S. data and using the calibrated model (a) reported in table 5. The change in inflation on the y-axis is reported in ratio to the equilibrium standard deviation. Hence, the standard deviation of inflation is calculated from the simulation of the model equilibrium dynamics, using the model (a) reported in table 5. The x-axis reports the debt-to-GDP ratio, whereas the three lines report the amount of portfolio rebalancing toward real debt, expressed in percentage of GDP. For example, the solid blue line shows the change in inflation derived using equation (48) when the real debt is increased (and nominal debt simultaneously decreased) by 100% of GDP.

In figure 7, we inspect the inflationary effect of a shift towards real debt in the amount of Δb_{t+1} up to 100% of GDP. The horizontal axis shows the relevant amount of exogenous government expenditures relative to GDP, and the vertical axis shows the change in the same period inflation in terms of standard deviations after recomposing the portfolio towards real debt. The results show that the higher is the portfolio share of real debt is associated with lower inflation. For example, in terms of magnitudes, if real debt increased, and thus nominal debt decreased, by 100% of GDP and the spending needs were around 20% of GDP, inflation would be almost one and a half standard deviation lower than in the current equilibrium. Moreover, the effects of portfolio rebalancing increase in the size of the spending needs. This indicates that, according to the model, higher TIPS shares effectively curb inflation.

5.3.2 Time-Consistent Optimal Policy with Long-term Nominal and Real Debt

For the sake of realism, we next augment the model by introducing long-term nominal government bond using perpetuities with decreasing coupon rates in the spirit of Hatchondo and

Martinez (2009) and Arellano and Ramanarayanan (2012). This payment structures allows us to span different maturity composition starting from a one-year bond. More specifically, we assume nominal and real bonds issued in period t promise an infinite stream of deterministic coupon payments that decline at an exogenous constant rate $\delta_B \in [0, 1]$ and $\delta_b \in [0, 1]$, respectively. In particular, one unit of debt issued in period t promises to repay a fraction $1 - \delta_B$ (or $1 - \delta_b$) of its outstanding value in each subsequent period. This structure implies that a fixed share of the remaining principal is repaid each period, akin to a consol or perpetuity with declining coupons. An advantage of this specification is that it collapses all future repayment obligations into a single state variable—the stock of long-term debt maturing in the current period. As a result, the law of motion for nominal debt is:

$$B_{t+1} = (1 - \delta_B) \frac{B_t}{\pi_t} + i_t^B,$$

where i_t^B denotes the new nominal bond issuance and π_t is gross inflation. The law of motion for real debt is:

$$b_{t+1} = (1 - \delta_b) b_t + i_t^b,$$

where i_t^b denotes the new real bond issuance.

Denoting the price of long-term nominal bonds by Q_t and that of long-term real bonds by q_t , and the associated coupons by (z_B, z_b) , the household's budget constraint becomes

$$c_t + Q_t i_t^B + q_t i_t^b + p_t S_{t+1} = (1 - \tau_t) w_t h_t + (\delta_B + (1 - \delta_B) z_B) \frac{B_t}{\pi_t} + (\delta_b + (1 - \delta_b) z_b) b_t + (p_t + d_t) S_t, \quad (49)$$

and the government budget constraint is

$$Q_t i_t^B + q_t i_t^b + \tau_t w_t h_t = g_t + (\delta_B + (1 - \delta_B) z_B) \frac{B_t}{\pi_t} + (\delta_b + (1 - \delta_b) z_b) b_t. \quad (50)$$

Households maximize utility (1) subject to (49). The intertemporal Euler equations for nominal and real bonds are:

$$u_c(c_t) Q_t = \beta \mathbb{E}_t \left[u_c(c_{t+1}) (\delta_B + (1 - \delta_B) (Q_{t+1} + z_B)) \frac{1}{\pi_{t+1}} \right], \quad (51)$$

$$u_c(c_t) q_t = \beta \mathbb{E}_t [u_c(c_{t+1}) (\delta_b + (1 - \delta_b) (q_{t+1} + z_b))]. \quad (52)$$

When $\delta_B = 1$ and $\delta_b = 1$, these reduce to the Euler equations for one-period bonds, as in the baseline model. For illustration, consider the case of a zero-coupon nominal bond, i.e.,

$z_B = 0$. Forward iteration of equation (64) then yields:

$$Q_t = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \beta^j \delta_B (1 - \delta_B)^{j-1} \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot \frac{1}{\prod_{k=1}^j \pi_{t+k}} \right],$$

highlighting that the price of long-term nominal bonds reflects the entire expected path of future inflation, discounted by both the maturity structure and marginal utility.

Combining the household and government budget constraints and imposing private-sector optimality yields the implementability condition:¹⁷

$$(\delta_B + (1 - \delta_B)z_B) \frac{B_t}{\pi_t} + (\delta_b + (1 - \delta_b)z_b) b_t = s_t + Q_t \left(B_{t+1} - (1 - \delta_B) \frac{B_t}{\pi_t} \right) + q_t (b_{t+1} - (1 - \delta_b) b_t), \quad (53)$$

where s_t denotes the primary surplus, defined as in the baseline. When nominal debt has duration ($\delta_B < 1$), the planner can use not only current inflation π_t but also the expected path of future inflation embedded in Q_t to relax the implementability constraint.

Quantitative Results. While we relegate the formal definition of the Markov-Perfect equilibrium to appendix B.2, we note that we tackle the problem numerically using an extension of the algorithm for the baseline model described in appendix C.3. For simplicity, we assume unit coupons, i.e., $z_b = z_B = 1$. All other parameters match those in the baseline calibration with short-term debt only, except for the maturity parameters (δ_B, δ_b) , which we vary across specifications. When $\delta_B = \delta_b = 1$, the model reduces to the baseline case with one-period bonds.

First, we consider the case in which the government issues TIPS with a one-year maturity, and report the results in table 6. Lustig, Sleet, and Yeltekin (2008) show that under full commitment, long-term nominal allows to achieve better hedging of fiscal shocks and is preferred to short-term debt. This is because the long-term debt price can be manipulated by increasing the expected inflation in all future periods up to its maturity. As the maturity increases, the planner can achieve the same variation in bond prices by at the same time spreading the inflationary distortions over longer horizons. In contrast, our framework assumes no commitment, which introduces an additional friction: the planner must not only smooth taxes and complete markets, but also manage the incentives that arise from the inability to commit to future policies. In this setting, longer nominal debt maturities exacerbate the incentives to use expected future inflation contained in the bond's price as a tool to reduce the real burden of outstanding nominal liabilities, which can be viewed as

¹⁷We leave the inter temporal bond Euler equations (64) and (65) as constraints, treating Q_t and q_t as choice variables due to their recursive nature.

an inflationary debt dilution motive. This mechanism is captured by the implementability constraint (67), where the term

$$\frac{B_t}{\pi_t} (\delta_B + (1 - \delta_B)(Q_t + z))$$

illustrates how both current and expected future inflation relax the government's budget constraint when $\delta_B < 1$.

As the average maturity of nominal debt increases, the incentive for inflationary dilution strengthens. To mitigate this, the planner shifts the composition of debt toward inflation-indexed securities. Indeed, the table shows that both the level and share of TIPS increase with nominal debt maturity. In this sense, TIPS serve as a commitment device that helps discipline inflation and address the time inconsistency problem. As nominal debt maturity rises, the planner increases the outstanding amount of TIPS, resulting in lower average inflation and inflation volatility, along with higher and more stable tax revenues.

Table 6: Comparative Moments by Average Maturity of Nominal Debt

Description	Moment	Avg. Maturity of Nominal Debt		
		1Y	2Y	5Y
Avg. Inflation (%)	$\mathbb{E}(\pi - 1) \cdot 100$	1.67	1.54	1.35
Avg. Tax Rate (%)	$\mathbb{E}(\tau) \cdot 100$	25.45	28.07	37.42
Std. Dev. of Inflation (%)	$\sigma(\log \pi) \cdot 100$	0.07	0.05	0.00
Std. Dev. of Log Tax Rate (%)	$\sigma(\log(1 + \tau)) \cdot 100$	1.42	1.28	0.93
Share of Real Debt	$\mathbb{E}(b^N / (b^N + B^N))$	0.07	0.46	0.77
Debt-to-GDP Ratio	$\mathbb{E}((B^N + b^N) / Y)$	0.33	0.36	0.64
Real Debt-to-GDP Ratio	$\mathbb{E}(b^N / Y)$	0.02	0.17	0.49

Notes: This table reports simulated model moments under four different values of the average maturity of nominal debt, corresponding to different values of the parameter δ_B . The columns report results for maturities of 1, 2 and 5 years, which correspond respectively to $\delta_B = \{1, 0.5, 0.2\}$. Reported moments include the average inflation rate, average tax rate, standard deviations of log inflation and log tax rate, the share of real debt in total debt, and the total and real debt-to-GDP ratios. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments. All moments are calculated in a simulation with $T = 10000$ periods.

Second, we consider the case in which the government issues both nominal and TIPS with maturities of two and five years. Table 7 reports the results. As the table shows, allowing for long-term TIPS in addition to long-term nominal debt increases both the share of real debt and the overall debt-to-GDP ratio. Long-term TIPS reinforce their role as a commitment device, since they insure households against the incentives of future governments to dilute outstanding debt. With longer maturities, it becomes more costly for future planners to rebalance the portfolio toward nominal debt: buybacks of non-matured TIPS are expensive, and their payoff is protected from inflationary dilution. In the implementability constraint

(67), the inclusion of long-term TIPS adds the term $(1 - \delta_b)q_t b_t$, which restricts portfolio rebalancing and where q_t is protected from inflationary debt dilution. As a result, long-term TIPS provide stronger discipline than their short-term counterparts, allowing the government to achieve commitment with a smaller overall share of real debt relative to the case with one-period TIPS in table 6.

Table 7: Comparative Moments by Joint Average Maturity of Nominal and TIPS Debt

Description	Moment	Avg. Debt Maturity		
		1Y	2Y	5Y
Avg. Inflation (%)	$\mathbb{E}(\pi - 1) \cdot 100$	1.67	1.40	1.25
Avg. Tax Rate (%)	$\mathbb{E}(\tau) \cdot 100$	25.45	31.51	47.28
Std. Dev. of Inflation (%)	$\sigma(\log \pi) \cdot 100$	0.07	0.09	0.00
Std. Dev. of Log Tax Rate (%)	$\sigma(\log(1 + \tau)) \cdot 100$	1.42	1.12	0.73
Share of Real Debt	$\mathbb{E}(b^N / (b^N + B^N))$	0.07	0.47	0.55
Debt-to-GDP Ratio	$\mathbb{E}((B^N + b^N)/Y)$	0.33	0.40	0.47
Real Debt-to-GDP Ratio	$\mathbb{E}(b^N/Y)$	0.02	0.19	0.26

Notes: This table reports simulated model moments under three configurations in which the average maturity of nominal debt and TIPS coincide (1, 2 or 5 years). Reported moments include the average inflation rate, average tax rate, standard deviations of log inflation and log tax rate, the share of real debt in total debt, and the total and real debt-to-GDP ratios. The simulation is initialized at $b = B = 0$ and we drop the first 100 periods before calculating moments. All moments are calculated in a simulation with $T = 10000$ periods.

6 Conclusion

In the wake of elevated inflation in the aftermath of unprecedented debt-financed stimulus packages, controlling inflation has again moved to the forefront of policymakers' attention. In this paper, we examine optimal government debt management in the presence of inflation concerns in a setting where (i) the government can issue long-term nominal and real (TIPS) non state-contingent bonds, (ii) the monetary authority sets short-term interest rates according to a Taylor rule, and (iii) inflation has real costs as prices are sticky. Nominal debt can be inflated away, but bond prices reflect elevated inflation expectations. Real bond prices are higher, but such debt constitutes a real commitment ex-post. Critically, the government can exploit a debt portfolio consisting of both nominal and real bonds to smooth fiscal distortions to the extent that inflation is sufficiently volatile, so that these securities are not perfect substitutes, thereby rendering markets more complete. We analyze the optimal policy under Full Commitment and the optimal time-consistent policy with No Commitment.

Intriguingly, our model specification with No Commitment provides a remarkably accurate quantitative description of U.S. data, quite in contrast to the specification with Full Commitment. In our quantitatively realistic specification with No Commitment, raising the

real share in the government’s debt portfolio effectively helps curb inflation, as it reduces future governments’ incentives to inflate away outstanding debt. Raising the real debt share thus helps governments to commit to low and stable inflation rates.

Our results suggest that a framework with No Commitment captures the relevant constraints actual governments face reasonably well. We thus view accounting for limited government commitment as a useful and natural starting point for relevant policy design. A more realistic setting would account for richer asset price dynamics (e.g., [de Lannoy, Bhandari, Evans, Golosov, and Sargent, 2024](#); [Gomez Cram, Kung, and Lustig, 2025](#)) and tackle the challenge of allowing for recursive preferences in optimal fiscal policy design problems.¹⁸ We leave these challenging extensions for further research.

References

- AIYAGARI, S. R., A. MARCET, T. J. SARGENT, AND J. SAPPALA (2002): “Optimal Taxation without State-Contingent Debt,” *Journal of Political Economy*, 110(6), 1220–1254.
- ALVAREZ, F., P. J. KEHOE, AND P. A. NEUMEYER (2004): “The Time Consistency of Optimal Monetary and Fiscal Policies,” *Econometrica*, 72(2), 541–567.
- ANGELETOS, G.-M. (2002): “Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure,” *Quarterly Journal of Economics*, 117(3), 1105–1131.
- ARELLANO, C., AND A. RAMANARAYANAN (2012): “Default and the Maturity Structure in Sovereign Bonds,” *Journal of Political Economy*, 120(2), 187–232.
- BANDI, F. M., L. BRETSCHER, AND A. TAMONI (2023): “Return predictability with endogenous growth,” *Journal of Financial Economics*, 150(3), 103724.
- BARRO, R. J. (1979): “On the Determination of the Public Debt,” *Journal of Political Economy*, 87(5), 940–971.
- (2003): “Optimal Management of Indexed and Nominal Debt,” *Annals of Economics and Finance*, 4, 1–15.
- BASSETTO, M., AND C. GALLI (2019): “Is Inflation Default? The Role of Information in Debt Crises,” *American Economic Review*, 109(10), 3556–84.
- BLOMMESTEIN, H., AND A. HUBIG (2012): “A Critical Analysis of the Technical Assumptions of the Standard Micro Portfolio Approach to Sovereign Debt Management,” *OECD Working Papers on Sovereign Borrowing and Public Debt Management*, OECD Publishing, Paris, (4).
- BOCOLA, L., AND G. LORENZONI (2020): “Financial Crises, Dollarization, and Lending of Last Resort in Open Economies,” *American Economic Review*, 110(8), 2524–57.
- BOHN, H. (1988): “Why do we have nominal government debt?,” *Journal of Monetary Economics*, 21, 127–140.
- BRETSCHER, L., A. HSU, AND A. TAMONI (2020): “Fiscal Policy Driven Bond Risk Premia,” *Journal of Financial Economics*, 138(1), 53–73.

¹⁸See [Karantounias \(2018\)](#) for recent progress.

- BUERA, F., AND J. P. NICOLINI (2004): “Optimal maturity of government debt without state contingent bonds,” *Journal of Monetary Economics*, 51, 531–554.
- CIESLAK, A., W. LI, AND C. E. PFLUEGER (2023): “Inflation and treasury convenience,” *USC Marshall School of Business Research Paper Sponsored by iORB*.
- CLARIDA, R., J. GALI, AND M. GERTLER (1999): “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature*, 37(4), 1661–1707.
- CLYMO, A., AND A. LANTERI (2020): “Fiscal Policy with Limited-Time Commitment,” *Economic Journal*, 130(627), 623–652.
- CLYMO, A., A. LANTERI, AND A. T. VILLA (2023): “Capital and Labor Taxes with Costly State Contingency,” *Review of Economic Dynamics*, 51, 943–964.
- CORHAY, A., T. KIND, H. KUNG, AND G. MORALES (2023): “Discount Rates, Debt Maturity, and the Fiscal Theory,” *Journal of Finance*.
- DE LANNOY, L. A., A. BHANDARI, D. EVANS, M. GOLOSOV, AND T. SARGENT (2024): “Managing Public Portfolios,” *Working Paper*.
- DEBORTOLI, D., AND R. NUNES (2013): “Lack of Commitment and the Level of Debt,” *Journal of the European Economic Association*, 11(5), 1053–1078.
- DEBORTOLI, D., R. NUNES, AND P. YARED (2017): “Optimal Time-Consistent Government Debt Maturity,” *The Quarterly Journal of Economics*, 132(1), 55–102.
- DEN HAAN, W., AND A. MARCET (1990): “Solving the Stochastic Growth Model by Parameterizing Expectations,” *Journal of Business and Economic Statistics*, 8, 34–38.
- DIAMOND, W., T. LANDVOIGT, AND G. SANCHEZ (2022): “Printing away the mortgages: Fiscal inflation and the post-covid housing boom,” *Jacobs Levy Equity Management Center for Quantitative Financial Research Paper*.
- DITTMAR, R., A. HSU, G. ROUSSELLET, AND P. SIMASEK (2025): “Default Risk and the Pricing of U.S. Sovereign Bonds,” *Journal of Finance, forthcoming*.
- ELENEV, V., M. FARIA-E CASTRO, AND D. L. GREENWALD (2019): “Quantitative Tightening,” *Working Paper*.
- ELENEV, V., T. LANDVOIGT, P. SHULTZ, AND S. V. NIEUWERBURGH (2022): “Can Monetary Policy Create Fiscal Capacity?,” *Working Paper*.
- ENGEL, C., AND J. PARK (2022): “Debauchery and Original Sin: The Currency Composition of Sovereign Debt,” *Journal of the European Economic Association*, 20(3), 1095–1144.
- FANG, X., Y. LIU, AND N. ROUSSANOV (2025): “Getting to the core: Inflation risks within and across asset classes,” *Discussion paper*.
- FARAGLIA, E., A. MARCET, R. OIKONOMOU, AND A. SCOTT (2013): “The Impact of Debt Levels and Debt Maturity on Inflation,” *The Economic Journal*, 123, 164–192.
- FARAGLIA, E., A. MARCET, R. OIKONOMOU, AND A. SCOTT (2019): “Government Debt Management: the Long and the Short of It,” *Review of Economic Studies*, 86, 2554–2604.
- FERNÁNDEZ-VILLAYERDE, J., P. GUERRÓN-QUINTANA, K. KUESTER, AND J. RUBIO-RAMÍREZ (2015): “Fiscal Volatility Shocks and Economic Activity,” *American Economic Review*, 105(11), 3352–3384.

- FLECKENSTEIN, M., H. LUSTIG, AND F. LONGSTAFF (2014): “The TIPS-Treasury Bond Puzzle,” *Journal of Finance*, 69, 2151–2197.
- FLEMING, M., AND N. KRISHNAN (2012): “The Microstructure of the TIPS Market,” *FRBNY Economic Policy Review*.
- GENNAIOLI, N., A. MARTIN, AND S. ROSSI (2014): “Sovereign Default, Domestic Banks, and Financial Institutions,” *The Journal of Finance*, 69(2), 819–866.
- GOMEZ CRAM, R., H. KUNG, AND H. N. LUSTIG (2025): “Can US treasury markets add and subtract?,” .
- HATCHONDO, J. C., AND L. MARTINEZ (2009): “Long-duration bonds and sovereign defaults,” *Journal of International Economics*, 79(1), 117–125.
- HILSCHER, J., A. RAVIV, AND R. REIS (2022): “Inflating Away the Public Debt? An Empirical Assessment,” *Review of Financial Studies*, 35(3), 1553–1595.
- HSU, A., E. LI, AND F. PALOMINO (2020): “Real and Nominal Equilibrium Yield Curves,” *Management Science*, 67(2), 1135–1158.
- KARANTOUNIAS, A. (2018): “Optimal Fiscal Policy with Recursive Preferences,” *Review of Economic Studies*, 85(4), 2283–2317.
- KLEIN, P., P. KRUSELL, AND J.-V. RIOS-RULL (2008): “Time-consistent public policy,” *The Review of Economic Studies*, 75, 789–808.
- KLEIN, P., P. KRUSELL, AND J. V. RÍOS-RULL (2008): “Time-Consistent Public Policy,” *Review of Economic Studies*, 75(3), 789–808.
- KRUSELL, P., F. M. MARTIN, AND J.-V. RIOS-RULL (2006): “Time Consistent Debt,” 2006 Meeting Papers 210, Society for Economic Dynamics.
- LEEPER, E. M., AND X. ZHOU (2021): “Inflation’s Role in Optimal Monetary-Fiscal Policy,” *Journal of Monetary Economics*, 124, 1–18.
- LI, E. X., T. ZHA, J. ZHANG, AND H. ZHOU (2022): “Does fiscal policy matter for stock-bond return correlation?,” *Journal of Monetary Economics*, 128, 20–34.
- LUCAS, R. E., AND N. L. STOKEY (1983): “Optimal Fiscal and Monetary Policy in an Economy without Capital,” *Journal of Monetary Economics*, 12, 55–93.
- LUSTIG, H., A. BERNDT, AND S. YELTEKIN (2012): “How Does the U.S. Government Finance Fiscal Shocks?,” *American Economic Journal: Macroeconomics*, 4, 69–104.
- LUSTIG, H., C. SLEET, AND S. YELTEKIN (2008): “Fiscal hedging with nominal assets,” *Journal of Monetary Economics*, 55, 710–727.
- MALIAR, L., AND S. MALIAR (2003): “Parameterized Expectations Algorithm and the Moving Bounds,” *Journal of Business and Economic Statistics*, 21(1), 88–92.
- MARCET, A., AND R. MARIMON (2019): “Recursive Contracts,” *Econometrica*, 87, 1589–1631.
- MARTIN, F. M. (2009): “A positive theory of government debt,” *Review of Economic Dynamics*, 12(4), 608–631.
- OTTONELLO, P., AND D. J. PEREZ (2019): “The Currency Composition of Sovereign Debt,” *American Economic Journal: Macroeconomics*, 11(3), 174–208.

- PERSSON, M., T. PERSSON, AND L. E. O. SVENSSON (2006): “Time Consistency of Fiscal and Monetary Policy: A Solution,” *Econometrica*, 74(1), 193–212.
- PFLUEGER, C., AND L. VICEIRA (2011): “Inflation-Indexed Bonds and the Expectations Hypothesis,” *Annual Review of Financial Economics*, 3, 139–158.
- ROTEMBERG, J. J. (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90(6), 1187–1211.
- SCHMITT-GROHE, S., AND M. URIBE (2004): “Optimal fiscal and monetary policy under sticky prices,” *Journal of Economics Theory*, 114, 198–230.
- SIU, H. E. (2004): “Optimal fiscal and monetary policy with sticky prices,” *Journal of Monetary Economics*, 114, 198–230.
- VALAITIS, V., AND A. VILLA (2024): “A machine learning projection method for macro-finance models,” *Quantitative Economics*, 15(1), 145–173.

ONLINE APPENDIX

A Motivating Evidence

In this section, we document some stylized facts regarding the composition of the U.S. government debt portfolio and its evolution over time, and link it to the macroeconomic environment. Figure 1 illustrates the evolution of the share of Treasury inflation-protected securities (TIPS) in the U.S. government debt portfolio. The U.S. Treasury started issuing TIPS in 1997 as a way of reducing its borrowing costs by taking on inflation risk and avoiding paying an inflation risk premium, offering investors a security that would allow them to hedge inflation, and providing access to a market-based measure of inflation expectations.¹⁹ After an initial expansion, the TIPS share stabilized at around 7% to 8% of the overall debt outstanding. Moreover, since the initial expansion, the TIPS share has been remarkably stable and persistent. Indeed, as we document in table A.1 below, the share displays an autocorrelation close to one.

In this paper, we are interested in understanding what determines this composition and evolution of the government debt portfolio. More specifically, we ask, how should governments optimally manage nominal and real bonds? Should governments issue more real bonds or less? And finally, and importantly, does it matter, and how? We examine these questions in the context of the U.S. macroeconomic environment that we illustrate in figure A.1.

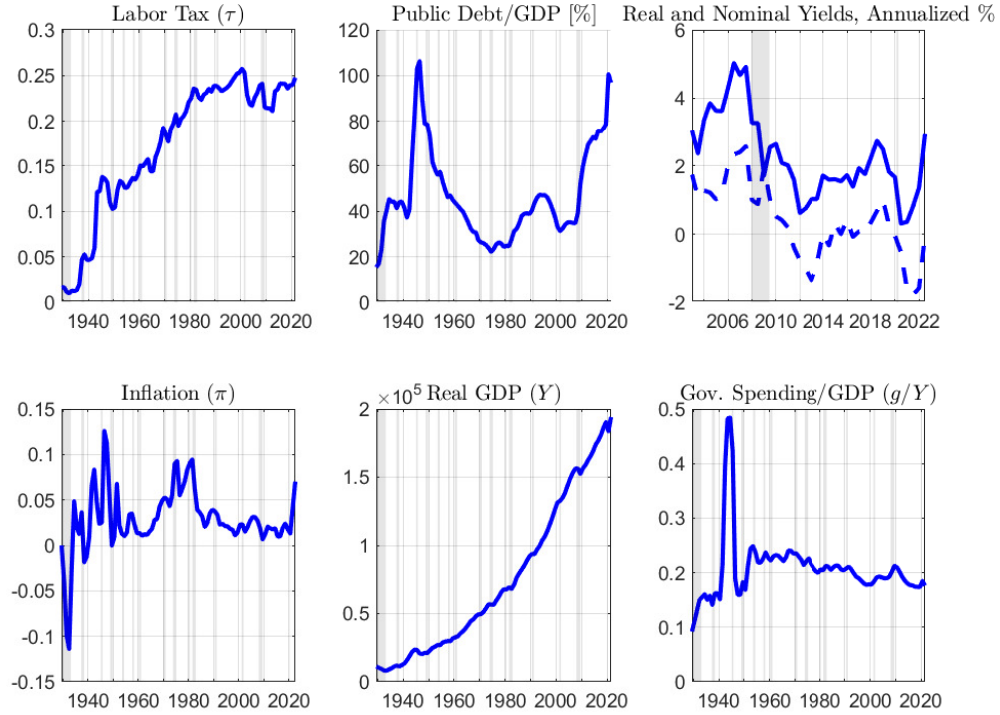
In figure A.1, we illustrate the evolution of inflation, government expenditures, taxes, nominal and real debt, nominal and real yields, macroeconomic growth, and the connection of the TIPS share, that is, the share of indexed debt in the government debt portfolio, with the macroeconomic experience in the U.S. For robustness, we consider a long sample from 1929-2021 and complement that with data on TIPS that became available in 1997 only.

In the top right panel, labor taxes have gone up rather steadily in our sample, and have stabilized at around 25 percent in the more recent experience, although they have exhibited some movements around that trend as well. The top middle panel shows a well-known and widely discussed pattern for the government debt held by the public expressed as a fraction of GDP, undergoing long and large swings over the past hundred years. Historically, the debt-to-GDP ratio had peaked at around 100 percent during World War II and hovered between around 40 and 60 percent of GDP before the financial crisis. In response to fiscal stimulus packages around the financial crisis and then the pandemic, the debt burden has recently reached World War II levels for the first time. Moreover, according to the CBO, under current policies it is projected to reach 200 percent of GDP by around 2050.²⁰ The rightmost

¹⁹See e.g. Fleming and Krishnan (2012) for a review of the history and the microstructure of the TIPS market.

²⁰See <https://www.cbo.gov/publication/58848>.

Figure A.1: U.S. MACROECONOMIC ENVIRONMENT



Notes: The figure shows the evolution of the U.S. macroeconomic environment from 1929 to 2021. The time series for labor tax, debt held by public, inflation (from output price deflator), real GDP, and government spending are constructed from the National Accounts (NIPA) tables provided by the Bureau of Economic Analysis (BEA). A detailed explanation about data construction can be found in [Clymo, Lanteri, and Villa \(2023\)](#). In the top-right panel, the continuous line represents nominal yields whereas the dashed line reports real yields. Both nominal and real yields are annualized and calculated using 5-year maturity bonds, reported by the US Department of Treasury, which can be accessed at the following link: https://home.treasury.gov/resource-center/data-chart-center/interest-rates/TextView?type=daily_treasury_yield_curve&field_tdr_date_value_month=202312.

top panel shows the evolution of the yields on 5-year nominal and real Treasury notes, and illustrates that these yields tend to move in parallel, with a difference that tends to be rather stable. That difference, sometimes referred to as *breakeven inflation*, is often interpreted as a market-based measure of inflation expectations, although differential liquidity premia and risk premia on these securities complicate this identification.

The leftmost lower panel documents the changing nature of inflation over our sample. While inflation had stayed low and stable since the early eighties, inflationary pressure has recently picked up again to levels not seen since the persistently high inflation rates in the 1970s. Indeed, after edging up to a 40-year high of 8.6 percent in 2022, the current annual inflation rate in the U.S. is still over 3 percent, substantially above the Federal Reserve's

target, while notably core inflation has recently been rising.²¹ The rightmost lower panel shows that government spending as a fraction of GDP has hovered around 20 percent ever since a peak in World War II, exhibiting little volatility in the recent U.S. macroeconomic environment.

Table A.1 presents portfolio statistics, as well as business cycle moments for some key variables. Since 1997, the real debt share in the U.S. government debt portfolio has amounted to 7 percent on average, leaving about 93 percent nominal debt. In line with the pattern displayed in figure 1, the real debt share is rather stable and persistent. The business cycle statistics in the lower panel show that government spending, taxes, as well as inflation all exhibited procyclical dynamics, although the latter only modestly so. Indeed, the conditional correlation between inflation and GDP has flipped sign repeatedly over time, leaving the unconditional correlation only mildly positive. While real debt has been quite evidently positively correlated with inflation since the inception of the TIPS market, nominal debt has exhibited a mildly negative correlation with inflation over the longer sample.

Table A.1: Relevant U.S. Data Moments

Description	Moments	Data
Avg. Inflation [%]	$\mathbb{E}(\pi) - 1$	2.87
Avg. Tax [%]	$\mathbb{E}(\tau)$	22.8
Real Portfolio Weight	$\mathbb{E}[b/(b + B)]$	0.07
Nominal Portfolio Weight	$\mathbb{E}[B/(b + B)]$	0.93
Autocorr. Real Portfolio Weight	$\rho_1(b/(b + B))$	0.94
Corr. Gov. Spending and GDP	$\rho(g, Y)$	0.23
Corr. Tax and GDP	$\rho(\tau, Y)$	0.35
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.05
Corr. Inflation and Real	$\rho(\pi, b)$	0.4
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.02

Notes: The table reports the portfolio weights of real and nominal bonds and the salient correlations among monetary and fiscal policy instruments. As shown in figure A.1, the real debt share in the U.S. government debt portfolio increased steadily before stabilizing at around 7 percent, leaving about 93 percent nominal debt. The table reports average taxes starting from the 1970, since when they have started to stabilize. All the correlations are computed using the entire available sample presented in figure A.1. Hence, all correlations, with the exception of $\rho(\pi, b)$, are computed from 1929 to 2021. $\rho(\pi, b)$ is computed starting from 1997, given the apparent recent adoption of TIPS. We detrend each variable (where necessary) before computing the correlations.

We take the moments reported in table A.1 as macroeconomic context for our quantitative analysis.

²¹See <https://www.bls.gov/cpi/>.

B Extensions

In this appendix, we collect a number of extensions and further supporting analysis. In particular, [B.1](#) reports additional material for the Full Commitment case, whereas [B.2](#) reports supporting material for the No Commitment case.

B.1 Full Commitment

In this appendix, we i) extend the one-period special case of section [3](#) under Full Commitment to allow for endogenous inflation, and ii) show that the main results from a Full Commitment model are robust to the maturity of government bonds, the degree of nominal rigidity, introduction of alternative shocks, the strength of interest rate response to inflation. We analyze these extensions one-by-one and show that, in all cases, it remains optimal to issue nominal bonds and to accumulate real assets and that the planner builds such leverage in good times, and then uses it to absorb negative shocks.

B.1.1 Endogenous Inflation in the Two-Date Setting

In this section, we further enrich the example in section [3](#) by allowing the government to choose not only the debt portfolio, but also inflation to examine the role of the Taylor rule constraint that appears in the dynamic setting. Accordingly, we consider a government that serves as an integrated fiscal and monetary authority. Assume, thus, that at date 1 there are two realizations of the exogenous government expenditure shock, i.e. a low state g_1^L and a high state g_1^H . Assume each realization realizes with a conditional probability $f(g_1|g_0)$. The problem is otherwise identical to the previous subsection, except that the Ramsey planner also chooses a state contingent plan for inflation in the second period. Hence, the optimal policy under full commitment requires to find

$$\{h_0(g_0), h_1(g_0, g_1), B_1(g_0), b_1(g_0), \mu_0(g_0), \mu_1(g_0, g_1), \pi_1(g_0, g_1)\},$$

such that welfare

$$c_0 - \frac{h_0^2}{4} + \beta \mathbb{E}_0 \left[c_1 - \frac{h_1^2}{4} \right] \tag{54}$$

is maximized.

First, we note that $\{h_0(g_0), h_1(g_0, g_1), B_1(g_0), b_1(g_0), \mu_0(g_0), \mu_1(g_0, g_1)\}$ are still optimally pinned down by expressions [\(16\)](#), [\(24\)](#), [\(25\)](#), [\(12\)](#), and [\(13\)](#), respectively. In particular, equations [\(24\)](#) and [\(25\)](#) continue to hold as long as $\pi_1(g_0, g_1^L) \neq \pi_1(g_0, g_1^H)$.

Second, when it comes to optimal inflation, we note that the objective function is continuous everywhere except when $\pi_1(g_0, g_1^L) = \pi_1(g_0, g_1^H)$, i.e. when there is zero-inflation volatility, since in that case nominal and real bonds become perfect substitutes. Hence, the objective function is not differentiable when $\pi_1(g_0, g_1^L)$ equals $\pi_1(g_0, g_1^H)$. For this reason a first-order condition approach cannot be used to pin down the optimal inflation. Intuitively, the planner faces the following trade-off.

On the one hand, if the planner chooses $\pi_1(g_0, g_1^L) = \pi_1(g_0, g_1^H) = \pi$, then it pays zero cost $\Phi(\pi_1) = 0$ but it needs to renounce the perfect tax smoothing benefits. In fact, in this case, it cannot achieve a complete markets equilibrium, since nominal and real bonds become perfect substitutes. On the other hand, if the planner chooses $\pi_1(g_0, g_1^L) \neq \pi_1(g_0, g_1^H)$, then the planner needs to pay a non-zero cost (either in the high state, low state, or both) but, in equilibrium, it can complete the markets and achieve perfect tax smoothing, as shown previously.

Figure A.2 illustrates this trade-off by plotting the welfare function in function of all possible combinations of $\pi_1(g_0, g_1^L)$ and $\pi_1(g_0, g_1^H)$. On the diagonal of the welfare function, when $\pi_1(g_0, g_1^L) = \pi_1(g_0, g_1^H)$, we see that welfare suddenly drops because the planner is unable to complete the market (at this point the welfare is computed using the one-bond solution). Such a choice therefore is not optimal. Accordingly, we note that (i) some level of inflation volatility is optimal in this setting, while costly, because it allows the Ramsey planner to complete the market, and (ii) there are two equivalent welfare-optimal combinations of inflation with symmetric deflationary and inflationary shocks, as indicated by the red dots. Intuitively, the planner finds it optimal to have minimal inflation volatility, either with inflationary or deflationary shocks, in order to simultaneously pay a minimal cost of nominal rigidities and complete the market by using leveraged positions of nominal and real bonds according to equations (24) and (25), provided that all debt limits are satisfied.

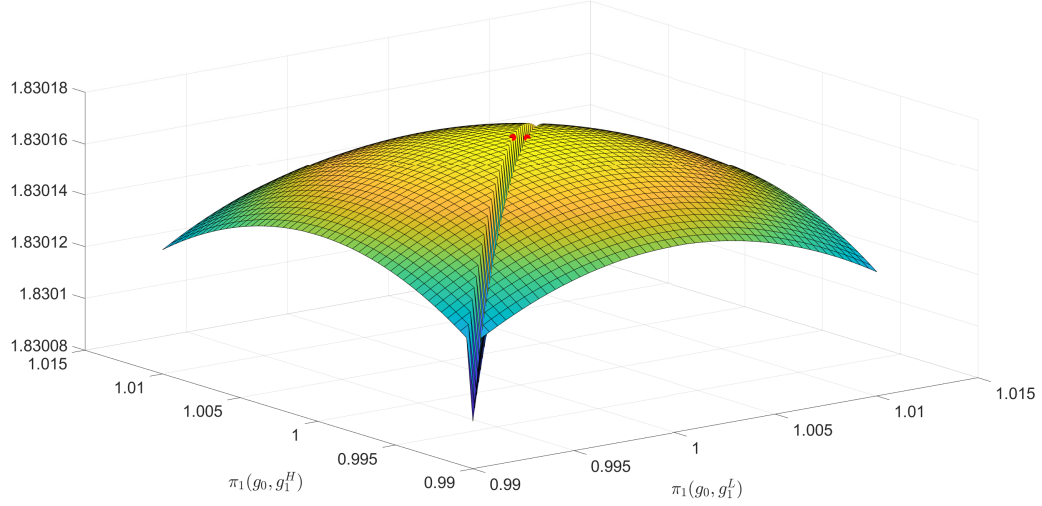
These considerations lead us to formulate the following preposition. Proposition 3 synthesizes the idea that, under incomplete markets and in presence of nominal and real debt, optimal fiscal and monetary policy requires a trade-off between the stabilization of inflation and the welfare-relevant output gap.

Proposition 3 (*Optimal Inflation and Stabilization of Output Gap*). *Given initial conditions B_0, b_0, g_0, π_0 , optimal monetary and fiscal policy is such that*

$$\pi_1^L = \pi_1^H \implies y_1^L - y_1^* \neq 0 \vee y_1^H - y_1^* \neq 0, \quad (55)$$

where y_1^* is the natural output and $y_1 - y_1^*$ is the welfare-relevant output gap. Natural output y_1^* is the output the planner can achieve under complete markets, i.e. the first-best when households and the government trade a full set of Arrow–Debreu securities.

Figure A.2: WELFARE FUNCTION



Notes: The figure shows welfare, i.e. equation (54), on a Cartesian grid for several combinations of $\pi_1(g_0, g_1^L)$ and $\pi_1(g_0, g_1^H)$ under the parameterization of subsection 3.1 and with $\pi = 1$. Red dots indicate the optimal inflation policy.

Proof. See appendix D.

Taylor Rule. These findings suggest that the optimal policy with integrated fiscal and monetary authorities features not only multiple equilibria, but also very large and opposite positions of real and nominal bonds with minimal inflation volatility. In order to allow for a more realistic setting reflecting policymakers' institutional arrangements, we enrich the quantitative model with an independent central bank that follows a Taylor rule. In this setting, inflation is still a choice variable for the planner, but subject to an additional constraint.²² The Taylor rule constraint implies a positive correlation between inflation and the nominal rate, in line not only with the data, but common to a wide range of economic settings, as noted by Bohn (1988).²³ In this sense, we view the addition of the Taylor rule constraint as one of the many possible equilibrium selection criteria and extensions that break the indifference between inflationary and deflationary responses to shocks from the perspective of the planner, which appears plausibly supported in the data. Indeed,

²²In other words, the government can engineer fiscal policies to shape a consumption growth profile such that it induces a desired inflation. Equivalently, since we are in general equilibrium, the government can choose inflation to engineer a consumption growth profile consistent with the Taylor rule and fiscal policies.

²³When the Taylor rule only responds to inflation, the consumption growth rate and inflation are likely to be positively correlated. That is, current consumption is low when inflation is high. Current consumption tends to fall when the realization of government expenditure is high. In that sense, under the Taylor rule, government expenditure shocks tend to be inflationary.

in terms of realistic institutional arrangements, it is worth noting that evidence from the Federal Reserve Board of Governors Tealbook highlights that central banks indeed follow pre-determined interest rate rules.²⁴

B.1.2 Robustness of the Optimal Portfolio

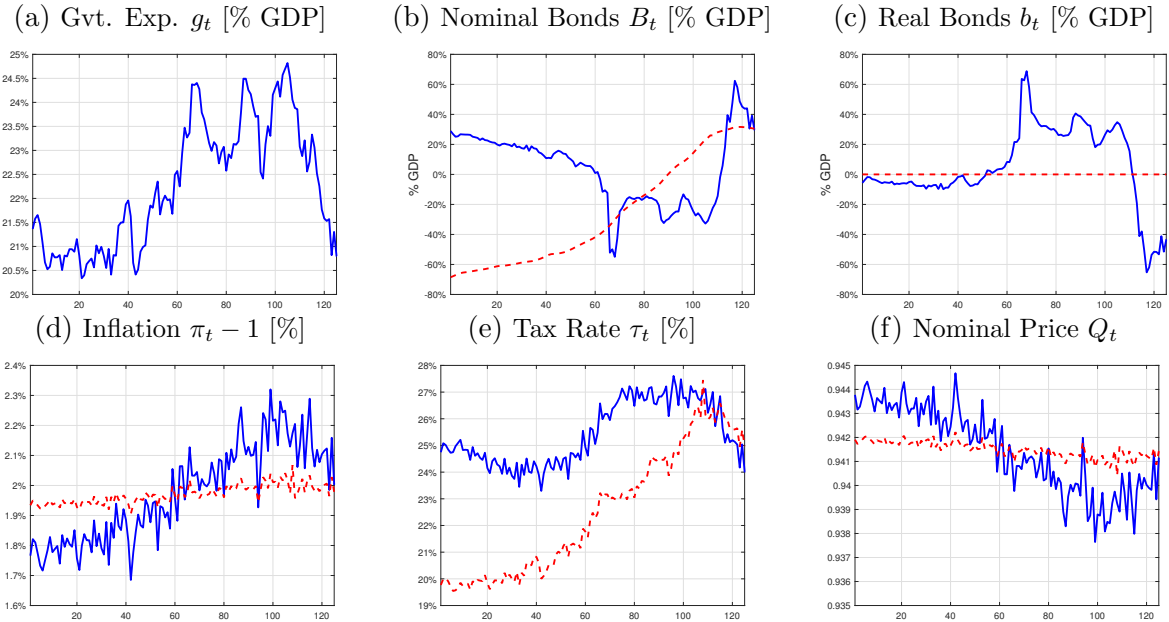
In this section, we confirm quantitatively that the main baseline results regarding the optimal portfolio of nominal and real bonds and the use of inflation under Full Commitment are robust to multiple extensions. Specifically, we show that the policy prescription to borrow in nominal bonds and accumulate real assets remains optimal when (i) the degree of nominal rigidity is increased, (ii) the bonds' maturity is extended, (iii) TFP shocks are introduced in the economy, and (iv) the strength of the interest rate response to inflation is changed. Each extension offers interesting economic insights. In section B.1.3 we show that a longer debt maturity allows the planner to spread inflation distortions over time, leading to a lower inflation volatility and even more leveraged bond positions. In section B.1.4 we find that the degree of nominal rigidity, as controlled by φ , matters for inflation volatility both in the baseline model and in the model with only nominal bonds. However, its effect is much greater in the model with only nominal bonds. In section B.1.5 we consider a model where the driving force is the total factor productivity shock and argue that it is optimal to issue nominal debt and accumulate real assets as long as inflation and discounted net present value of future government surpluses are negatively correlated. Lastly, in section B.1.6 we show that when monetary policy becomes more responsive to inflation, i.e. ϕ_π increases, inflation volatility falls and, like in extension (i), an even more leveraged bond position becomes optimal.

Stochastic Simulations. Before moving to robustness checks, in figure A.3, we first illustrate the simulated paths from the baseline Full Commitment setting from section 5. The figure displays a period of rising expenditures followed by a decrease, which is useful to highlight the difference in optimal policy over the expenditure cycle. The top left panel shows the exogenous process for government expenditures, which initially increase and reach a higher level at around period 60 and start to decline after period 100. The other five panels show policy variables in the baseline model (solid blue line) and the model without TIPS bonds (dashed red line). In the beginning when government expenditures are low, the planner starts with a leveraged portfolio position. As expenditures increase, the government responds by deleveraging and, in fact, temporarily issuing real bonds and accumulating

²⁴For instance, see page 5 in the “Report to the FOMC on Economic Conditions and Monetary Policy” at the following link: <https://www.federalreserve.gov/monetarypolicy/files/FOMC20170201tealbooka20170123.pdf>.

nominal assets, thereby actively rebalancing its portfolio. This is because the planner also optimally responds to increasing expenditures by engineering a persistent increase in inflation. Given the leveraged portfolio position, persistent inflation allows to relax the budget constraint and reduces the need to increase labor taxes as much as in the one-bond model. However, this comes at a cost. Due to persistently higher inflation, the nominal bond price falls by more than in the one-bond model. In light of such falling prices, the planner finds it cheaper to substitute to borrowing in real bonds. Once expenditures begin to fall, the planner chooses lower inflation and rebalances the portfolio back to nominal liabilities and real assets. This stands in contrast to the one-bond model. Here a rise in expected inflation increases the cost of new debt issuance. Therefore, the planner optimally keeps inflation stable and responds to rising government expenditure by further issuing nominal bonds. The baseline model thus prescribes an active role for policy across the business cycle, whereby in periods of high inflation it is optimal to rebalance the portfolio to include more real debt. Such active rebalancing in the face of movements in inflation render the real debt share volatile with minimal persistence, in sharp contrast to the U.S. experience.

Figure A.3: SIMULATION: POLICY VARIABLES



Notes: The figure shows an excerpt from the simulation of the Full Commitment model equilibrium dynamics. X-axes report time t . Solid blue line: baseline model. Dashed red line: benchmark model without TIPS. Both models are simulated with the same realization of government expenditure shocks. The same simulation was used to calculate moments in table 2.

B.1.3 Role of Maturity

In the main text, we introduced the model with short-term bonds. In this appendix, we augment the model by introducing long-term nominal and real bonds when both instruments exhibit a generic, but same, maturity N .²⁵ We then proceed to formulate the Ramsey problem and characterize the optimal policy.

Environment. The model is identical to section (2) except that the representative household saves through: (i) a N -period non-contingent nominal debt B_t^N traded at a price Q_t^N and (ii) a N -period non-contingent inflation-protected debt b_t^N traded at a price q_t^N . The government issues both types of debt, collects revenues in the current period and repays debt at maturity. In particular, the government repays nominal maturing debt at a unitary price and real maturing debt at a price $\prod_{j=1}^N \pi_{t-j+1}$. As before, the government levies a distortionary labor tax τ_t on labor income. The representative household, conjointly with government financial needs, make savings decisions in long-term nominal and real debts.

In every period t , the representative household receives labor and investment income according to the following budget constraint

$$c_t + Q_t^N B_t^N + q_t^N b_t^N + p_t S_{t+1} = (1 - \tau_t) w_t h_t + \frac{B_{t-N}^N}{\prod_{j=1}^N \pi_{t-j+1}} + b_{t-N}^N + (p_t + d_t) S_t. \quad (56)$$

Household Optimality. Households maximize utility (1) subject to their budget constraint (56). The intratemporal labor-consumption margin and the firm's stock pricing equation are identical to those of section (2). The Euler equations that price long-terms bonds are

$$u_c(c_t) \cdot Q_t^N = \beta \mathbb{E}_t u_c(c_{t+N}) \cdot \left(\prod_{j=1}^N \pi_{t+j} \right)^{-1}, \quad (57)$$

$$u_c(c_t) \cdot q_t^N = \beta \mathbb{E}_t u_c(c_{t+N}). \quad (58)$$

²⁵Note that the model with long-term bonds collapses to the short-term formulation when $N = 1$. Alternatively, we could have introduced maturities through long-term perpetuities with decreasing coupon rates. With our approach with $N = 5$ the problem requires to keep track of 26 state variables and solve for 10 policy functions. With perpetuities it would have required 8 state variables and 14 policy functions. With perpetuities, the additional 4 policy functions for bonds prices and associated Lagrange multipliers are required, since nominal and real bonds prices are expressed recursively and would not be substitutable directly in the implementability constraint. We chose our methodology since the stochastic simulation approach we adopted is scalable in function of the state variables but less effective and stable the more policies need to be solved jointly at each time step. Note also that with 8 state variables a stochastic simulation approach would still be needed.

Government. The government needs to finance exogenous spending g_t using labor income taxes and bonds, subject to the following budget constraint:²⁶

$$Q_t^{N-1} \frac{B_{t-1}^N}{\pi_t} + q_t^{N-1} b_{t-1}^N = \tau_t A h_t w_t - g_t + Q_t^N B_t^N + q_t^N b_t^N. \quad (59)$$

Implementability. Substitute τ , Q_t^N , and q_t^N in equation (59) using equations (4), (57), and (58) to get sequences of implementability constraints

$$\mathbb{E}_t \left[\frac{u_c(c_{t+N-1})}{\prod_{j=1}^{N-1} \pi_{t+j}} \right] \frac{B_{t-1}^N}{\pi_t} + b_{t-1}^N \mathbb{E}_t [u_c(c_{t+N-1})] = u_c(c_t) s_t + B_t^N \mathbb{E}_t \left[\frac{u_c(c_{t+N})}{\prod_{j=1}^N \pi_{t+j}} \right] + b_t^N \mathbb{E}_t [u_c(c_{t+N})], \quad (60)$$

where s_t is surplus as defined in subsection 2.5.

Optimal Policy with Full Commitment. We now consider optimal debt management and fiscal policy under the assumption that the government has Full Commitment and issue long-term nominal and real bonds. The government chooses stochastic sequences of allocations $\{c(g^t), h(g^t)\}_{t=0}^\infty$ and prices $\{w(g^t), \pi(g^t)\}_{t=0}^\infty$, and stochastic sequences of nominal and real non state-contingent bonds $\{B^N(g^{t-1}), b^N(g^{t-1})\}_{t=0}^\infty$ to maximize the household's utility (1) subject to the resource constraint (2), with associated Lagrange multiplier λ_t , the implementability constraint (60), with multiplier μ_t , the New-Keynesian Phillips Curve (8), with multiplier λ^π , the Taylor Rule (10), with multiplier λ^T , the bounds $(\bar{B}, \underline{B}, \bar{b}, \underline{b})$, with multipliers $(\bar{\Lambda}, \underline{\Lambda}, \bar{\lambda}, \underline{\lambda})$.

The first order conditions with respect to nominal bond B_t and real bond b_t are

$$\mu_t \cdot \mathbb{E}_t [\prod_{j=1}^N \pi_{t+j}^{-1} \cdot u_c(c_{t+N})] = \mathbb{E}_t [\mu_{t+1} \cdot u_c(c_{t+N}) \cdot \prod_{j=1}^N \pi_{t+j}^{-1}] + \beta^{-1} (\bar{\Lambda}_t - \underline{\Lambda}_t), \quad (61)$$

$$\mu_t \cdot \mathbb{E}_t [u_c(c_{t+N})] = \mathbb{E}_t [\mu_{t+1} \cdot u_c(c_{t+N})] + \beta^{-1} (\bar{\lambda}_t - \underline{\lambda}_t). \quad (62)$$

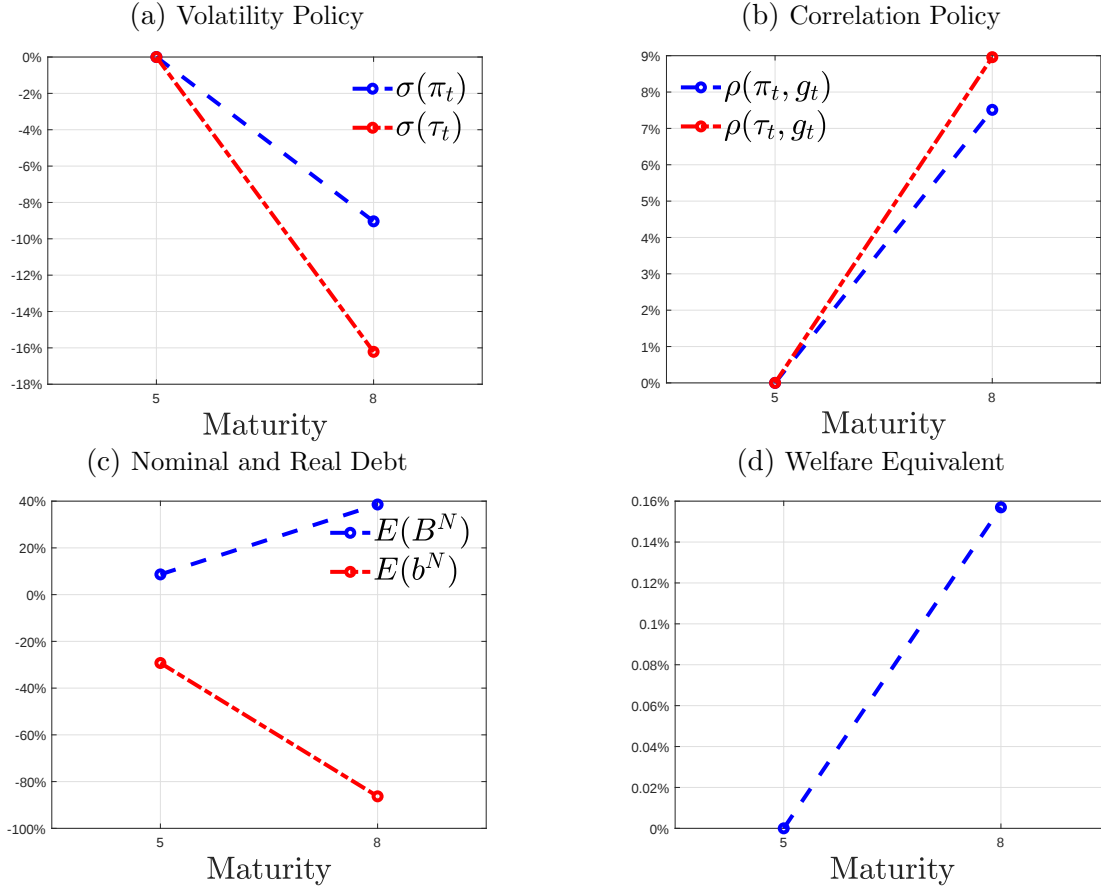
Note that equations (61) and (62) collapse to (30) and (31) when $N = 1$. The first order condition with respect to wage is identical to equation (32). The remaining first-order conditions with respect to consumption c_t and inflation π_t , together with further details about the computational methodology, can be found in appendix C.

Next, we analyze the role of maturity on optimal inflation, taxes and debt portfolio. In general, longer maturity brings greater benefits of using inflation. As the maturity increases, both inflation and taxes become less volatile, as shown in the left panel of figure

²⁶We implicitly assume that the government can buy back both nominal and real bonds from the private sector. As documented in the OECD report by [Blommestein and Hubig \(2012\)](#), more than 80 percent of countries engage in some forms of debt buyback and some of them they do so on a regular basis.

A.4. Intuitively, with longer available maturity the planner can inflate the nominal liability by increasing inflation by less but during a longer time horizon and in this way to minimize the cost of inflation coming from the nominal rigidity. As a consequence, both inflation and taxes become less volatile but more responsive to shocks as the available maturity increases from 5 to 8 years. This is associated with the consumption equivalent welfare gain of 0.16 percent.²⁷ Less volatile inflation makes the nominal and real bonds more substitutable as their prices become more positively correlated. This means that an even more leveraged position is optimal, as shown in bottom left panel of figure A.4.

Figure A.4: ROLE OF MATURITY



Notes: The figure shows comparative statics when the bond maturity is exogenously increased from five to eight years in our baseline model. Each panel describes the relative values of respective moments relative to the counterpart in the model where maturity is five years. The top left panel shows the volatility of inflation (dashed blue line) and the volatility of taxes (dotted-dashed red line). The top right panel shows the correlation of inflation with government expenditures (dashed blue line) and the correlation between taxes and government expenditures (dotted-dashed red line). The bottom left panel shows the average position of real and nominal bonds. The bottom right panel shows the welfare increase relative to the model, where the bond maturity is five years.

²⁷We set N equal to 5 years for both nominal and real bonds and use it as a benchmark. This is close to the average maturity of U.S. federal debt, which is 5.5 years.

B.1.4 Role of Nominal Rigidities

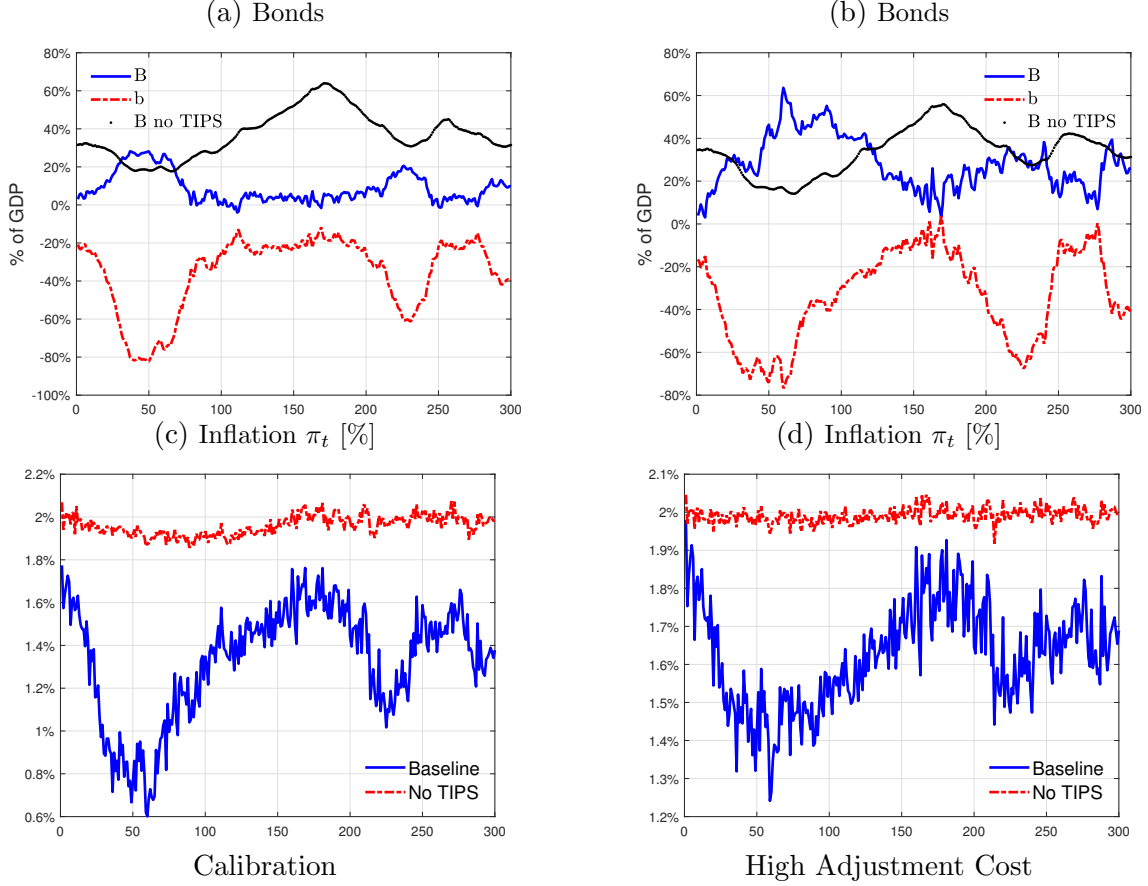
Next, we study the role of nominal rigidities for bonds positions and inflation volatility. As shown in [Siu \(2004\)](#), if the model is calibrated to match the empirically realistic degree of price rigidity, the real cost of inflation on firms pricing decisions begins to outweigh the benefits of relaxing the budget constraint and there is little incentive to use inflation in a model where only nominal bonds are available. Our results are consistent with [Siu \(2004\)](#). In figure [A.5](#) we compare the bond positions and inflation in our baseline model and the model with only nominal bonds. In addition to that, we analyze a counterfactual where we resolve both models with a much higher degree of nominal rigidity, controlled by the parameter φ . We find that, indeed, the size of nominal rigidity affects inflation volatility in both models but its' role is much more pronounced in the one-bond model. Compared to the baseline calibration, in the counterfactual with high inflation adjustment costs, inflation volatility falls by 44 percent in the baseline model and by 73 percent in the one-bond model. On the other hand, the degree of nominal rigidity has little effect on the optimal portfolio allocation.

B.1.5 Alternative Shocks

In this subsection, we analyze how the findings depend on the type of shock driving the economy. As discussed, the government expenditure shocks analyzed in the previous subsections are *inflationary*. That is, periods a high government expenditure are associated with high inflation. We consider an alternative model where the only source of stochasticity is given by total factor productivity shocks A_t , which are typically *deflationary*. We assume that A_t follows an AR(1) process in logs. Table [A.2](#) shows the relevant moments of the alternative model together with the baseline model with government expenditure shocks. Columns 4 and 5 in table [A.2](#) reveal a similar pattern. In the model with TFP shocks, average real bond to GDP ratio is still negative and nominal bond to GDP ratio is still positive. Moreover, the two bonds remain negatively correlated and inflation follows a similar pattern as in the baseline model in terms of how it co-moves with nominal and real bonds. Another way to understand the logic behind such pattern is to think in terms of the correlation of inflation with the net present value of government future surpluses as in [Angeletos \(2002\)](#).²⁸ In the spirit of [Angeletos \(2002\)](#), the positions of nominal and real bonds that would achieve perfect insurance depends on the correlation of inflation and the net present values of discounted

²⁸ [Angeletos \(2002\)](#) shows that portfolio that achieves perfect insurance against aggregate shocks in a model without state-contingent debt is the one that equates the net present value of future government surpluses state by state to the model with state-contingent debt. In our model the source of state-contingency is inflation and the position of nominal and real bonds depends on the correlation of inflation with the net present value of future surpluses.

Figure A.5: ROLE OF NOMINAL RIGIDITIES



Notes: The figure shows an excerpt from the simulation of model equilibrium dynamics. X-axes report time t . Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds. Both models are simulated with the same realization of government expenditure shocks.

future government surpluses. In the baseline model with exogenous government expenditure shocks, inflation typically increases in response to positive shocks. High expenditure also implies lower private consumption, which increases the current marginal utility and the discounts. Increasing discounts mean that net present values of future surpluses need to fall. As a consequence, in the model with exogenous government expenditure shocks net present values and inflation are negatively correlated. Now consider an economy where the only shock is the TFP shock. A negative TFP shock leads to lower current consumption and consequently, rising discounts and falling net present value of future surpluses. In order for the Taylor rule constraint to be satisfied, equation (10), nominal interest rate and inflation need to increase. From this analysis we can conclude that inflation and net present values are also negatively correlated in an economy with TFP shocks, which helps to understand why it is optimal to hold a qualitatively similar portfolio of nominal and real bonds with

both *inflationary* and *deflationary* shocks.

Table A.2: *g* AND TFP SHOCKS

Description	Moments	No TIPS <i>g shocks</i>	Baseline <i>g shocks</i>	Baseline <i>TFP shocks</i>
Avg. Real to GDP	$\mathbb{E}(b^N/Y)$	-	-0.3	-0.31
Avg. Nominal to GDP	$\mathbb{E}(B^N/Y)$	0.36	0.09	0.72
Corr. Tax and GDP	$\rho(\tau, Y)$	0.54	0.3	-0.69
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.39	0.39	-0.22
Corr. Tax and Inflation	$\rho(\tau, \pi)$	0.84	0.96	0.53
Corr. Inflation and Real	$\rho(\pi, b^N)$	-	0.93	0.29
Corr. Inflation and Nominal	$\rho(\pi, B^N)$	0.68	-0.69	-0.3
Corr. Real and Nominal	$\rho(b^N, B^N)$	-	-0.84	-0.90

Notes: The table shows the relevant moments from the model with TFP shocks only compared with two models with government expenditure shocks only. The third column (*No TIPS*) corresponds to the model with government expenditures shocks when nominal bonds are not available. The fourth column corresponds to the *baseline* model with both types of bonds and government expenditures shocks. The fifth column corresponds to the *baseline* model with both types of bonds and TFP shocks.

B.1.6 Monetary Policy Tightness

In the main body of the paper we impose a functional form for Taylor rule and a specific value for the coefficient controlling the central banks' response to inflation ($\phi_\pi = 1.2$). In this section we explore how different values of ϕ_π affect our main findings. Specifically, we solve the model with increasingly stronger central banks' response to inflation. Results presented in table A.3 indicate that the main finding of negatively correlated and leveraged position of real and nominal bonds holds. In addition to this, we observe that inflation volatility falls with higher ϕ_π , which is intuitive. With higher ϕ_π the same variation in expected consumption growth rate is related to smaller changes in inflation.

Table A.3: Role of ϕ_π

	$\rho(b_t^N, B_t^N)$	$\rho(B_t^N - b_t^N, g_t)$	$\mathbb{E}(B_t^N/Y_t)$	$\mathbb{E}(b_t^N/Y_t)$	$\sigma(\pi_t)$
$\phi_\pi = 1.2$	-0.8545	-0.8046	0.0912	-0.3054	0.0040
$\phi_\pi = 1.25$	-0.9171	-0.8332	0.4972	-0.2780	0.0032

Notes: Table shows unconditional moments from model solution for different values of Taylor rule's coefficient on inflation ϕ_π . Moments are calculated using the same realization of government expenditure process.

B.2 No Commitment

In this section, we present further details regarding the extension of the No Commitment case to long-term bonds, including a formal definition of the Markov-perfect Equilibrium (MPE).

B.2.1 Time-Consistent Optimal Policy with Long-term Nominal and Real Debt

In this appendix we augment the model by introducing long-term nominal government bond using perpetuities with decreasing coupon rates in the spirit of as in [Hatchondo and Martinez \(2009\)](#) and [Arellano and Ramanarayanan \(2012\)](#). This payment structures allows us to span different maturity composition starting from one-year bond.

Environment. We consider a stochastic production economy identical to the baseline, except that we assume nominal and real bonds issued in period t promise an infinite stream of deterministic coupon payments that decline at an exogenous constant rate $\delta_B \in [0, 1]$ and $\delta_b \in [0, 1]$, respectively. Specifically, one unit of debt issued in period t promises to repay a fraction $1 - \delta_B$ (or $1 - \delta_b$) of its outstanding value in each subsequent period. This structure implies that a fixed share of the remaining principal is repaid each period, akin to a consol or perpetuity with declining coupons. An advantage of this specification is that it collapses all future repayment obligations into a single state variable—the stock of long-term debt maturing in the current period. As a result, the law of motion for nominal debt is:

$$B_{t+1} = (1 - \delta_B) \frac{B_t}{\pi_t} + i_t^B,$$

where i_t^B denotes the new nominal bond issuance and π_t is gross inflation. The law of motion for real debt is:

$$b_{t+1} = (1 - \delta_b) b_t + i_t^b,$$

where i_t^b denotes the new real bond issuance.

The household's budget constraint becomes:

$$c_t + Q_t i_t^B + q_t i_t^b + p_t S_{t+1} = (1 - \tau_t) w_t h_t + (\delta_B + (1 - \delta_B) z_B) \frac{B_t}{\pi_t} + (\delta_b + (1 - \delta_b) z_b) b_t + (p_t + d_t) S_t, \quad (63)$$

where Q_t is the price of long-term nominal bonds, q_t is the price of long-term real bonds, and (z_B, z_b) are the coupons. The other terms follow the baseline.

Households maximize utility (1) subject to (49). The intertemporal Euler equation for

nominal and real bonds are:

$$u_c(c_t)Q_t = \beta \mathbb{E}_t \left[u_c(c_{t+1}) (\delta_B + (1 - \delta_B) (Q_{t+1} + z_B)) \frac{1}{\pi_{t+1}} \right], \quad (64)$$

$$u_c(c_t)q_t = \beta \mathbb{E}_t [u_c(c_{t+1}) (\delta_b + (1 - \delta_b) (q_{t+1} + z_b))]. \quad (65)$$

When $\delta_B = 1$ and $\delta_b = 1$, these reduce to the Euler equations for one-period bonds, as in the baseline model. For illustration, consider the case of a zero-coupon nominal bond, i.e., $z_B = 0$. Forward iteration of equation (64) then yields:

$$Q_t = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \beta^j \delta_B (1 - \delta_B)^{j-1} \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot \frac{1}{\prod_{k=1}^j \pi_{t+k}} \right],$$

highlighting that the price of long-term nominal bonds reflects the entire expected path of future inflation, discounted by both the maturity structure and marginal utility.

The government budget constraint is:

$$Q_t i_t^B + q_t i_t^b + \tau_t w_t h_t = g_t + (\delta_B + (1 - \delta_B) z_B) \frac{B_t}{\pi_t} + (\delta_b + (1 - \delta_b) z_b) b_t. \quad (66)$$

Combining the household and government budget constraints and imposing private-sector optimality yields the implementability condition:²⁹

$$(\delta_B + (1 - \delta_B) z_B) \frac{B_t}{\pi_t} + (\delta_b + (1 - \delta_b) z_b) b_t = s_t + Q_t \left(B_{t+1} - (1 - \delta_B) \frac{B_t}{\pi_t} \right) + q_t (b_{t+1} - (1 - \delta_b) b_t), \quad (67)$$

where s_t denotes the primary surplus, defined as in the baseline. When nominal debt has duration ($\delta_B < 1$), the planner can use not only current inflation π_t but also the expected path of future inflation embedded in Q_t to relax the implementability constraint.

MPE. We now proceed to redefine the optimal time consistent policy, given the presence of long-term nominal bond. As in the baseline, let all future governments set their policy according to functions $\tilde{c}(x), \tilde{h}(x), \tilde{w}(x), \tilde{Q}(x, B', b'), \tilde{q}(x, B', b'), \tilde{B}(x), \tilde{b}(x)$ and $\tilde{\pi}(x)$. Similarly to the baseline, let $\tilde{W}(x)$ be the present discounted value of government utility (1) as associated with the policy functions introduced above, given the state of the economy x . Using this notation, we can state the optimization problem of a government as to choose allocations

²⁹We leave the inter temporal bond Euler equations (64) and (65) as constraints, treating Q_t and q_t as choice variables due to their recursive nature.

and prices (c, h, w, Q, q) , as well as policies (B', b', π) to maximize

$$u(c) + v(l) + \beta \mathbb{E} [\tilde{W}(x')|x], \quad (68)$$

subject to the resource constraint

$$A \cdot h - c + g + \Phi(\pi) = 0, \quad (69)$$

the implementability constraint

$$s + Q \left(B' - (1 - \delta_B) \frac{B}{\pi} \right) + q \left(b' - (1 - \delta_b) b \right) - \left((\delta_B + (1 - \delta_B) z_B) \frac{B}{\pi} + (\delta_b + (1 - \delta_b) z_b) b \right) = 0, \quad (70)$$

the intertemporal Euler equation for nominal bond price

$$u_c(c) \cdot Q = \beta \mathbb{E} u_c(\tilde{c}(x')) \cdot \left[\delta_B + (1 - \delta_B) \left(\tilde{Q}(x', \tilde{B}(x'), \tilde{b}(x')) + z_B \right) \right] \cdot \tilde{\pi}(x')^{-1}, \quad (71)$$

the intertemporal Euler equation for real bond price

$$u_c(c) \cdot q = \beta \mathbb{E} u_c(\tilde{c}(x')) \cdot \left[\delta_b + (1 - \delta_b) \left(\tilde{q}(x', \tilde{B}(x'), \tilde{b}(x')) + z_b \right) \right], \quad (72)$$

the New-Keynesian Phillips Curve

$$u_c(c) \left(Y \cdot \left(\frac{\nu - 1}{\nu} + \frac{w}{A} \right) - \Phi_\pi(\pi) \pi \right) + \mathbb{E} [\beta u_c(\tilde{c}(x')) \cdot \Phi_\pi(\tilde{\pi}(x')) \tilde{\pi}(x') | x] = 0, \quad (73)$$

and the Taylor Rule

$$\pi^{1-\phi_\pi} \pi(x)^{\phi_\pi} \mathbb{E} [u_c(\tilde{c}(x')) \cdot \tilde{\pi}(x')^{-1} | x] - u_c(c) = 0. \quad (74)$$

Formally, a symmetric Markov-Perfect equilibrium, is a set of policy functions for allocations and long-term nominal bond price $c(x) = \tilde{c}(x)$, $h(x) = \tilde{h}(x)$, $w(x) = \tilde{w}(x)$, $Q(x) = \tilde{Q}(x)$, $q(x) = \tilde{q}(x)$, for bonds and inflation $B'(x) = \tilde{B}(x)$, $b'(x) = \tilde{b}(x)$, $\pi(x) = \tilde{\pi}(x)$ that maximizes (68) taking (69)-(74) as constraints. We tackle this problem numerically using an extension of the algorithm for the baseline model described in appendix C.3 where, on top of all previous guesses, we also need to guess the functions $\tilde{Q}(x, B', b')$ and $\tilde{q}(x, B', b')$ and iterate on equations (71)-(72).

C Computational Appendix

In this appendix we describe the computational procedure we used to solve the model under Full Commitment and No Commitment.

C.1 Algorithm under Full Commitment

We solve the model under Full Commitment using a generalization of the Parameterized Expectations Algorithm (den Haan and Marcet, 1990) proposed by Valaitis and Villa (2024). In this appendix, we describe how to adapt the methodology introduced by Valaitis and Villa (2024) in this context. At every instant t the information set is

$\mathcal{I}_t = \{g_t, \{B_{t-k}^N\}_{k=0}^{N-1}, \{b_{t-k}^N\}_{k=0}^{N-1}, \{\mu_{t-k}\}_{k=1}^N, \{\lambda_{t-k}^T\}_{k=1}^N, \{\lambda_{t-k}^\pi\}_{k=1}^N\}$.³⁰ Consider projections of the forward looking terms in the model onto $\mathcal{I}_t$. We model these relationships using a single hidden-layer artificial neural network $\mathcal{ANN}(\mathcal{I}_t)$ with 9 neurons in the hidden layer and as many neurons as many inputs and outputs in the input and output layers, respectively. Moreover, the activation functions we use are *hyperbolic tangent sigmoid* and the training algorithm is *Adaptive Gradient Descent*.

Before proceeding, we calculate the remaining first-order conditions with respect to consumption and inflation under Full Commitment, which were omitted in the main text.

The first-order condition with respect to consumption c_t is

$$\begin{aligned} & u_{cc}(c_t) - v_l(l)A^{-1} + \mu_t \left(u_{cc}(c_t)s_t + \frac{\partial s_t}{\partial c_t} u_c(c_t) \right) + \tilde{b}_t u_{cc}(c_t)(\mu_{t-1} - \mu_t) \\ & + \lambda_t^\pi \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A\nu} - \frac{u_{cc}(c_t)}{u_c(c_t)^2} \beta \mathbb{E}_t [u_c(c_{t+1}) \Phi_\pi(\pi_{t+1}) \pi_{t+1}] \right) + \\ & \lambda_{t-1}^\pi \frac{u_{cc}(c_t)}{u_c(c_{t-1})} \Phi_\pi(\pi_t) \pi_t - \lambda_t^T \frac{1}{\pi} \left(\frac{\pi_t}{\pi} \right)^{-\phi_\pi} u_{cc}(c_t) + \lambda_{t-1}^T u_{cc}(c_t) \frac{1}{\beta \pi_t} = 0. \end{aligned} \quad (75)$$

The first-order condition with respect to inflation π_t is³¹

$$\frac{v_l(l_t)}{u_c(c_t)} \frac{\Phi_\pi(\pi_t)}{A} = \mu_t \frac{\partial s_t}{\partial \pi_t} + B_t \frac{\mu_t - \mu_{t-1}}{\pi_t^2} + \lambda_t^\pi \mathcal{H}_t + \frac{\lambda_{t-1}^\pi \mathcal{K}_t}{u_c(c_{t-1})} + \left(\frac{\pi_t}{\pi} \right)^{-\phi_\pi - 1} \frac{\lambda_t^T \phi_\pi}{\pi^2} - \frac{\lambda_{t-1}^T}{\beta \pi_t^2}. \quad (76)$$

We describe the procedure for a generic maturity N . In particular, when maturity $N \geq 2$,

³⁰For example, with $N = 5$ the problem requires to keep track of 26 state variables and solve for 10 policy functions.

³¹Define $\mathcal{H}_t \equiv \left(\frac{\nu-1}{\nu} + \frac{w_t}{A\nu} \right) \Phi_\pi(\pi_t) - \mathcal{K}_t$ and $\mathcal{K}_t \equiv \varphi(2\pi_t - \pi)$.

then we approximate all the following terms:

$$\begin{aligned}
\mathcal{ANN}_1 &= \mathbb{E}_t \left[\frac{u_c(c_{t+N})}{\prod_{j=1}^N \pi_{t+j}} \right], \\
\mathcal{ANN}_2 &= \mathbb{E}_t \left[\frac{\mu_{t+1} u_c(c_{t+N})}{\prod_{j=1}^N \pi_{t+j}} \right], \\
\mathcal{ANN}_3 &= \mathbb{E}_t [u_c(c_{t+N})], \\
\mathcal{ANN}_4 &= \mathbb{E}_t [\mu_{t+1} u_c(c_{t+N})], \\
\mathcal{ANN}_5 &= \mathbb{E}_t [\varphi(\pi_{t+1} - \pi) \pi_{t+1}], \\
\mathcal{ANN}_6 &= \mathbb{E}_t [u_c(c_{t+N-1})], \\
\mathcal{ANN}_7 &= \mathbb{E}_t \left[\frac{u_c(c_{t+N-1})}{\prod_{j=1}^{N-1} \pi_{t+j}} \right], \\
\mathcal{ANN}_8^k &= \mathbb{E}_t \left[u_c(c_{t+N-k}) \left(\prod_{j=1}^{N-1} \pi_{t-k+j+1} \right)^{-1} \right], \quad \text{for } k \in \{1, 2, \dots, N-1\}, \\
\mathcal{ANN}_9 &= \mathbb{E}_t \left[u_c(c_{t+1}) \frac{1}{\pi_{t+1}} \right],
\end{aligned}$$

The solution procedure is summarized by the following algorithm. Given starting values $\mathcal{I}_0 = \{g_0, \{B_{-k}^N\}_{k=0}^{N-1}, \{b_{-k}^N\}_{k=0}^{N-1}, \{\mu_{-k}\}_{k=1}^N, \{\lambda_{-k}^\pi\}_{k=1}^N, \{\lambda_{-k}^T\}_{k=1}^N\}$ and initial weights for the $\mathcal{ANN}$, perform a stochastic simulation $\{c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^\pi, w_t\}_{t=1}^T$ as follows.³²

1. Impose the Maliar moving bounds, see [Maliar and Maliar \(2003\)](#), on debt. These bounds are particularly important and need to be tight and open slowly since the ANN at the beginning can only make accurate predictions around zero debt - that is our initialization point. Proper penalty functions are used to approximate the behavior of the Lagrange Multipliers (Λ, λ) which avoid out of bound solutions while the Maliar moving bounds are opening, see [Faraglia, Marcet, Oikonomou, and Scott \(2019\)](#) for more details.³³
2. At every instant t , given the information set $\mathcal{I}_t$ and the prediction $\mathcal{ANN}(\mathcal{I}_t)$, solve for $c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^\pi$, and w_t such that all the following equations are satisfied: the resource constraint (2), the implementability constraint (60), the New-Keynesian Phillips Curve (8), the Taylor Rule (10), the planner first-order condition with respect to nominal debt (61), the planner-first order condition with respect to real debt (62), the planner-first order condition with respect to wage (32), the planner-first order condition with respect to consumption (75), and the planner-first order condition

³²The network can be initially trained imposing $\{b_t\} = 0$.

³³We also find that including Λ and λ terms explicitly in the training set improves prediction accuracy.

with respect to inflation (76). Note that simply substituting predictions of the neural network in equations (61) and (62) such as

$$\begin{aligned}\mu_t &= \mathcal{ANN}_1(\mathcal{I}_t)^{-1} \left[\mathcal{ANN}_2(\mathcal{I}_t) + \frac{\bar{\Lambda}_t}{\beta} - \frac{\Lambda_t}{\beta} \right], \\ \mu_t &= \mathcal{ANN}_3(\mathcal{I}_t)^{-1} \left[\mathcal{ANN}_4(\mathcal{I}_t) + \frac{\bar{\lambda}_t}{\beta} - \frac{\lambda_t}{\beta} \right],\end{aligned}$$

render the system over-identified. We tackle this problem by using a Forward-States approach, as described in Faraglia, Marcet, Oikonomou, and Scott (2019). This involves approximating the expected value terms with the state variables that are relevant at period $t + 1$ and invoking the law of iterated expectations.³⁴ For example, equations (61) and (62) using the Forward-States approach are:

$$\begin{aligned}\mu_t &= [\mathbb{E}_t \mathcal{ANN}_1(\mathcal{I}_{t+1})]^{-1} \left[\mathbb{E}_t \mathcal{ANN}_2(\mathcal{I}_{t+1}) + \frac{\bar{\Lambda}_t}{\beta} - \frac{\Lambda_t}{\beta} \right], \\ \mu_t &= [\mathbb{E}_t \mathcal{ANN}_3(\mathcal{I}_{t+1})]^{-1} \left[\mathbb{E}_t \mathcal{ANN}_4(\mathcal{I}_{t+1}) + \frac{\bar{\lambda}_t}{\beta} - \frac{\lambda_t}{\beta} \right].\end{aligned}$$

3. If the solution error is large, or a reliable solution could not be found, the algorithm automatically restores the previous period ANN and tries to proceed with a reduced Maliar bound.³⁵
4. If the solution calculated shrinking the bound at iteration $i - 1$ is not satisfactory, the algorithm does not go back another iteration but uses the same ANN and tries to lower the $Bound_{i-1}$ again towards $Bound_{i-2}$. Once a reliable solution is found, the algorithm proceeds to calculate the solution for iteration i again, but with $Bound_i = Bound_{i-1} + (Bound_{i-1} - Bound_{i-2})$. In this way, if an error is detected multiple times we guarantee that both $Bound_i$ and $Bound_{i-1}$ keep shrinking toward $Bound_{i-2}$ and there must exist a point close enough to $Bound_{i-2}$ such that the system can be reliably solved with both $Bound_{i-1}$ and $Bound_i$.
5. If the solution found at iteration i is satisfactory, the ANN enters the learning phase supervised by the implied model dynamics, the Maliar bounds are increased and a new iteration starts again.

Keep repeating until the ANN prediction errors converge below a certain small threshold

³⁴For a detailed description of the procedure using polynomial regressions see Faraglia, Marcet, Oikonomou, and Scott (2019). Here we follow the same logic using the neural network.

³⁵If the unreliable solution has been detected in iteration i the algorithm restore the $i - 1$ environment and tries to proceed with $Bound_{i-1} = \alpha Bound_{i-1} + (1 - \alpha) Bound_{i-2}$.

and the simulated sequences for $c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^\pi$, and w_t converge among iterations and the difference between predicted and forecasted series is small. ³⁶

C.1.1 Implementation

Here we describe the implementation details of the baseline model with $N = 1$. Long maturity extensions with $N > 1$ are implemented very similarly.

Information Set. Although formally $\bar{\Lambda}_t, \underline{\Lambda}_t, \bar{\lambda}_t, \underline{\lambda}_t$ are not relevant state variables, we find that it helps to achieve smaller forecast errors if they are included in the information set $\mathcal{I}_t$ that the neural network is using. Then the information set $\mathcal{I}_t = \{g_t, B_{t-1}b_{t-1}, \mu_{t-1}, \lambda_{t-1}^T, \lambda_{t-1}^\pi, \bar{\Lambda}_{t-1}, \underline{\Lambda}_{t-1}, \bar{\lambda}_{t-1}, \underline{\lambda}_{t-1}\}$.

Neural Network. We use a single-layer a single hidden-layer artificial neural network $\mathcal{ANN}(\mathcal{I}_t)$ with 9 neurons in the hidden layer and as many neurons as many inputs and outputs in the input and output layers, respectively. We use the *hyperbolic tangent sigmoid* activation function. To initialize the network, we first train it using *Levenberg-Marquardt Backpropagation* and during the simulation we train it using *Adaptive Gradient Descent* algorithm. The reason is that the *Adaptive Gradient Descent* is more stable and prevents the algorithm from diverging.

Simulation. We set $T=5000$ and drop initial 150 periods in the training set. We stop the algorithm when the simulated sequences and the neural network weights stabilize. If prediction errors are small enough, we conclude that the model is solved.

Prediction Errors. When $N=1$, the neural network needs to approximate 5 expectations, namely:

$$\begin{aligned}\mathcal{ANN}_1 &= \mathbb{E}_t \left[\frac{u_c(c_{t+1})}{\pi_{t+1}} \right], \\ \mathcal{ANN}_2 &= \mathbb{E}_t \left[\frac{\mu_{t+1} u_c(c_{t+1})}{\pi_{t+1}} \right], \\ \mathcal{ANN}_3 &= \mathbb{E}_t [u_c(c_{t+1})], \\ \mathcal{ANN}_4 &= \mathbb{E}_t [\mu_{t+1} u_c(c_{t+1})], \\ \mathcal{ANN}_5 &= \mathbb{E}_t [\varphi(\pi_{t+1} - \pi) \pi_{t+1}].\end{aligned}$$

³⁶Note that the under uncertainty, the forecast errors will never go to 0. It is important that there are not systematic under or over prediction.

Figure A.6 shows a sample of predicted and simulated sequences.

C.2 Algorithm under No Commitment

We now describe the key steps of the algorithm we use to compute the NC equilibrium of the baseline model with exogenous g . We solve the model using global methods and, specifically, an algorithm similar in spirit to [Clymo and Lanteri \(2020\)](#). Recall that the state space is $x \equiv (B, b, g)$.

1. We discretize the sets of B and b with 11 nodes (linearly distributed) each. Moreover, we discretize the AR(1) process for g with Rouwenhorst with 11 nodes.
2. We guess the future government policy functions $B'(x)$, and $b'(x)$ as three-dimensional tensors with $11 \times 11 \times 11$ nodes and piece-wise linear interpolation. That is, $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$, with $1 \leq i \leq 11$, $1 \leq j \leq 11$, $1 \leq w \leq 11$. Evaluations of the policies outside of the specified indices are obtained through 3-D linear interpolation.
3. We define policy functions for inflation $\pi(x, x^g)$ and labor $h(x, x^g)$ on an augmented state space that includes both x and the additional space $x^g \equiv (B', b')$, that we use to evaluate all possible strategic interactions between current and future government. Note that given $\pi(x, x^g)$ and $h(x, x^g)$, it is possible to back-out the associated policy for consumption

$$c(x, x^g) = Ah(x, x^g) - g(x, x^g) - \Phi(\pi(x, x^g)), \quad (77)$$

from the resource constraint equation (2), for wage

$$w(x, x^g) = \Phi_\pi(\pi(x, x^g)) \frac{\pi(x, x^g)}{h(x, x^g)} - \frac{1}{h(x, x^g)} \mathbb{E} \left[\beta \frac{u_c(c(x', x^{g'}))}{u_c(c(x, x^g))} \cdot \Phi_\pi(\pi(x', x^{g'})) \pi(x') \right] - A \cdot \left(\frac{\nu - 1}{\nu} \right), \quad (78)$$

through the NKPC equation (8), and for labor tax

$$\tau(x, x^g) = 1 - \frac{c(x, x^g)^{\eta_c}}{(1 - h(x, x^g))^{\eta_w} w(x, x^g)}, \quad (79)$$

from the intra-temporal consumption-labor substitution margin equation (4).

4. Given the guesses for the linearly-interpolated future government policy functions $g(x) \simeq \tilde{g}(B_i, b_j, g_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$ and expressions (77),

(78), and (79); we solve with projection the implementability constraint, equation (37), and the Taylor Rule, equation (39) in order to find *augmented* policy functions for inflation $\pi(x, x^g)$ and labor $h(x, x^g)$ approximated using the following polynomial

$$\begin{aligned} P(x, x^g; \phi) \equiv & \phi(1) + \phi(2) \cdot B + \phi(3) \cdot b + \phi(4) \cdot B' + \phi(5) \cdot b' + \phi(6) \cdot g + \\ & + \phi(7) \cdot B^2 + \phi(8) \cdot b^2 + \phi(9) \cdot B'^2 + \phi(10) \cdot b'^2 + \phi(11) \cdot g^2 + \\ & + \phi(12) \cdot B \cdot g + \phi(13) \cdot b \cdot g + \phi(14) \cdot B' \cdot g + \phi(15) \cdot b' \cdot g + \phi(16) \cdot B \cdot b + \\ & + \phi(17) \cdot B' \cdot b' + \phi(18) \cdot B \cdot B' + \phi(19) \cdot B \cdot b' + \phi(20) \cdot b \cdot B' + \phi(21) \cdot b \cdot b', \end{aligned}$$

with different sets of parameters ϕ^π and ϕ^h , respectively.

5. Given updated guess for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$, and an initial guess for the value function $\tilde{W}(x')$, and given all the other policies given by expressions (77), (78), and (79), solve the government problem described in equation (35) using one iteration of Value Function Iteration in order to find updated best responses for all government policies $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$. Note that multiple iterations on the Value Function can be done, since we look for a symmetric MPE where all best responses and value functions converge to a fixed point, i.e. all governments are symmetric.
6. Use the updated best responses for all government policies $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$, to restart from point 4. Iterate till convergence. At this step we use all policies to simulate an equilibrium sequence of T=10.000 periods. We declare the algorithm has converged when the maximum absolute errors of the simulated sequences for consumption, labor, and bonds between two consecutive iterations is in the order of 10^{-3} or lower.
7. At convergence, the *augmented* policy functions for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$ can be reduced to standard policy functions just in function of the state space x by plugging the converged government optimal policies: $\pi(x) = P(x, (\tilde{B}(x), \tilde{b}(x))); \phi^\pi$ and $h(x) = P(x, (\tilde{B}(x), \tilde{b}(x))); \phi^h$.

C.3 Algorithm under No Commitment with Long-term Bonds

We now describe the key steps of the algorithm we use to compute the NC equilibrium of the baseline model with exogenous g and long-term government bond. We solve the model using global methods and, specifically, an algorithm similar in spirit to [Clymo and Lanteri \(2020\)](#). Recall that the state space is $x \equiv (B, b, g)$.

1. We discretize the sets of B and b with 11 nodes (linearly distributed) each. Moreover, we discretize the AR(1) process for g with Rouwenhorst with 11 nodes.
2. We guess the future government policy functions $B'(x)$, and $b'(x)$ as three-dimensional tensors with $11 \times 11 \times 11$ nodes and piece-wise linear interpolation. That is, $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$, with $1 \leq i \leq 11$, $1 \leq j \leq 11$, $1 \leq w \leq 11$. Evaluations of the policies outside of the specified indices are obtained through 3-D linear interpolation.
3. We define policy functions for inflation $\pi(x, x^g)$, labor $h(x, x^g)$, and bond prices $(Q(x, x^g), q(x, x^g))$ on an augmented state space that includes both x and the additional space $x^g \equiv (B', b')$, that we use to evaluate all possible strategic interactions between current and future government. Note that, similarly to the case without long-term bond, given $\pi(x, x^g)$ and $h(x, x^g)$, it is possible to back-out the associated policy for consumption exactly as explained in the algorithm with short-term nominal bond.
4. Given the guesses for the linearly-interpolated future government policy functions $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$ and expressions (77), (78), and (79); we solve with projection the implementability constraint, equation (70), the Taylor Rule, equation (74), and the nominal bond intertemporal Euler, equation (64), in order to find *augmented* policy functions for inflation $\pi(x, x^g)$, labor $h(x, x^g)$, $Q(x, x^g)$, and $q(x, x^g)$. The first two are approximated using the following polynomial

$$\begin{aligned}
P(x, x^g; \phi) \equiv & \phi(1) + \phi(2) \cdot B + \phi(3) \cdot b + \phi(4) \cdot B' + \phi(5) \cdot b' + \phi(6) \cdot g + \\
& + \phi(7) \cdot B^2 + \phi(8) \cdot b^2 + \phi(9) \cdot B'^2 + \phi(10) \cdot b'^2 + \phi(11) \cdot g^2 + \\
& + \phi(12) \cdot B \cdot g + \phi(13) \cdot b \cdot g + \phi(14) \cdot B' \cdot g + \phi(15) \cdot b' \cdot g + \phi(16) \cdot B \cdot b + \\
& + \phi(17) \cdot B' \cdot b' + \phi(18) \cdot B \cdot B' + \phi(19) \cdot B \cdot b' + \phi(20) \cdot b \cdot B' + \phi(21) \cdot b \cdot b',
\end{aligned}$$

with different sets of parameters ϕ^π and ϕ^h , respectively. The *augmented* policy function $Q(x, x^g)$ and $q(x, x^g)$ are approximated on a mesh using piece-wise linear interpolation. We treat $(\pi(x, x^g), h(x, x^g))$ and $(Q(x, x^g), q(x, x^g))$ differently because equations (64)-(65) give us a closed form solution for the implied prices given all guesses.

5. Given updated guess for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$, $Q(x, x^g)$, and $q(x, x^g)$, and an initial guess for the value function $\tilde{W}(x')$, and given all the other policies given by expressions (77), (78), and (79), solve the government problem described in equation (35) using one iteration of Value Function Iteration in order to find updated best responses for all government policies $g(x) \simeq \tilde{g}(B_i, b_j, g_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$. Note that multiple iterations on the Value Function can be done,

since we look for a symmetric MPE where all best responses and value functions converge to a fixed point, i.e. all governments are symmetric.

6. Use the updated best responses for all government policies $B'(x) \simeq \tilde{B}(B_i, b_j, g_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, g_w)$, to restart from point 4. Iterate till convergence. At this step we use all policies to simulate an equilibrium sequence of $T=10.000$ periods. We declare the algorithm has converged when the maximum absolute errors of the simulated sequences for consumption, labor, bonds, and long-term nominal bond price between two consecutive iterations is in the order of 10^{-3} or lower.
7. At convergence, the *augmented* policy functions for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$, $Q(x, x^g)$, and $q(x, x^g)$ can be reduced to standard policy functions just in function of the state space x by plugging the converged government optimal policies: $\pi(x) = P(x, (\tilde{B}(x), \tilde{b}(x)); \phi^\pi)$, $h(x) = P(x, (\tilde{B}(x), \tilde{b}(x)); \phi^h)$, $Q(x, (\tilde{B}(x), \tilde{b}(x)))$, and $q(x, (\tilde{B}(x), \tilde{b}(x)))$.

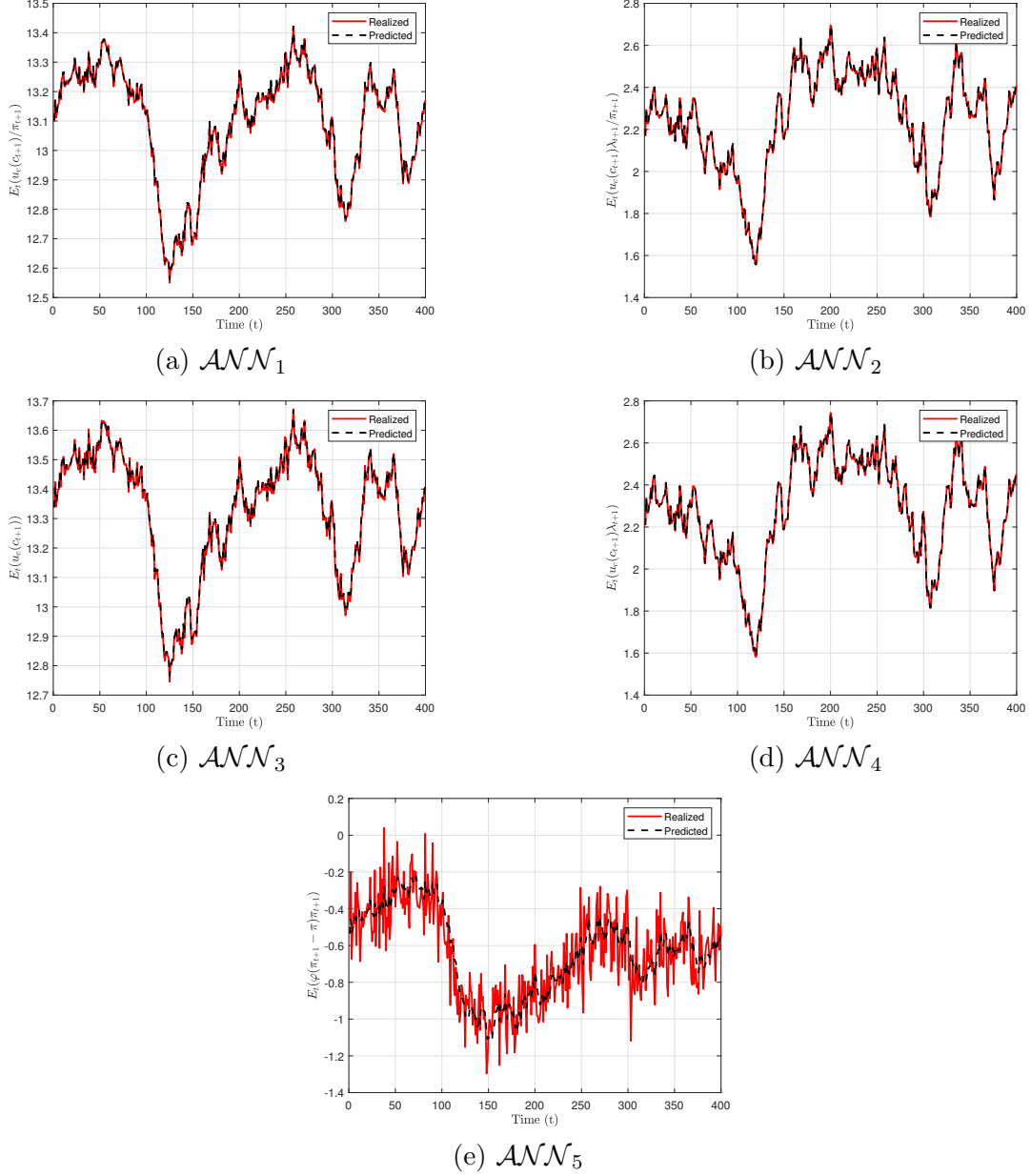
D Mathematical Appendix

Proof of Proposition 1. Result (17) follows from equation (14), combined with a formula for $\mu_t(h_t)$, which can be derived directly from equation (16). Apply the definition of expectation to get $l_1^H = l_1^L \kappa$, with $\kappa = \frac{f^H}{f^L} \cdot \frac{\mathbb{E}_0 \frac{1}{\pi_1} - \frac{1}{\pi_1^H}}{\frac{1}{\pi_1^L} - \mathbb{E}_0 \frac{1}{\pi_1}} = 1$, given that $f^L + f^H = 1$. Similarly, result (18) follows from equation (15), combined with the formula for μ . Apply the definition of expectation to get $l_1^L = l_0 \eta$, with $\eta = f^L + f^H \frac{1}{\kappa} = 1$. ■

Proof of Proposition 2. Result (24) follows directly from equation (21). Result (25) follows directly from equation (23). ■

Proof of Proposition 3. When $\pi_1^L = \pi_1^H$, the planner cannot achieve smoothing of the Lagrange multiplier across states $\mu_1^H = \mu_1^L$, since nominal and real debt become perfect substitutes. To see this, use the first-order conditions with respect to nominal and real debt, i.e. equations (14) and (15). When $\pi_1^L = \pi_1^H$, equations (14) and (15) effectively collapse to the same equation, i.e. $\mu_0 = \mathbb{E}_0[\mu_1]$, as the two bonds become perfect substitutes. This is confirmed by expression (24) for B_1 , which yields a singularity when $\pi_1^L = \pi_1^H$. Hence, $\pi_1^L = \pi_1^H \implies \mu_1^H \neq \mu_1^L$. Since, there is a one-to-one mapping between μ_1 and h_1 , given by equation (16), and since $y_1 = Ah_1$, it follows that $\pi_1^L = \pi_1^H \implies y_1^H \neq y_1^L$. Since y_1^* is defined as the output under complete markets, perfect smoothing commands $y_1^* = y_1^{*H} = y_1^{*L}$. Hence, it must be the case that either $y_1^L - y_1^* \neq 0$ or $y_1^H - y_1^* \neq 0$. ■

Figure A.6: Predicted and Realized Sequences.



Notes: The figure shows the realized and the predicted sequences for the relevant terms in the model with $N = 1$. Solid red line shows the realized sequences and dashed black line shows the predicted sequences.