Housing Illiquidity, Asset Prices, and the Amplification of Macroeconomic Shocks*

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Abstract

How does housing illiquidity affect household risk-aversion and saving behavior? This paper shows that when housing services provide utility, the risk over the relative consumption ratio of nondurables and houses determines household relative risk aversion and drives asset prices. I show in a calibrated heterogeneous-agents model that accounting for the relative consumption risk (i) helps to explain challenging asset pricing facts, such as the countercyclical market price of risk and a stable risk-free rate, (ii) greatly amplifies the business cycle fluctuations, (iii) illuminates the new source of business cycle costs (iv) helps to understand the endogenous variation in uncertainty.

Keywords: Search and Matching, Labor Income Risk, Portfolio Choice, Business Cycles.

JEL classification: E24, E32, G11, D14.

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1 Introduction

An extensive literature has documented that households hold assets that differ in their liquidity, and that a majority of household wealth is invested in illiquid assets.[1] Accounting for this asset illiquidity has helped economists to understand the responsiveness of individual consumption to economic policies and the effects of aggregate shocks on real economic quantities.[2] Because of the literature's typical focus on household consumption and the methodological difficulties involved, the understanding of how housing illiquidity interacts with income risk and investment in the stock market has remained limited. In this paper, I examine how illiquidity in the housing market matters for household discount factor, and through this, for the aggregate economy.

I model housing both as a component of wealth and as a consumption good that enters the utility function. As long as nondurable consumption and houses enter utility nonseparably, the nondurable-to-house ratio becomes a second factor in the household stochastic discount factor in addition to the standard consumption growth. The idea is intuitive: households dislike the variation in the nondurable-to-house ratio but optimally choose to adjust their house infrequently and in large quantities due to illiquidity and indivisibilities that define the housing markets. Such representation bears similarities with the Campbell and Cochrane (1999a) habits model: the relative risk aversion and therefore sensitivity of the discount factor to shocks depends on the state of the economy, summarized by the nondurable-to-housing ratio of the stock owners. This provides rich asset pricing implications that can be tested using micro-data. The main contribution of this paper is to show that housing illiquidity is an important driver of asset prices and provides a parsimonious way to understand challenging asset pricing facts, such as why households require higher return for risk in recessions.

In order to understand the role of illiquidity on discounts, I proceed in two steps. First, in Section 2 I present an illustrative model that provides conditions when illiquidity is irrelevant for household discounts and then shows analytically how it enters the risk-free rate and the equity premium. Second, to gauge the quantitative importance of illiquidity on discounts, in Section 3 I build a general equilibrium heterogeneous agent incomplete markets model that can account for limited stock market participation and differences in asset portfolio composition across the wealth distribution. The model also accounts for differences in sources of income and the types of income risk that households face depending on their asset portfolio. All these features are crucial to match the wealth and income profile of the marginal investor pricing the assets.

The quantitative model includes household portfolio choice problems based on liquidity as well as on risk. Households can save in risk-free bonds and risky shares, both of which are liquid, and houses that are risk-free but indivisible and subject to trading frictions. Such an elaborate asset

[1] In the data, median-wealth households hold more than 70% of the wealth in illiquid assets and households in the top wealth percentiles more than 30%. Source: PSID. See Appendix E for more information.

[2] See Kaplan and Violante (2014); Kaplan, Moll, and Violante (2018); Berger and Vavra (2015); and Bayer, Luetticke, Pham-Dao, and Tjaden (2019), among many others.

structure renders the model broadly consistent with a set of cross-sectional facts: high concentration of wealth and stock ownership, limited stock market participation, and the fact that the share of household wealth invested in real estate declines with wealth and the share of wealth invested in stocks increases with wealth. The intuition is simple. The share of wealth invested in real estate decreases with wealth because houses are essential consumption goods. Since stocks provide financial returns but houses do not, richer households receive larger returns on their portfolios and this differential drives the wealth dispersion. While stocks are traded in Walrasian markets, the housing market is characterized by trading frictions. In order to buy or sell houses, households engage in competitive search by posting prices and quantities and interact with each other through real estate brokers. On the production side, firms engage in random search in the labor market and post vacancies taking the household stochastic discount factor as given. Search in the labor market allows to have a setting that generates endogenous variation in both labor and dividend income that is relevant for how households discount the future. In Section 4, I show that the calibrated model features a qualitatively different behavior from the model where illiquidity is absent. The main illiquidity model can explain the large and countercyclical market price of risk, whereas in the standard model without illiquidity it is small and procyclical. Consequently, the business cycle variation in prices, unemployment, and output and greatly amplified. For instance, unemployment volatility increases by 100% only because of the presence of illiquidity in the discount factor. Achieving such amplification under standard calibration of the search-and-matching model is known to be challenging Shimer (2005).

I show that housing illiquidity is the dominant transmission channel from income uncertainty to asset prices. As firms reduce their hiring in recessions, unemployment increases, creating more uncertainty about both labor and dividend income. In Section 4.2.1, I explain that models' ability to generate endogenous variation in uncertainty is an overlooked feature, present in a standard search-and-matching model. It arises due to congestion externality, which makes vacancies to be more responsive to shocks in a slack labor market and implies more uncertain dividends. This uncertainty gets priced in by the household who now face more uncertainty about the future consumption and its' allocation among nondurables and housing. Because of housing illiquidity and indivisibility households know that their nondurable-to-housing ratio is likely to change either because they will keep the same house and reduce the nondurable consumption or will make a large adjustment in their housing without similar changes to nondurable consumption.[3] As long as preferences over nondurables and housing are nonseparable, households dislike such variation and demand higher return for risk, thus lowering the asset prices. As the stock market discounts increase, firms take this into account in their investment decisions and hire even fewer workers, thus creating even more income uncertainty in the economy. Therefore, the model features a novel general equilibrium mechanism where both micro and macro uncertainty rise endogenously, in

[3]This is a typical mechanism in the liquid-illiquid asset framework (Kaplan and Violante, 2014).

response to standard productivity level shocks. The main driver of this endogenous amplification is the household concern about the nondurable-to-housing ratio and its role on the stochastic discount factor. Quantitatively, I show that this mechanism is important even after accounting for the concentration of stock ownership by the rich households and matching their portfolio allocation into houses and other assets. While the idea that households dislike the variance of their nondurable consumption relative to housing has many similarities with the habits model, I show that the illiquidity channel, unlike the habits model, is quantitatively strong in a general equilibrium model with consumer optimization and after accounting for limited stock market participation.

Since the business cycles are greatly amplified, in Section 5 I study the implications for welfare and policy. Specifically, in Section 5.1 I show that welfare costs of business cycles are greatly amplified relative to a one-asset model without illiquid housing. Calculations show that the welfare difference in terms of consumption equivalent is 13.27%. Welfare costs arise both due to the fact that households in the baseline model tend to have more illiquid wealth portfolios and due to the fact that they dislike when their nondurable consumption varies relative to their consumption of housing services. Motivated by this, in Section 5.2 I study how policies intended to insure labor income help to alleviate the business cycle costs. Specifically, I find that my model provides novel implications for policies that affect wage rigidity, such as unemployment insurance, termination, or minimum wage laws. From an aggregate point of view, rigid wages are associated with a countercyclical labor share and provide implicit insurance for workers at the expense of capital owners. However, the more rigid wages become, the more unemployment rises in recessions. This means that insurance for those who remain employed comes at the cost of a larger risk of losing a job from the individual's point of view. I show that in my framework where households dislike variation in nondurable-to-house ratio, the equilibrium relation between household welfare and wage rigidity is nonmonotonic. As wages become more rigid, concerns about individual risk begin to outweigh the benefits of insurance at the aggregate level. In contrast, such a trade-off is absent in a standard one-asset heterogeneous-agent model.

The importance of housing illiquidity in this paper depends crucially on whether households view nondurables and houses as complements or substitutes. In the quantitative part, I show that cross-sectional data is informative about household preference regarding these two consumption goods. I endogenously calibrate the intratemporal elasticity of substitution to match the relative consumption ratio between nondurables and houses of the top percentiles of the US wealth distribution. In addition to that, leveraging on the CoreLogic house transaction data, I provide novel estimates of the intratemporal elasticity of substitution that exploits the house price variation across the US counties. I show that the elasticity that I endogenously calibrate is broadly consistent with the empirical estimates, as both approaches point to the two goods being relative complements.

Related Literature

The paper builds on the literature that studies how household portfolio allocation between liquid and illiquid assets affects real quantities (Kaplan and Violante, 2014; Berger and Vavra, 2015; Kaplan, Moll, and Violante, 2018; Bayer, Luetticke, Pham-Dao, and Tjaden, 2019) and the literature on housing market dynamics and consumption expenditure (Kaplan, Mitman, and Violante, 2019; Hedlund and Garriga, 2020; Berger, Guerrieri, Lorenzoni, and Vavra, 2017). This paper shows that portfolio allocation to assets that differ in liquidity affects household attitudes toward risk and examines how liquidity frictions affect the real economy through the risk attitudes of stock owners.

In doing so, the paper follows a literature studying the relationship between housing consumption and asset prices. Flavin and Nakagawa (2008) use cross-sectional data on housing consumption to estimate a model in which housing and nondurable consumption are complements and houses subject are to fixed adjustment costs. They show that such a model exhibits a time-varying risk aversion and provides a structural interpretation of Campbell and Cochrane's (1999a) habits model. In a contemporaneous paper to Flavin and Nakagawa (2008), Piazzesi, Schneider, and Tuzel (2007) propose a model in which consumption and freely adjustable housing services enter the CES utility as relative substitutes and show that when disciplined to time-series data on housing expenditure, the model introduces a new channel called composition risk. The authors show that through this channel, the share of housing expenditure predicts excess stock returns over long horizons. My paper follows Flavin and Nakagawa (2008) by embedding their mechanism in a general equilibrium model with a realistic degree of heterogeneity and time-varying income risk. This paper is also related to Duffie and Constantinides (1996), who show that time-varying individual income risk can drive asset prices. I show that this channel is quantitatively much more important when the consumption of housing services matters for the discount factor.

In terms of implications, my paper contributes to the burgeoning literature on the relation between stock and labor markets (Hall, 2017; Petrosky-Nadeau, Zhang, and Kuehn, 2018; Kehoe, Lopez, Midrigan, and Pastorino, 2022; Borovicka and Borovickova, 2018; Clymo, 2020; Kilic and Wachter, 2018; Liu, Miao, and Zha, 2016). Hall (2017) shows that time variation in discounts is a quantitatively important channel in generating unemployment volatility in a Diamond-Mortensen-Pissarides type search-and-matching model. Hall (2017) does this by feeding the data on stock market discounts into a search-and-matching model with credible bargaining. Petrosky-Nadeau, Zhang, and Kuehn (2018) argue that a comovement between discounts and unemployment can arise endogenously if the vacancy posting costs are countercyclical. In contrast to my paper, the main mechanism in these papers works through the risk-free rate channel, and since Hall (2017) treats stock market discounts as exogenous, there is no general equilibrium amplification between stock and labor markets. Clymo (2020) studies how the complementarity between capital and labor increases the elasticity of vacancies to profits —and therefore the unemployment —while taking discounts from the stock market as given. A contemporaneous paper by Kehoe, Lopez, Midrigan, and Pastorino (2022) argues that transferable and endogenously accumulated human capital is crucial to increasing the above-mentioned elasticity. They show that the model with standard preferences from the asset-pricing literature and endogenous human capital accumulation delivers unemployment volatility of plausible magnitude. Kilic and Wachter (2018) propose an exogenous and time-varying disaster risk as the driver of the comovement between asset prices and unemployment. Liu, Miao, and Zha (2016) shows that falling land prices reduce firms' collateral values and reduce their ability to invest in both capital and vacancies, thus amplifying unemployment volatility. My paper contributes to the literature by proposing a new mechanism that can be disciplined using cross-sectional data. In addition, rich cross-sectional heterogeneity allows me to study the differential effects of unemployment volatility across the wealth distribution.

This paper differs from the literature that examines how time-varying idiosyncratic risk affects output in demand-driven economies (Bayer, Luetticke, Pham-Dao, and Tjaden, 2019; Den Haan, Rendahl, and Reigler, 2018; Ravn and Sterk, 2017; Graves, 2023; Larkin, 2023). In these papers, high unemployment risk causes people to increase their savings or to substitute to safe but unproductive assets. This, in combination with New Keynesian frictions, implies that increasing idiosyncratic risk results in falling consumption demand and, therefore, output. In contrast, the mechanism in my paper does not rely on any of the New Keynesian features, but instead, the risk attitudes of the stock owners affect the aggregate output and individual incomes of the poor through the stochastic discount factor in firms' vacancy posting problem.

My model features a similar interaction between the household sector and firms when the labor market is frictional as in Gornemann, Kuester, and Nakajima (2021); However, theirs is essentially a one-asset model that focuses on differential effects of monetary policy across the wealth distribution. This paper introduces search frictions in the housing market, similar to Hedlund and Garriga (2020) and Hedlund (2016), but focuses on how such housing market frictions interact with the stock and labor markets, which are absent in their papers. Lastly, Ludvigson, Favilukis, and Nieuwerburgh (2017) study the effects of a relaxation of collateral constraints in a model with housing, bonds, and stock markets, yet in a model without involuntary unemployment and in which the intratemporal elasticity of substitution between nondurable consumption and houses is one.

Lastly, this paper contributes to the literature studying the role of uncertainty for the business cycle fluctuations that starts with Bloom (2009). One strand of the literature studies the role of time-varying uncertainty or time-varying volatility by taking them as exogenous processes (Bloom, Floetotto, Jaimovichm, Saporta-Eksten, and Terry, 2018; Vavra, 2014; Basu and Bundick, 2017; Leduc and Liu, 2016; Gilchrist, Sim, and Zakrajšek, 2014; Arellano, Bai, and Kehoe, 2019). Another strand of literature studies instead, how uncertainty can arise endogenously as a byproduct of other shocks (Baley and Blanco, 2019; Bachman and Moscarini, 2012; Fajgelbaum, Schaal, and Taschereau-Dumouchel, 2017; Straub and Ulbricht, 2023). This paper finds a new channel through which aggregate uncertainty and stochastic volatility arise endogenously in response to the first moment shocks in total factor productivity.

The paper is organized as follows. Section 2 provides the illustrative model to guide intuition, and Section 3 presents the quantitative model. Section 4 discusses the quantitative implementation and compares the baseline results with counterfactual models. Section 5 presents implications for policy, Section 6 provides empirical evidence of household preferences and Section 7 concludes.

2 Illustrative Model

This section presents an illustrative model whose purpose is to isolate the effect of housing illiquidity on household perception of risk and on the stochastic discount factor. It starts by describing the environment, then moves on to characterize the role of illiquidity on the relative risk aversion and makes a comparison with the Campbell and Cochrane (1999b) habits model. Lastly, it illustrates how housing illiquidity changes the properties of the stochastic discount factor relative to standard CRRA preferences.

2.1 Environment

The economy is populated by representative households that derive utility from consumption of nondurables C and housing service H.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{((1-\phi)C^{1-\alpha} + \phi H^{1-\alpha})^{\frac{1-\gamma}{1-\alpha}}}{1-\gamma}$$

β is the subjective time discount factor, γ is the elasticity of intertemporal substitution and $1/\alpha$ is the intratemporal elasticity of substitution between nondurables and houses. It is convenient to define the consumption to housing ratio as $R^H \equiv C/H$. While households optimally choose to smooth nondurable consumption over time and and to keep the consumption to housing ratio stable, it is assumed that they do not choose H and therefore they can only affect R^H by choosing C. Such assumption corresponds to an extreme case where adjustment cost on housing is infinite and it is never optimal to change H. Next, I specify the endowment processes. Nondurable consumption and R^H follow an i.i.d. log-normal processes.

$$c_t = g + c_{t-1} + \epsilon_t^c \tag{1}$$

$$r_t^H = \epsilon_t^r \tag{2}$$

where $c_t = ln(C_t)$, $r_t^H = ln(R_t^H)$ and g is the consumption growth rate. Shocks to c_t and r_t^H are distributed normally with an arbitrary correlation ρ.

$$\begin{pmatrix} \epsilon_t^c \\ \epsilon_t^r \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ \mu^r \end{pmatrix} \begin{pmatrix} \sigma_c^2 & \sigma_c\sigma_r\rho \\ \sigma_c\sigma_r\rho & \sigma_r^2 \end{pmatrix}\right)$$

Specifying c and r^H as separate processes allows to isolate the role of intra-temporal risk on the stochastic discount factor and to benchmark it against standard CRRA preferences. Household optimization yields the stochastic discount factor described by equation 3, which consists of two factors: the standard nondurable consumption growth rate and the second factor related to the intra-temporal risk, whose importance depends on the difference between the elasticity of substitution in time and across goods, or simply $\alpha - \gamma$. Proposition 1 establishes the main conditions under which the intra-temporal risk is not relevant.

$$M_{t,t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{1-\phi+\phi(R^H_{t+1})^{\alpha-1}}{1-\phi+\phi(R^H_t)^{\alpha-1}}\right)^{\frac{\alpha-\gamma}{1-\alpha}} \tag{3}$$

Proposition 1. *Intra-temporal risk is irrelevant if as least one of the following is true: 1.* $\alpha = \gamma$*, 2.* $\sigma^r = 0$*, 3.* $\phi = 0$*.*

Proof. See Appendix □

First, housing illiquidity does not matter for the SDF if the substitutability in time is the same as the substitutability across goods, in which case utility still depends on both C and H but is separable. Second, $\sigma^r = 0$ can be interpreted as the case when households are able to smooth the intratemporal risk, such that R^H is constant. That would happen if it was possible to change the housing service consumption in arbitrarily small quantities without incurring any cost. Such a case is only hypothetical though, as the housing market is highly illiquid and houses are known to be indivisible. While households have a preference to consume nondurables and housing services in constant proportion, housing illiquidity is a technological constraint preventing that. The last case says that housing service consumption is irrelevant if households have no preference for it, in which case the stochastic discount factor collapses to the standard CRRA case. I next turn to analyze to role of intra-temporal risk on household risk aversion and the stochastic discount factor.

2.2 Illiquidity and Relative Risk Aversion

$$RRA^{CES} = \alpha - (\alpha-\gamma)\frac{(1-\phi)}{(1-\phi)+\phi(R^H)^{\alpha-1}} \tag{4}$$

Proposition 2. $RRA^{CES} > \gamma$ *iff* $\alpha > \gamma$*.*

Proof. See Appendix □

Proposition 2 establishes that the relative risk aversion is constant with CRRA preferences and is always higher than the CRRA when the goods are complements, in the sense that $\alpha > \gamma$ and is lower when $\alpha < \gamma$.[4] Intuitively, the higher the complementarity, the more households care

[4] In the rest of the paper I refer to goods as being complements when $\alpha > \gamma$ and $\gamma > 1$

about how their total consumption is allocated between the two goods. In that case, in addition to disliking uncertainty about their total consumption, agents also dislike uncertainty about variation in the nondurable-to-house ratio, R^H. The idea that the variance of C is painful because it is evaluated relative to H resembles the habits model (Campbell and Cochrane, 1999b). The main difference is that in this paper, the stable consumption of housing services arises as an optimal household choice in the presence of housing illiquidity.

Proposition 3. *The RRA decreases in the nondurable-to-house ratio, R^H when $\alpha < \gamma$ and $\alpha > 1$, or $\alpha > \gamma$ and $\alpha < 1$.*

Proof. See Appendix ☐

Relative risk aversion also depends on the current state, summarized by H, and varies with the economy. Whether the relative risk aversion increases or falls with nondurable-to-housing ratio again depends on how α relates to γ, as summarized in proposition 3. Understanding such variation is important for two reasons. First, it means that the model can generate variation in risk premium without any variation in underlying consumption risk. Under the specification described in 3 risk aversion, and therefore the risk premium, would increase if the nondurable-to-house ratio fell, which, keeping H fixed would happen when nondurable consumption C is low. Such specification is qualitatively similar to Campbell and Cochrane (1999b) habits model, where risk aversion increases when consumption is low relative to the habit.[5]

Second, note that SDF can be rewritten in terms of RRA. This shows that variation in the RRA translates into variation in the second factor of the SDF, which means that uncertainty over tomorrow's R^H will translate into more volatility of the SDF, and if such uncertainty varies over the business cycle, it will result in more variation in the equity prices and will lead to more precautionary saving.

$$M_{t,t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{\alpha - RRA_t}{\alpha - RRA_{t+1}}\right)^{\frac{\alpha-\gamma}{1-\alpha}} \tag{5}$$

Figure 1 summarizes the relation between RRA and R^H from propositions 2 and 3 depending on the values of α and γ that span all possible cases. The takeaway is that the size and the relation between RRA and R^H depend on whether $\alpha > \gamma$ and whether nondurables and houses are substitutes or complements in the sense of whether α is greater or smaller than 1. Note however that when the cost of adjusting H is not infinite, the consumption to housing ratio varies due to both C and H and it is ex-ante not obvious how R^H varies over the business cycle. The next subsection analyzes the role of both C and R^H on the risk-free rate and the market price of risk in more detail.

[5] In Campbell and Cochrane (1999b) relative risk aversion is related to habit by $RRA = \gamma/S$, where S is the surplus consumption ratio $S = \frac{C-X}{C}$ X is the consumption history.

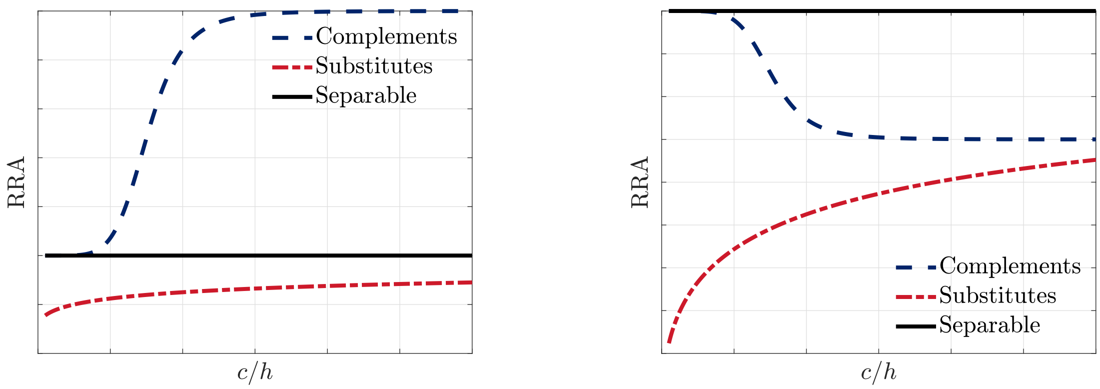


Figure 1: Relation between the Relative Risk Aversion and R^H

Notes: The figure shows the relative risk aversion RRA as a function of R^H for different utility parameterizations The left panel is the case when $\gamma = 2$ and $\alpha = 8$ (Complements), $\alpha = 0.5$ (Substitutes) and $\alpha = 2$ (Separable). The right panel is the case when $\gamma = 8$ and $\alpha = 8$ (Separable), $\alpha = 5$ (Complements) and $\alpha = 0.5$ (Substitutes). In both panels $\phi = 0.5$.

2.3 Illiquidity and the Stochastic Discount Factor

Hansen and Jagannathan (1991) show that conditional moments of the stochastic discount factor describe the slope of the conditional mean–standard deviation frontier such that the Sharpe ratio of any asset is bounded by

$$\frac{E_t(R^e)}{\sigma_t(R^e)} \leq \frac{\sigma_t(M_{t+1})}{E_t(M_t)}$$

where the term on the right is referred to as the market price of risk. The variation and the size of the market price of risk define the size and the variation in the equity premium. In the time series data, the risk-free rate $(1/E_t(M_{t+1}))$ is low and relatively stable, which suggests that most of the variation in the market price of risk as well as the equity premium must be accounted by the variation in the conditional variance of the SDF, $\sigma_t(M_{t+1})$. It is informative to inspect how housing illiquidity matters for both. First, under log-normality assumptions in Equation 2 it is possible to express the inverse of the risk-free rate in terms of current allocations and future shocks.

$$E_t(M_{t+1}) = e^{-\gamma g}\left(1-\phi+\phi(R_t^H)^{\alpha-1}\right)^{\frac{\alpha-\gamma}{\alpha-1}}\mathbb{E}_t\left(e^{-\gamma\epsilon_{t+1}^c}(1-\phi+\phi e^{(\alpha-1)\epsilon_{t+1}^h})^{\frac{\alpha-\gamma}{1-\alpha}}\right)$$

Under the special case when shocks are not correlated ($\rho = 0$), it is possible to isolate the role of housing illiquidity on the risk-free rate, as shown in Equation 6. [6]

$$r_t^f = \underbrace{\gamma g}_{\substack{\text{consumption}\\ \text{growth}}} - \underbrace{\gamma^2\sigma^2/2}_{\substack{\text{precautionary}\\ \text{motive}}} + \underbrace{\frac{\alpha-\gamma}{1-\alpha}\left(ln(1-\phi+\phi(R_t^H)^{\alpha-1})-\mu_x\right)}_{\text{Variation in RRA}} - \underbrace{\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2/2}_{\substack{\text{Intratemporal}\\ \text{risk}}} \tag{6}$$

The first two terms are the standard under CRRA preferences. The first term reflects the intertemporal substitution motive: households are willing to save more when the consumption growth rate is low. Low intertemporal elasticity of substitution (high γ) means that a risk-free rate will be highly responsive to small changes in consumption growth rate. This is the source of the risk-free rate puzzle and is the reason researchers have started using preferences that separate the risk aversion from the elasticity of intertemporal substitution (Bansal and Yaron, 2004) or where the effect on the risk-free rate due to intertemporal smoothing is exactly outweighed by the precautionary motive (Campbell and Cochrane, 1999a). The precautionary saving motive in the second term reflects the idea that households want to insure against uncertainty over nondurable consumption. The last two terms come from housing illiquidity. The third term reflects the fact that the local curvature of the marginal utility depends on today's consumption-to-housing ratio. The quantitative importance of this channel and its direction depends on the difference between α and γ and whether nondurables and houses are complements or substitutes, as summarized in Figure 1. The last term is the intra-temporal risk channel - households dislike uncertainty about tomorrow's consumption-to-housing ratio and would like to save to insure against such risk. While in Campbell and Cochrane (1999a) model the risk-free rate is stable because the first two terms push it in the opposite directions, in this model with illiquid houses it can potentially be stable depending on how R_t varies over the business cycle and, depending on the size and the sign of $\alpha-\gamma$.

$$ln(\sigma_t(M_{t+1})) = -\underbrace{\gamma g}_{\substack{\text{Consumption}\\ \text{Growth}}} + \underbrace{\frac{\alpha-\gamma}{\alpha-1}(ln(1-\phi+\phi R_t^{\alpha-1})-\mu_x)}_{\text{Variation in RRA}} + \underbrace{\gamma^2\sigma_c^2/2 + \left(\frac{\alpha-\gamma}{\alpha-1}\right)^2\frac{\sigma_x^2}{2}}_{\text{Consumption Risk}} + \underbrace{\frac{1}{2}ln(e^{\gamma^2\sigma_c^2+\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2}-1)}_{\text{Intratemporal Risk}} \tag{7}$$

Next, the conditional standard deviation of $M_{t,t+1}$, as shown in Equation 7 depends on the same forces. Increase in uncertainty, either through a rise in σ_c or σ_x unambiguously increases the $\sigma(M_{t,t+1})$, however, the degree to which the intra-temporal risk matters again depends on the difference between α and γ. The first two terms show that $\sigma(M_{t,t+1})$ also depends on the current state through g and R_t. In recessions consumption growth rate is typically high, pushing $\sigma(M_{t,t+1})$. The role of R_t, as in the equation for the risk-free rate, depends on the cyclical properties of R_t and on the difference between α and γ. While the cyclical properties of $\sigma_t(M_{t,t+1})$ are ex-ante not obvious, it is possible to show that it is always larger compared to the standard model under CRRA

[6] If variable X is log normal with parameters μ and σ, $Y=X+c$ is a three-parameter log normal with parameters μ, σ and c, such that $E(Y)=E(X)+c$, $Var(Y)=Var(X)$ and $ln(Y)\sim N(\mu_y),\sigma_y^2$. Accordingly, $ln(1-\phi+\phi(R_t^H)^{\alpha-1})\sim N(\mu_x,\sigma_x^2)$. More details can be found in Appendix A.

preferences, as shown in Proposition 4. This means that everything else kept constant, the model with illiquid housing would imply a higher Sharpe ratio and risk premium.

Proposition 4. *Assume further that* ϵ_t^c *and* ϵ_t^r *are independent. Then* $\sigma_t(M_{t+1}^{CES}) \geq \sigma_t(M_{t+1}^{CRRA})$

Proof. See Appendix A. □

Given 6 and 7, we can see that the Hansen-Jagannathan bound depends only on the sum of uncertainty about tomorrow's consumption and the intra-temporal risk, summarized by σ_x. This says that the bound is always larger as long as α is not equal to γ and there is some intre-temporal risk (σ_x is positive). Additionally, the bound, will increase in recessions if uncertainty increases in recessions. This suggests that the model with illiquid housing will feature an on average higher equity premium and the premium will increase in recessions if recessions are times when intra-temporal risk is high.

$$\frac{E_t(R^e)}{\sigma_t(R^e)} \leq \frac{\sigma_t(M_{t,t+1})}{E_t(M_{t,t+1})} = \sqrt{e^{\gamma^2\sigma_c^2 + \left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2} - 1} \tag{8}$$

The analysis above is possible by treating c_t and $R_t \equiv c_t/h_t$ as separate processes. While this simplification yields many insights, it misses the idea that changes in nondurable consumption also have an effect on the consumption-to-housing ratio and does not allow to isolate the role of c_t and h_t. Another way to understand the role of housing illiquidity is to isolate the effect of c_t and h_t on the terms in the Sharpe ratio, which is done quantitatively in Figure 2. It plots the effect of c_t, h_t and uncertainty about future nondurable and housing consumption on $\sigma_t(M_{t,t+1})$ and $E_t(M_{t,t+1})$ for the case when $\alpha > \gamma$ and the goods are relative complements.[7] Solid lines represent outcomes in the model with CES preferences and illiquid houses and dashed lines represent outcomes in a standard model with CRRA preferences. Two important points can be made here. First, holding everything else fixed, the effect of changes in nondurable consumption, both c_t and $\sigma(c_t)$, is greatly amplified relative to the standard model. This is due to the fact that variation in nondurable consumption now also distorts the optimal consumption-to-housing ratio and amplifies the variation in SDF through the intra-temporal risk channel. Second, changes in the current consumption of housing service (h_t) affects the SDF in the opposite direction than c_t. Intuitively, low h_t means that now the consumption-to-housing ratio is high and households are willing to reduce their nondurable consumption to bring the ratio back to their optimal level. This creates incentives to save and is the reason why $E_t(M_{t,t+1})$ increases (risk-free rate falls) in the top left panel.

[7]Such parameter combination is an outcome of the calibration exercise (Section 4) and finds support in the micro-data (Section 6).

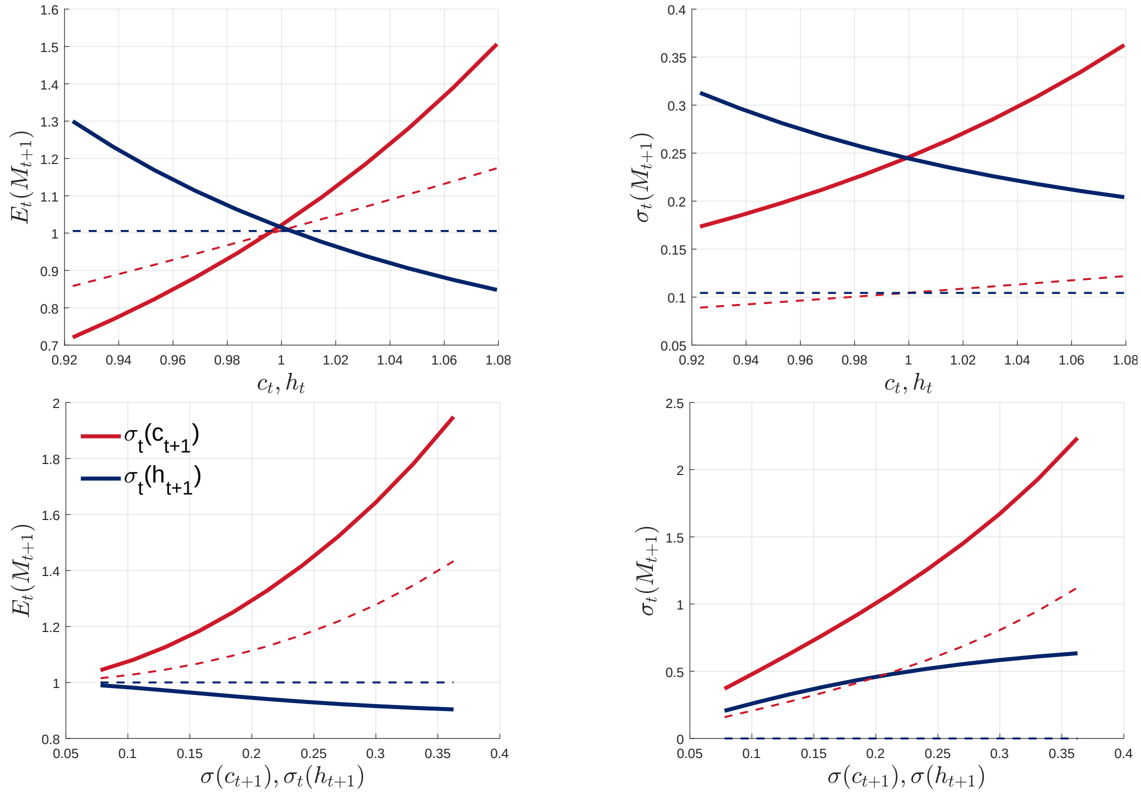


Figure 2: Properties of the Stochastic Discount Factor

Notes: The figure shows how the mean and the variance of the stochastic discount factor depend on the current and future consumption of nondurables and housing services. Solid lines represent the stochastic discount factor under CES preferences, dashed lines - under CRRA preferences. In this case $\alpha = 7.66$, $\gamma = 2$, $\phi = 0.5$. c_t, h_t are today's consumption of nondurables and houses, respectively. $\sigma_t(c_{t+1})$ and $\sigma_t(h_{t+1})$ represent uncertainty about tomorrow's consumption.

As can be seen, the role of intra-temporal risk depends not only on whether household view nondurables and houses as substitutes or complements, but also on the cyclical properties of c_t and h_t of the marginal investor. In the next section, I proceed by laying out a quantitative model that contains salient features allowing to quantify the role of intra-temporal risk and to study its' implications for the business cycle.

3 Quantitative Model

This section presents a recursive formulation of the model that is used to quantify the role of housing illiquidity on the discounts and the business cycles. The model contains several key features that allow to study the role of housing illiquidity quantitatively. It contains both aggregate and idiosyncratic income risk. Idiosyncratic income risk gives rise to wealth inequality and limited stock market participation, which allows to identify the marginal investor that prices the assets. Stock market participation is limited because houses are essential goods and low-wealth households are disproportionately more invested in houses. Houses provide utility and are illiquid assets. The model also allows for time-varying income uncertainty, which then translates into consumption uncertainty. The source of it is the standard search friction in the labor market. The section begins with a general description of the model environment and then proceeds by presenting housing and labor markets and the household problem. Lastly, it provides a description of the equilibrium conditions.

3.1 Environment

This is discrete. The economy is populated by a continuum of infinitely-lived households who have preferences $u(c, h)$ over consumption of nondurables c and housing h. Nondurable consumption can be adjusted by infinitesimally small quantities, whereas houses are indivisible and belong to a discrete set $h \in H$. This indivisibility implies that the nondurable-to-house ratio will vary in time and will differ across households. Houses are traded in decentralized markets, where households direct their search by posting bid p^b and ask p^s prices defined relative to the aggregate house price p^h. In addition, households can save in risky shares of the representative firm and risk-free bonds. Shares a yield dividends d and are traded at a market price p^a in Walrasian markets. Risk-free bonds b are traded in open financial markets at an exogenous rate r^b. In addition, unsecured borrowing in bonds is available up to an exogenously set limit $\underline{b}$. Households are subject to uninsurable shocks to their labor market status. They are either employed or unemployed. Employed households supply labor inelastically to a representative firm and receive income in the form of wages w and face an exogenous job separation probability. Unemployed households receive unemployment insurance equal to μw and find jobs with probability $\lambda^w(\theta)$, which is an equilibrium object determined in the labor market but is taken by households as given. The production sector is characterized by search and matching frictions as in Mortensen and Pissarides (1994), with the addition that with risk-averse households stock market discounts matter for firm vacancy posting. Labor market frictions mean that recessions will be characterized by low job-finding rates and high unemployment. Housing indivisibilities and borrowing constraints then imply that, due to market incompleteness, recessions will affect households differently depending on their portfolio allocation to different assets. Three markets clear in equilibrium: aggregate house price p^h, stock price p^a, and labor market tightness θ clear the housing, stock, and labor markets, respectively.

3.2 Housing Market

Houses are sold in a decentralized market, where heterogeneous households direct their search by house size, h', and list price, p^s, in the case of selling and bid price, p^b, in the case of buying. To make this two-sided matching problem tractable I introduce real estate brokers who act as a counterparty for the households willing to sell or buy houses from other households. Seller and buyer brokers collect prices and quantities from households and then trade with each other in a centralized market, where supply and demand determine the aggregate housing market price p^h. To become a broker one needs to pay fixed entry cost κ_b and κ_s per unit of transaction to enter buying and selling markets. By the free entry condition brokers enter every sub-market until profits go to zero, as shown in equation 9.

$$\underbrace{(p^b - p^h \Delta h')}_{\text{Broker Profit}} \underbrace{\alpha_b(\theta(p_b, \Delta h')}_{\text{Match Probability}} \geq \underbrace{\kappa_b \Delta h'}_{\text{Broker Cost}} \tag{9}$$

Broker surplus depends on the profit they get by executing transactions with households and the probability that they get matched. Brokers are matched to households via the constant returns to scale matching function $M = br^{\eta_h} hh^{1-\eta_h}$.[8] Let θ_b and θ_s denote the market tightness in the buying and selling markets, respectively. The probability that a selling household is matched with a broker is then $\lambda_h(\theta_s) = \theta_s^{-\eta_h}$ and the probability that a broker is matched with a selling household is $\alpha_h = \theta_s^{1-\eta_h}$. These probabilities are determined by posted prices and quantities; therefore, the housing market is made of submarkets indexed by $(p_s, \Delta h)$ and $(p_b, \Delta h)$.[9]

Substituting 9 into the expression for $\lambda_h(\theta_b)$ gives the probability, and a household willing to buy a house is matched with the real estate broker expressed in terms of prices and quantities.

$$\lambda_h(\theta_b(p_b, \Delta h', p^h)) = \left(\frac{\kappa_b \Delta h'}{p^b - p^h \Delta h'}\right)^{\frac{\eta_h}{\eta_h - 1}} \tag{10}$$

Such a formulation of the housing market implies a price-liquidity trade-off. By posting higher buying or lower selling prices, households increase the probability of executing the transaction, but at the cost of buying at a premium or selling at a discount. High selling prices or low buying prices, on the other hand, reduce the monetary cost of the transaction, but increase the expected time it takes to execute it. The maximum bid and discount needed to execute the transaction in the same period are controlled by parameters κ_b and κ_s respectively. Specifically, the premium that a household would have to pay to meet the broker with a 100% probability is equal to $(\kappa - p^h)/p^h$. This means that the friction is time-varying as falling house prices endogenously increase the discount and the premium required to meet the real estate brokers. In addition to that, such a formulation implies that the structure is block recursive (Menzio and Shi, 2010) in that the household wealth distribution matters for household housing market policies only to the extent that it affects the aggregate house price p^h.

[8] hh^x denotes the household, br^x denotes the broker and $x \in \{b, s\}$. $\theta_s = \frac{hh^s}{br^s}$, $\theta_b = \frac{hh^b}{br^b}$. 'b' and 's' stand for buying and selling

[9] By the broker free entry condition, sub market for every price and quantity exists

3.3 Labor Market

The production sector consists of a representative firm that is owned by the households. Aggregate output is produced with a constant returns to scale technology, labor, N, being the only input:

$$Y = zN.$$

The value of the firm is the present discounted value of the future dividends, which are equal to aggregate production net of wage payments and the cost of posting vacancies.

$$d = (z - w)(1 - u) - \kappa v \tag{11}$$

Future profits are discounted with the aggregate discount factor $M(X, X')$, which is the weighted average of the marginal rates of substitution of all the firm owners (Equation 6), weighted by their demand for the shares of the firm. Note that such discounting is consistent with the individual stock price valuations and with the market clearing stock price p^a, which is the value of the firm, as I show in the Appendix B.5.[10]

$$M(X, X') = \beta \int a'(\omega, h, e, X) \frac{u_c(c'(\omega', h', e', X'), h'(\omega, h, e, X))}{u_c(c(\omega, h, e, X), h)} d\Lambda(\omega, h, e, X)^{11} \tag{12}$$

Search in the labor market is random and workers are matched to firms via the following matching function, which ensures that the matching probabilities for both workers and firms are bounded between zero and one.

$$M(\nu, u) = \frac{uv}{(u^\eta + v^\eta)^{\frac{1}{\eta}}}$$

where is ν_t is the number of vacancies and u is the unemployment rate. The job filling and job finding rates are then

$$\lambda^f = (1 + \theta^\eta)^{\frac{-1}{\eta}}; \qquad \lambda^w = (1 + \theta^{-\eta})^{\frac{-1}{\eta}}$$

where the labor market tightness $\theta = \frac{\nu}{u}$.

[10] Brav, Constantinides, and Geczy (2002) also provides evidence that stock prices are consistent with the stochastic discount factor calculated as the weighted average of the individual marginal rates of substitution. More prominent examples using the same assumption include Ludvigson, Favilukis, and Nieuwerburgh (2017) and Gornemann, Kuester, and Nakajima (2021)

[11] Variable e is introduced here to summarize the labor market state. In the rest of the paper it does not show up since value and policy functions are defined separately for each state

Finally the value of a vacancy, $V(X)$ and a filled job, $J(X)$, are given by

$$V(X) = -\kappa + \mathbb{E}\left[M(X,X')(\lambda^f(\theta)J(X') + (1-\lambda^f(\theta))V(X'))|X\right]$$
$$J(X) = z - w + \mathbb{E}\left[M(X,X')(\sigma V(X') + (1-\sigma)J(X'))|X\right]$$

Competitive entry by firms implies that the firm equates the fixed vacancy posting cost κ with the discounted value of future job surplus adjusted by the probability that the posted vacancy gets filled.

$$\frac{\kappa}{(1-\sigma)\lambda^f(\theta)} = \mathbb{E}[(M(X,X')J(X'))|X] \tag{13}$$

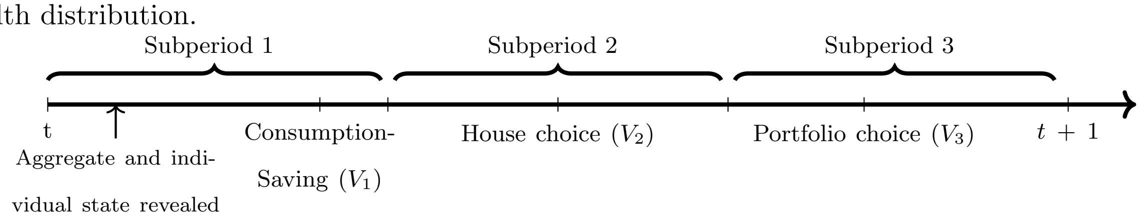
This is the key equilibrium condition relating the household risk attitudes, as summarized by the discount factor, with the firm vacancy posting decision and therefore, with the aggregate labor market condition. I elaborate on this more in Section 4.2 when I explain the workings of the model. For numerical tractability, I do not introduce a wage bargaining problem, but instead, close the model by assuming that aggregate wages are related to aggregate productivity via the following functional form.

$$w = \varphi z^{\zeta}$$

Here the parameter φ controls the wage level and the aggregate labor share, while the exponent ζ controls the wage rigidity. From the worker's perspective, this wage rule is always feasible in equilibrium since w is always positive as long as φ is positive and workers do not receive disutility from working. From the firms' perspective, this is a feasible outcome as long as it results in positive dividends. I verify that in equilibrium it is always the case.

3.4 Household problem

At the beginning of the period, households learn the aggregate state, their cash on hand $\omega(X)$, employment status, and the outcome of their housing market choice from the last period. The aggregate state $X = (u, z, \lambda)$ consists of the unemployment rate, aggregate productivity, and the wealth distribution.



Each period consists of three sub-periods. In sub-period 1 households solve the consumption-saving problem. Sub-period 2 is when households post prices and quantities in the housing market.

In sub-period 3 households solve the portfolio allocation problem between risky stocks and risk-free bonds, given the risk-return trade-off. Now I work through the household value functions associated with each sub-period moving backward.

3.4.1 Sub Period 3

The beginning of sub-period household wealth consists of the cash on hand that has not been spent on consumption or house purchases in the first two sub-periods (ω_3). Households solve the portfolio choice problem of allocating ω_3 to risky shares and bonds, where the optimal choice is determined by the trade-off between return and risk. Stocks provide larger returns, but at a higher risk as tomorrow's dividends and stock prices are only determined by the market clearing conditions in the next periods. In addition, households face uncertainty about the next period's wages and their labor market state and this drives their precautionary saving motives. To smooth the shocks, borrowing up to an exogenously specified limit $\underline{b}$ is allowed. Employed households[12] face an exogenous job separation probability σ and unemployed households are matched at an endogenous rate $\lambda^w(\theta)$. Of employed households that experience a separation shock, a fraction $\lambda^w(\theta)$ find a match within the same period and do not go through the unemployment state. The remaining fraction $1-\lambda^w(\theta)$ become unemployed.[13]

$$
\begin{aligned}
V_3^E(\omega_3, h, X) = &\max_{a',b'} \mathbb{E}((1-\sigma(1-\lambda^w(\theta)))V_1^E(\omega', h', X') + \sigma(1-\lambda^w(\theta))V_1^U(\omega', h', X')|X) \\
&\text{s.t.} \\
&\omega' = a'(p^{a\prime} + d') + b'(1+r^b) \\
&b' = \omega_3 - a'p^a \\
&b' \geq \underline{b}
\end{aligned}
$$

3.4.2 Sub Period 2

The second sub-period is when households decide whether and by how much to adjust their housing consumption. The beginning of sub-period wealth consists of cash on hand that has not been spent on consumption in the first sub-period, ω_2, and the housing wealth, h. Households willing to purchase a house optimally choose the bid price p^b and the quantity to be purchased (h'). What is not spent is transferred into sub-period 3. By posting higher prices households exploit the trade-off between liquidity and the purchase premium, with higher p^b implying higher probability. Low match probability means that households are more likely to be stuck for one more period with the

[12] The main body of the paper presents the employed household value functions and only the value functions when a household chooses to buy a house in the second sub-period. The full household problem is presented in Appendix B.

[13] A standard approach is to assume that a fraction σ become unemployed and $(1-\sigma)$ remain employed. This assumption that some households find a match within the same period is made to have a steady state employment rate of 6.23% in the quarterly calibration of the model.

same housing consumption. On the other hand, a high purchase premium means that the household has less funds available to invest in assets that provide financial returns, namely stocks and bonds.

$$
\begin{aligned}
V_2^{E,buy}(\omega_2, h, X) = \max_{p^b, h' \in H} & \Big(\lambda^b(p^b, \Delta h', p^h) V_3^E(\omega_3', h', X) + (1 - \lambda^b(p^b, \Delta h', p^h)) V_3^{f,E}(b^f, \omega_3', h, X)\Big) \\
& \text{s.t.} \\
& \omega_2 = hp^h + p^b + \omega_3' \\
& \omega_3' \geq \underline{b} \\
& b^f = p^b
\end{aligned}
$$

In the case when a household does not find a match in the housing market, the wealth that has been allocated to the house purchase needs to be allocated to bonds in the sub-period 3 and is carried to the next sub-period in the form of b^f.[14]

3.4.3 Sub Period 1

The first sub-period is a standard consumption-saving problem. First households realize the aggregate and individual states and the outcome of the housing market transaction in the previous period. Given the information on dividends and prices, they know their cash-on-hand, ω. Their wealth consists of cash-on-hand, housing wealth, h, and the labor income w in the case of employed households. Households consume the housing service provided by h and choose the consumption of a non-durable good. The remaining wealth is carried to sub-period 2 in the form of ω_2.

$$
\begin{aligned}
V_1^E(\omega, h, X) = & \max_{c, \omega_2'} U(c, h) + V_2^E(\omega_2', h, X) \\
& \text{s.t.} \\
& \omega + hp^h + w = c + \omega_2' \\
& \omega_2' \geq \underline{b}
\end{aligned}
$$

This saving decision takes into account the housing market indivisibilities and the individual and aggregate risk through the curvature of V_2^E. In other words, conditional on their choices in sub-period 1, they know their choice in sub-period 2 and likewise, given the sub-period 2 choice, they know their sub-period 3 choice and make their saving decision based on this information.

3.5 Equilibrium

Recursive competitive equilibrium in the economy consists of household and firm value and their associated policy functions, where agents take aggregate prices as given and, in addition, firms take

[14] The assumption that wealth from unsuccessful housing market transactions needs to be allocated to bonds is made to ensure that this stochastic housing market outcome does not affect the household Euler equation for stocks.

the household stochastic discount factor as given. There are three markets that clear endogenously in equilibrium determining the labor market tightness θ, house price p^h, and stock price p^a.

- Aggregate house price p^h is determined by the market clearing in the housing market.

$$\mathcal{D}(p^h, X) = \mathcal{S} \tag{14}$$

  where demand $\mathcal{D}(p^h, X)$ is the sum of successful purchase and selling transactions and supply $\mathcal{S}$ is as an endogenously calibrated parameter.

$$D(p^h, X) = \int h'\alpha_b(\theta(p_b, \Delta h', p^h)\lambda(\omega, h, e, X) \perp (h' \geq h) + \int h'\alpha_s(\theta(p_s, \Delta(-h'), p^h)\lambda(\omega, h, e, X) \perp (h' < h)$$

- Stock price p^a satisfies the stock market clearing condition.

$$1 = \int a'(\omega, h, e, X) d\lambda(\omega, h, e, X) \tag{15}$$

- Labor market tightness θ is pinned down by the firm free entry condition

$$\frac{\kappa}{(1-\sigma)\lambda^f(\theta)} = \mathbb{E}(M(X, X')J(X)|X)$$

Appendix B.4 provides a full equilibrium definition.

4 Quantitative Results

This section presents the quantitative results. It first describes the calibration strategy and compares the model against cross-sectional moments. Then it presents the main results and discusses the business cycle implications.

4.1 Calibration

The model is solved nonlinearly and globally using a modification of Krusell and Smith (1998) algorithm.[15] The model is calibrated to quarterly frequency. The calibrated parameters can be grouped into three categories: one part is assigned externally, one part is chosen to match the time series moments of the US economy in the steady state of the model without aggregate shocks, and one part is chosen to match the features of the US household wealth distribution in the model with aggregate shocks. Now I describe the calibration strategy in detail.

Externally calibrated parameters The model is calibrated to quarterly frequency. Utility over consumption and houses is CES, where γ controls the risk aversion, $1/\alpha$ the elasticity of

[15]See Appendix D for a detailed explanation of the solution method.

intratemporal substitution, and ϕ the taste for housing consumption.

$$U(c,h) = \frac{[(1-\phi)c^{1-\alpha} + \phi h^{1-\alpha}]^{\frac{1-\gamma}{1-\alpha}}}{1-\gamma}$$

I set γ to a standard value of 1.8, which is also used in Flavin and Nakagawa (2008) and calibrate the remaining utility parameters endogenously. Most of the labor market parameters are standard. I set the job separation rate σ to 0.1 as in the Job Openings and Labor Turnover Survey (JOLTS). The worker outside option μ is set to 0.4 as in Shimer (2005). One important difference from Shimer (2005) is that now the outside option is not constant, but 0.4 of the wage level, which is time-varying and procyclical, consistent with Chodorow-Reich and Karabarbounis (2016). I close the labor market by assuming that wages are related to aggregate productivity through the following function:

$$w_t = \varphi z_t^{\zeta}$$

where ζ controls the wage rigidity and φ the labor share. If ζ is 1, wages are fully flexible and the labor share is constant and equal to $\varphi/(1-u)$. On the other hand, $\zeta = 0$ means that wages are constant and the labor share is countercyclical. I follow Haefke, Sonntag, and van Rens (2013) and set ζ to 0.7, which is the estimate for wage rigidity in new jobs. Aggregate technology follows an AR(1) process in logs

$$ln(z_t) = \rho ln(z_{t-1}) + \epsilon_t; \quad \epsilon_t \sim N(0, \sigma^2)$$

where I estimate ρ and σ^2 from the BLS data on aggregate labor productivity in the US. Specifically, I use output per person in the nonfarm business sector in 1947:Q4-2019:Q4 and detrend it using an HP filter with a smoothing parameter of 10^5 following Shimer (2005). At a quarterly frequency, this gives values of 0.889 and 0.008 for ρ and σ^2, respectively.

Finally, the real estate broker bargaining weight η_h is set to 0.5 implying a linear trade-off between list house price and the transaction probability. The interest rate on bonds is set to the quarterly rate on the US treasuries. Table 1 summarizes externally calibrated parameters.

Assigned		
Parameter	Value	Source
γ	1.8	Risk Aversion
ζ	0.7	wage rigidity, Haefke, Sonntag, and van Rens (2013)
σ	0.1	quarterly separation probability, JOLTS
μ	0.4	unemployment insurance, Shimer (2005)
η_h	0.5	broker bargaining weight
ρ, σ^2	$0.889, 0.008$	quarterly labor productivity, BLS
r^b	0.0025	quarterly rate on US treasuries

Table 1: Externally Calibrated Parameters.

Time Series The next set of parameters is chosen to match the time series properties of the US economy in the steady state of the model without aggregate shocks. First I choose the labor vacancy posting cost κ to match the average US unemployment rate 1980:Q1-2018:Q4, which gives a value of 2.3884. Then I choose ξ to match the average postwar US labor share, which gives a value of 0.56.[16] Lastly I target labor matching elasticity η such that the quarterly vacancy filling rate is 0.71, as in den Haan, Ramey, and Watson (2000). Parameters κ_s and κ_b controlling the degree of housing market friction are chosen to match the maximum bid premium in Gruber and Martin (2003) and the maximum selling discount observed on RealtyTrac as in Hedlund and Garriga (2020).

Parameter	Value	Model	Data	Target
Wealth Distribution				
$\mathcal{S}$	0.9071	2.6054	2.649	Median wealth to income, PSID
β	0.9695	0.7516	0.72	Wealth share by top 10%, PSID
ϕ	0.41	0.4048	0.41	Houses to total assets by top 10%, PSID
α	10.4	0.7657	0.73	Consumption of nondurables to houses by top 10%, PSID
Labor Market				
κ	2.3884	6.23	6.23	Steady state unemployment, BLS
η	1.5485	0.72	0.71	Vacancy filling rate, (den Haan, Ramey, and Watson, 2000)
ξ	0.56	0.61	0.61	Labor share, BLS
Housing Market				
κ^b	0.0293	3%	3%	Maximum big premium,Gruber and Martin (2003)
κ^s	0.1463	15%	15%	Maximum list discount, RealtyTrac

Table 2: Internally Calibrated Parameters.

Wealth Distribution The remaining set of parameters $(\alpha, \beta, \phi, \mathcal{S})$ are calibrated internally to match the properties of the US wealth distribution. Data targets are summarized in Table 2. Specifically, I target the median wealth-to-income ratio and concentration of stock ownership, the

[16]Note that ξ is related to labor share as $\frac{\xi}{1-u}$

share of household wealth invested in real estate, and the relative consumption ratio by the top decile of the US wealth distribution. Targeting moments of the top wealth decile is chosen to ensure that the model is consistent with the wealth portfolio and consumption profile of the households that own most of the stock market and therefore, are most relevant for the firm vacancy posting decision.[17] Median wealth to income is chosen to ensure a realistic response of median wealth households to unemployment shocks. The mapping between parameters and data moments is not unique as all parameters affect all moments at least to some extent; however, the relation between parameters and some moments is strong and intuitive.

In my calibration, I look for the value of β to match the share of wealth held by the top wealth decile. Since houses are an essential consumption good but yield no financial return, richer households tend to be relatively less invested in real estate and more invested in stocks and therefore, receive larger financial returns on their wealth portfolio. Intuitively, a lower β means that a smaller share of the firm surplus is spent on posting vacancies and a larger share goes to dividends and this drives the difference between financial return on wealth between the rich and the poor. The large difference in return is then associated with a large wealth inequality. The taste for housing parameter ϕ controls how much of the housing service the average household wishes to consume and therefore, determines what fraction of their wealth gets invested in houses. I calibrate ϕ to match the average share of total wealth invested in real estate by the top wealth decile in order to match the liquidity of the portfolio of households that own most of the stock market.[18] I look for the value of α that matches the relative consumption of nondurables to houses by the top decile of the wealth distribution. For the measure of nondurable consumption, I use PSID expenditure data on all categories except housing vehicles. Measuring housing consumption is more challenging using expenditure data alone because consumption of housing services by homeowners does not show up in their expenditures. My measure includes all the items that appear in the housing expenditure and utilities categories and, in addition, for households that own their primary residence, I calculate their implied expenditure on rents using the value of their primary residence. I do this by converting the value of the primary residence to the implied rent expenditure using the price-to-rent ratio from Zillow housing data and match it with PSID observations using the state identifiers.[19] In the model α determines how complementary houses and nondurables are. Since houses have no depreciation and provide a service flow equal to their value, it is cheaper for households to derive utility for housing than nondurable consumption. As a consequence, if the goods were perfect substitutes, households would be deriving most of their utility from housing consumption and rich households would be deriving a larger share of their total consumption from houses compared to poor households. The calibration exercise suggests that in order to match the relative consumption ratio of 0.73 households must

[17] In the data, households in the top wealth decile own 90% of the stocks.

[18] The data target is the share of wealth invested in all types of real estate as opposed to the primary residence. The model does not distinguish between these two.

[19] Zillow housing data is free and available online at https://www.zillow.com/research/data. Appendix E.2 contains a more detailed description of the dataset.

view the two goods as complements. Lastly, I look for the total house supply $\mathcal{S}$ to match the median wealth-to-income ratio. Intuitively, the utility parameters ϕ and α determine the demand for houses. Then lower (higher) supply means that the aggregate house price p^h needs to increase (decrease). Higher prices, given the demand for houses, mean that median-wealth households end up consuming the same amount of housing service, but pay a higher price for it and therefore, are on average wealthier.

Untargeted Moments In the calibration, I target moments for households in the top wealth decile. Figure 3 compares the same moments across the wealth distribution in the model and the PSID data. The top left panel shows portfolio allocation to real estate by wealth percentile. The model is fairly consistent with real estate share for households in the 90-98th and top 2 percentiles and predicts that the real estate share decreases in household net wealth. This is because in the model housing is an essential consumption good and as households become richer, they start to be more invested in stocks, in order to finance their nondurable consumption using dividend income. While in the model this pattern is monotonic, in the data real estate share is decreasing for households above and is nonmonotonic below the 50th wealth percentile. This is because the pattern for poor households is driven by the homeownership rate. While most of the households above the 50th percentile own their primary residence, this share is increasing in wealth below the 50th wealth percentile. Nevertheless, the model captures the pattern for the homeowners.[20] The model could be easily extended to include an option to rent. However, the purpose of this paper is to study how housing illiquidity affects the risk attitudes of the stock owners, most of whom own their residence, and, quantitatively, renting option would have little effect on aggregate dynamics. The bottom left panel shows the share of total wealth held by respective wealth percentiles. The fact that households become increasingly less invested in real estate as they get richer means that they become increasingly more invested in stocks. Stocks provide financial returns in the form of dividends, while houses in the model only provide the utility flow. This means that richer households tend to receive larger financial returns on their total wealth and this return differential drives differences in wealth. The bottom right panel shows the share of total stocks owned by respective wealth deciles. Because richer households are more invested in stocks, stock ownership is more concentrated than ownership of wealth, both in the model and in the data.

The top right panel shows the average relative consumption ratio in respective wealth percentiles. The model matches the ratio for the top 20 percentiles, while undershoots for households in the bottom 80 percentiles. This is a consequence of the fact that CES preferences are homothetic.[21] Because this moment is mainly relevant to determine the risk aversion of the marginal investor, it is important that the model matches the data for households that own most of the stocks and in the calibrated model and in the data, households below the 80th wealth percentile

[20] See Appendix E.2 for more detailed statistics on home ownership across the wealth distribution.

[21] The setting can easily be extended to nonhomothetic preferences between nondurables and houses, for example, using an additive logarithmic utility introduced in Wachter and Yogo (2010).

own less than 12% of the stock market. While there is little difference in the relative consumption ratio comparing averages within the wealth percentiles, there is still large heterogeneity among households in the same wealth category, also in line with the data.

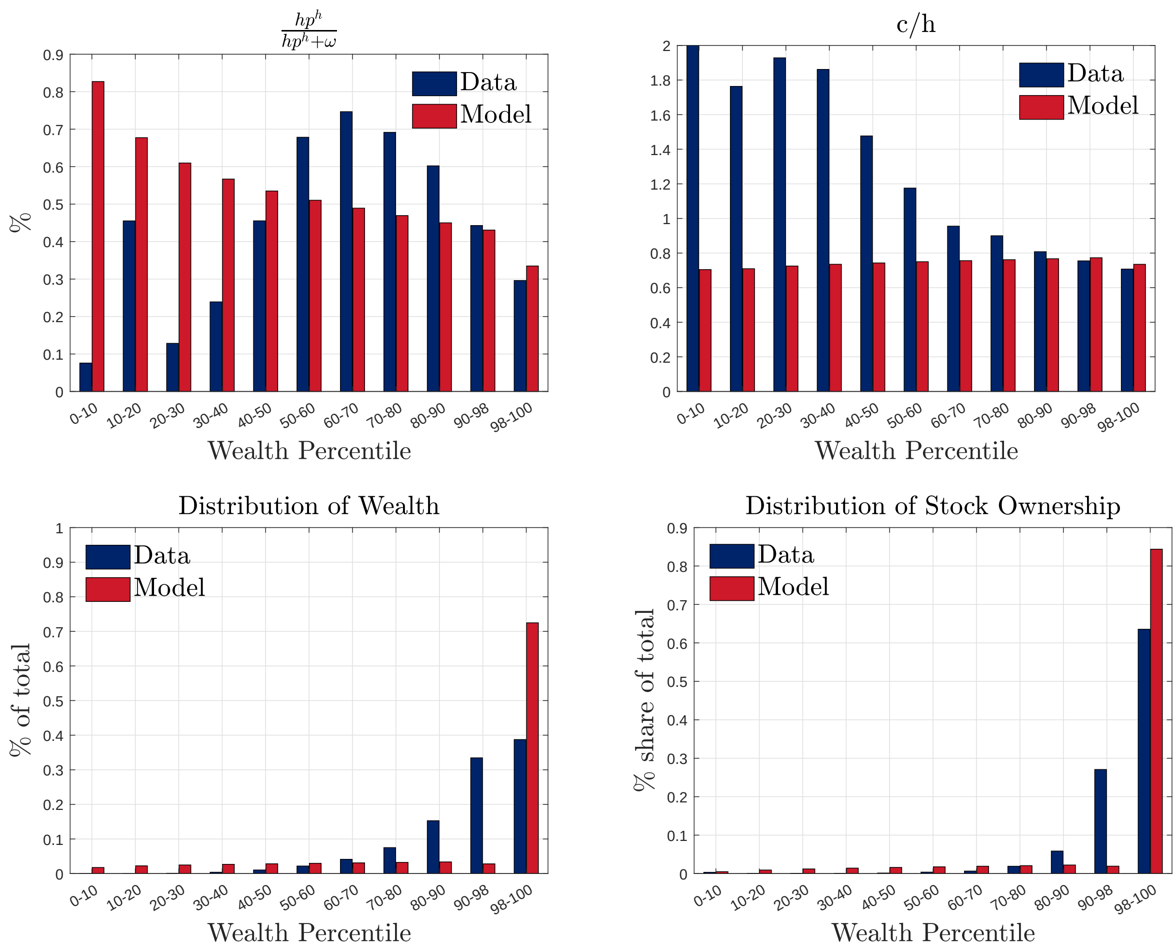


Figure 3: Cross-Sectional Distribution, Model Versus Data

Notes: Figure shows the relevant cross-sectional moments in the calibrated model and in the PSID data. Top left: real estate to total assets, bottom left: concentration of wealth, bottom right: concentration of stock ownership, top right: relative consumption ratio. Data moments calculated from the PSID household panel waves 2013-2017. See Appendix E for a detailed description of the data.

4.2 Findings

Next, I turn to evaluate the model performance in terms of aggregate untargeted moments. First, I present model implied relevant asset pricing moments, then I discuss the interaction between asset prices and unemployment and compare the model against most relevant benchmarks.

Table 3 shows the relevant untargeted aggregate moments in the baseline model and the relevant benchmark models. *No Illiquidity* is a counterfactual, where houses are treated as liquid and divisible assets. This means that houses still inter the utility function as complements to nondurables, but have no effect on the discount factor. This allows for a benchmark where the stochastic discount factor is as if preferences were CRRA but household portfolio allocations and wealth distribution are comparable to the baseline. *No GE* is another benchmark where firms use risk-neutral discount factor. The rest of the model being the same, this helps to understand the role of equilibrium feedback between household discounts and firm decisions. *Risk Neutral* is a case where households are not risk averse and have no preference for housing. Then the model collapses to a standard search-and-matching model with risk-neutral consumers. This benchmark helps to understand how the prices would behave if they did not depend on household discounts.

Description	Moment	Data	Baseline	No Illiquidity	No GE	Risk Neutral
Real Variables						
S.d. Unemployment	$\sigma(u_t)$	0.79	0.52	0.32	0.28	0.28
S.d. Job-finding rate %	$\sigma(log(\lambda^w(\theta)))$	6.67	5.86	4.18	3.14	3.19
S.d. Output	$\sigma(log(Y_t))$	1.31	2.30	2.06	2.09	2.09
Relative Consumption Volatility	$\frac{\sigma(log(c_t))}{\sigma(log(y_t))}$	0.83	0.70	0.60	0.80	1.03
Prices						
Mean log price-dividend ratio	$\mathbb{E}log(\frac{p_t}{d_t})$	2.14	2.38	2.67	2.35	1.99
Price−Dividend Volatility	$\frac{\sigma(log(p_t))}{\sigma(log(d_t))}$	1.31	1.09	0.16	0.57	0.48
Dividend Volatility	$\sigma(log(d_t))$	4.56	2.55	10.05	6.56	6.13
Correlation of dividends and output	$\rho(log(d_t), log(y_t))$	0.32	0.75	0.60	0.92	0.92

Table 3: Aggregate Untargeted Moments in the Model and the Data

Notes Table shows untargeted aggregate moments in the data and in the baseline and the benchmark models. *No Illiquidity* is a counterfactual model, where houses are treated as liquid assets. *No GE* is a model where firms discount profits using risk neutral discounting such that there is no equilibrium interaction between household discounts and firm decisions. *Risk Neutral* is a model with risk-neutral consumers. In all the counterfactuals the remaining parameters are the same as in the baseline calibration.

Looking at the asset prices, the baseline model does a good job in generating asset price volatility compared to the data. The price-dividend volatility in the baseline is 1.09, compared to 1.31 in the data. Comparing this number to the price-dividend volatility in the *No Illilquidity* benchmark helps to understand the role of housing illiquidity on asset prices, as this is the benchmark, where the illiquidity does not matter for the stochastic discount factor, which then collapses to the standard CRRA benchmark. Comparing 1.09 and 0.16 suggests that the presence of housing illiquidity in the SDF increases the price-dividend volatility almost seven-fold.

A consequence of volatile prices is the an increased volatility of the real variables at the business cycle frequency, specifically output and unemployment. Here it is instructive to compare the baseline model with *No GE* and *Risk Neutral* benchmarks, where stock price volatility has no effect on real quantities. Compared to this, unemployment volatility increases twice (0.52 versus 0.26) and goes a long way in resolving the Shimer (2005) puzzle.[22] As a consequence, output becomes

[22]In the model, all variation in unemployment volatility comes from variation in the job finding rate whereas in the data separation rate varies as well. To address this I follow Shimer (2012) and use a constant-separation unemployment series coming from the BLS data between 1948-2007 based on the following law of motion:

10% more volatile. Relative consumption volatility, on the other hand, is roughly similar in all models, except the *Risk-Neutral* case, suggesting that discounts in the baseline model are volatile not because of volatile consumption but because of higher aversion to such volatility. Another way to grasp the role of housing illiquidity on discounts and real quantities is to compare this with the *No Illiquidity* benchmark. In this case, firms discount the future using the standard SDF that does not contain the housing illiquidity term. Comparing the numbers (0.52, 0.32 and 0.26) one can observe that 1. The role of housing illilquidity is large enough to result in 63% more volatile unemployment 2. Model with standard preferences does not provide meaningful equilibrium interaction because the SDF in this case is not volatile and has counterfactual properties that in fact, dampen the interaction between the discounts and the real economy. The next subsection elaborates on this last point. It directly compares the behavior of the stochastic discount factor in the counterfactual models and explains why discounts have little effect on the real economy in the standard model.

Asset Prices and Unemployment Expansion of the right-hand side of the equilibrium firm free entry condition in Equation 16 shows that discounts affect firms future surpluses either through the inverse of the risk-free rate $E_t(M_{t+1})$ or through the variance of the discount factor $\sigma_t(M_{t+1})$. Firms post fewer vacancies if $E_t(M_{t+1})$ is low, so aggregate dynamics are amplified if $E_t(M_{t+1})$ is volatile and procyclical. Looking at the second term, firm surpluses are typically highly correlated with aggregate consumption, the $\rho_t(M_{t+1}, J_{t+1})$ term is always close to minus one. This means that in order to amplify aggregate dynamics, $\sigma_t(M_{t+1})$ needs to be volatile and countercyclical. Procyclical $\sigma_t(M_{t+1})$ instead, would dampen the dynamics. Additionally, vacancy posting is affected by variation in the level of J_{t+1} and in $\sigma_t(J_{t+1})$. Firm value J_t depends on aggregate productivity and the stock market discounts. Since the aggregate productivity is specified as a homoscedastic process, $\sigma_t(J_{t+1})$ only increases in recessions if $\sigma_t(M_{t+1})$ is countercyclical. In the data [23], the risk-free rate is acyclical and $\sigma_t(M_{t+1})$ and Sharpe ratio are large and countercyclical. This suggests that to be consistent with the asset pricing data, it is the conditional variance of the stochastic discount factor $\sigma_t(M_{t+1})$ that should drive the equilibrium dynamics.

$$E_t(M_{t+1}J_{t+1}) = E_t(M_{t+1})E_t(J_{t+1}) + \rho_t(M_{t+1}, J_{t+1})\sigma_t(M_{t+1})\sigma_t(J_{t+1}) \tag{16}$$

Table 4 compares the behavior of the discount factor in the baseline and in the counterfactual models. The stochastic discount factor in the baseline model is consistent with the standard, yet challenging asset pricing facts. The market price of risk is countercyclical (-0.64) and this variation comes through the right channel. The risk-free rate is almost acyclical (0.1) while the $\sigma_t(M_{t+1})$ is countercyclical (-0.62). Results also suggest that unsurprisingly, the standard model with CRRA

$u_{t+1} = \sigma(1 - u_t) + \lambda_t^w u_t$ with $\sigma = 0.028$. See Shimer (2012) for more information. The data is available at https://sites.google.com/site/robertshimer/research/flows.

[23] Cochrane (2001) chapter 21 explains that in order to explain the equity premium puzzle, $\sigma_t(M_{t+1})$ needs to be large and countercyclical and $E_t(M_{t+1}$ should by acyclical.

preferences (*No Illiquidity* column) fails to generate a countercyclical price of risk (0.97). The correlation between the $\sigma_t(M_{t+1})$ and the TFP in this model is highly procyclical, which acts as a force that dampens fluctuations in real variables. To this extent, the presence of housing illiquidity in the stochastic discount factor makes the baseline model not just quantitatively but also qualitatively different from the standard CRRA model. This qualitative difference in the stochastic discount factor then leads to quantitative differences in the dynamics of real variables and, as will be shown later, offers new insights regarding the costs of business cycles and stabilization policies.

Description	Moment	Baseline	No Illiquidity	No GE	Risk Neutral
S.d. Unemployment	$\sigma(u_t)$	0.52	0.32	0.26	0.26
Relative Consumption Volatility	$\sigma(C_t)/\sigma(Y_t)$	0.48	0.41	0.55	0.71
Correlation of dividends and TFP	$\rho(d_t, z_t)$	0.66	0.63	0.80	0.88
Correlation of Price of risk and TFP	$\rho(\textit{Price of Risk}, z_t)$	-0.64	0.97	0.57	-
Cprrelation of SDF variance and TFP	$\rho(\sigma_t(M_{t+1}), z_t)$	-0.62	0.95	0.57	-
Correlation of Risk-free rate and TFP	$\rho(E_t(M_{t+1}), z_t)$	0.1	0.09	0.1	-

Table 4: Aggregate Moments in the Data and in the Counterfactual Models

Notes Table shows untargeted aggregate moments in the data and in the baseline and the benchmark models. *No Illiquidity* is a counterfactual model, where houses are treated as liquid assets. *No GE* is a model where firms discount profits using risk neutral discounting such that there is no equilibrium interaction between household discounts and firm decisions. *Risk Neutral* is a model with risk-neutral consumers. In all the counterfactuals, the remaining parameters are the same as in the baseline calibration. *Price of Risk* is defined as $\frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}$.

Results in Table 4 also show that the *No GE* benchmark also fails to deliver the right discount properties of the discount factor: both market price of risk and the $\sigma_t(M_{t+1})$ term or procyclical. This happens despite the fact that housing illiquidity matters for the SDF in this benchmark, suggesting that the presence of a general equilibrium structure also changes the equilibrium dynamics in a non-trivial way. As will be explained in the next section, this result is due to the fact that high unemployment is related to endogenously higher aggregate uncertainty, which then leads to a higher price of risk and higher unemployment. Interestingly, such equilibrium feedback is not present in the standard model presented by *No Illiquidity* benchmark.

To further understand the quantitative effects of intra-temporal risk, Figure 4 shows the behavior of unemployment and the stochastic discount factor in the function of the technology shock. The left panel shows that the baseline model shows a qualitatively different behavior compared to the *No GE* and *No Illiquidity* benchmarks. The change in the correlation between the discounts and the technology shocks has quantitatively important effects on the volatility of unemployment, as shown in the right panel. This simple illustration suggests that the countercyclical price of risk in the model is due to both housing illiquidity and the general equilibrium effects. As shown in Section 2.3, housing illiquidity makes the risk-free rate less sensitive to changes in income and the variance of the discount factor reacts more to changes in income uncertainty. In this model, uncertainty is endogenous because idiosyncratic income risk in the form of unemployment rises in recessions, and recessions are associated with greater uncertainty about equity payouts. Given that the model matches a high concentration of stock ownership, the marginal investor is quite well insulated from the idiosyncratic but not from the aggregate risk. The next section explains how the general equilibrium effects between household discounts and firms' decisions lead to endogenous rises in aggregate uncertainty.

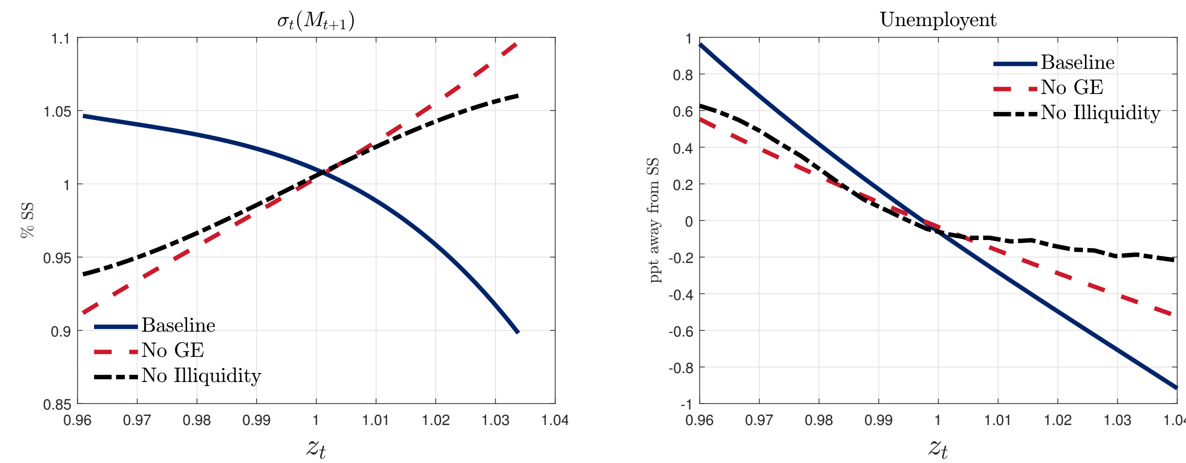


Figure 4: Dynamics of $\sigma_t(M_{t+1})$ and Unemployment

Notes Figures shows the dynamics of $\sigma_t(M_{t+1})$ and unemployment in the baseline model and the *No GE* and *No Illiquidity* benchmarks. x-axis - state of aggregate productivity. *SS* denotes the steady state. Figures plot relevant variables against z_t obtained from a simulation of 10000 periods.

4.2.1 Endogenous Time-Varying Uncertainty

It is known that unemployment fluctuates asymmetrically over the business cycle in the standard search-and-matching model (Mortensen and Pissarides, 1994). As explained in Ferraro (????), this is due to constant returns to scale matching technology between workers and firms, which implies that the job finding probability is concave in market tightness. This asymmetry has implications for the behavior of vacancies. The congestion externality in the labor market means that vacancies are typically being filled fast when unemployment is high meaning that vacancy posting costs are disproportionately low. Firms exploit this externality and by responding aggressively to shocks when unemployment is high. Another way to see it is to express vacancies from the firm vacancy posting condition and take the derivative with respect to productivity, as done in Equation 17. It is easy to see vacancies respond to technology shocks more strongly when unemployment is high.

$$\frac{\partial v_t}{\partial z_t} = (u_t(1-\sigma)+\sigma)\frac{\partial(\lambda^f)^{-1}\left(\frac{\kappa}{\beta\mathbb{E}_t(M_{t+1}J_{t+1})}\right)}{\partial z_t} \tag{17}$$

While cyclical asymmetry is a well-known and general feature of the search-and-matching models, research has overlooked its' implications for uncertainty. The implications can be seen more clearly by decomposing the dividend volatility (Equation 18) into several terms. It shows that dividend volatility depends on the volatility of profits and the volatility of vacancies. The fact that vacancies are more responsive to shocks in recessions also implies that they are more volatile and, consequently, more uncertain. This shows how congestion externality in the labor market can also give rise to countercyclical uncertainty about dividend payouts and can generate an endogenous variation in uncertainty.

$$var_t(d_{t+1}) = var_t(\pi_{t+1}) + \kappa^2 var_t(v_{t+1}) - \kappa cov_t(\pi_{t+1}, v_{t+1})^{24} \tag{18}$$

This is what happens in a quantitative model. Figure 5 shows how uncertainty about dividends and vacancies (left) and the conditional variance of the SDF (right) as a function of the unemployment rate. The left panel shows that while uncertainty about profits barely moves, while the middle term in equal 18 drives the endogenous uncertainty about dividends. The right panel shows that it then translates into a greater variance of the SDF, which implies greater Sharpe ratios and greater discounts.

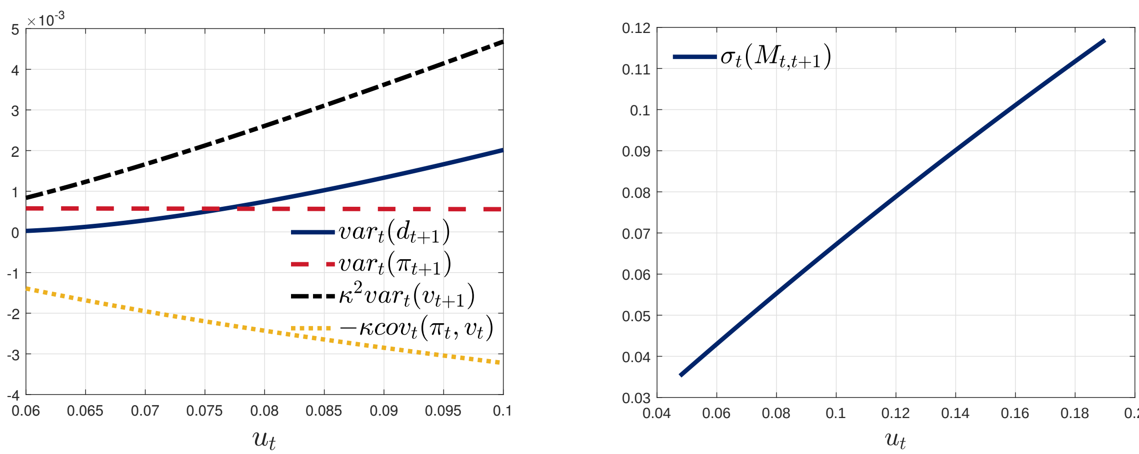


Figure 5: Unemployment and Endogenous Uncertainty

Notes: Figure plots perceived risk about labor market vacancies and dividends, and conditional variance of the stochastic discount factor as functions of aggregate unemployment other aggregate state variables being fixed at their steady state values. Numbers are obtained using the equilibrium laws of motion of the aggregate variables from the baseline calibration.

Overall, this highlights an overlooked ability of the search-and-matching model to endogenously generate countercyclical variation in uncertainty. The uncertainty rises endogenously with the unemployment rate and is priced in by the households owning the firm. The combination of the search-and-matching friction in the labor market together with non-risk-neutral discounting gives rise to a general equilibrium feedback loop between unemployment and stock market discounts. Figure 6 summarizes the equilibrium feedback and the relevant aspects. Unemployment increases both individual income risk (labor) and aggregate risk (dividends). Quantitatively, the second

[24] $\pi_t \equiv (z_t - w_t)(1 - u_t)$

channel is more important because the model generates a large dispersion of wealth and matches the wealth profile of the marginal investor. That means that the firm is priced by households that are well-insured against individual risks.

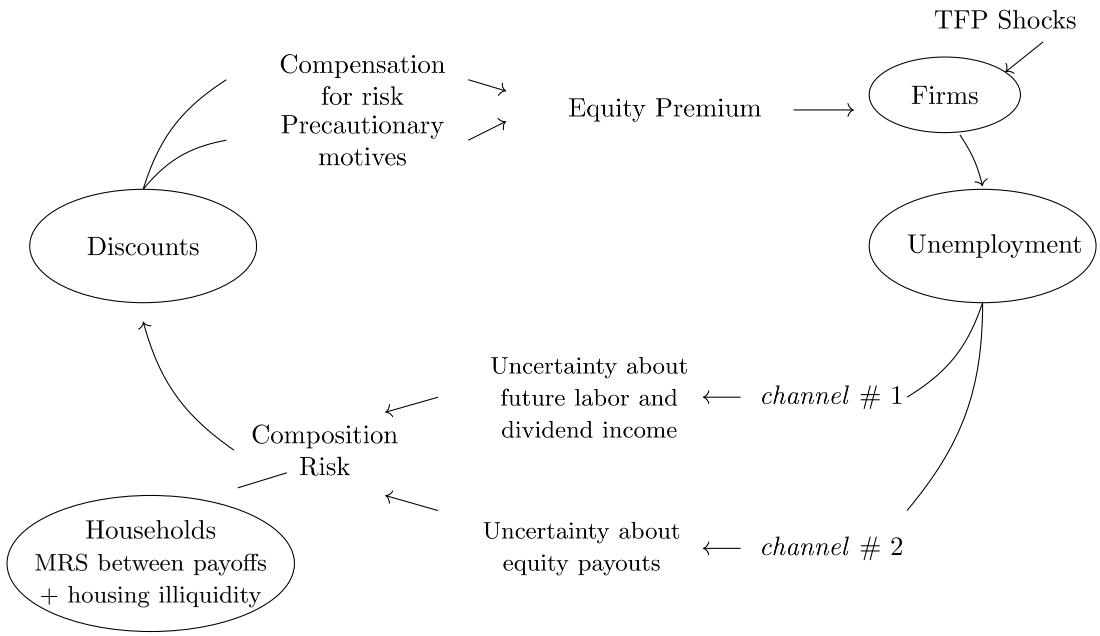


Figure 6: TFP Shocks and Endogenous Uncertainty

Notes: Figure illustrates the model general equilibrium dynamics that give rise to endogenous variation in uncertainty in response to TFP level shocks.

5 Aggregate Implications

This section shows the implications of housing illiquidity on the cost of business cycles and on stabilization policies. The first part explains how the intra-temporal risk arising from housing illiquidity changes the costs of business cycles and how their distribution depends on the household wealth portfolio. The second part shows the implications for policies aimed at mitigating these costs, namely, policies that mitigate aggregate versus individual risk.

5.1 Business Cycle Implications

The seminal paper by Robert E. Lucas (1987) found that the costs of business cycles are small. Building on this, people subsequently found larger costs in models with uninsurable income risk (Imrohoroğlu, 1989; Krusell and Smith, 1999; Krusell, Mukoyama, Şahin, and Smith, 2009). Housing illiquidity affects this calculation through both partial and general equilibrium effects. First, in illiquid housing models, households tend to have less liquid portfolios, which means that their nondurable consumption becomes more responsive to shocks, such as what happens in Kaplan and Violante (2014). Additionally, when housing consumption is a complement to nondurable consumption, variation in nondurable consumption is additionally costly in terms of utility because households dislike the variation in the c/h ratio. Illiquidity also affects household discounts, which feed into firms' decisions and affect prices and the labor market. This is the general equilibrium effect. In order to understand both effects I compare household welfare in the baseline model to two counterfactuals. *No GE* is the same model as the counterfactual considered in section 4.2. The *One-Asset* is a counterfactual where households have no preference for housing and only save by investing in stocks, whereas the first still uses the household discount factor in making the vacancy posting decisions. This is intended to highlight the partial equilibrium effects.

	Baseline	*No GE*	*One-Asset*
Welfare cost %	0	0.18	13.27
Average Unemployment %	7.27	7.54	7.23
Unemployment volatility	0.52	0.28	0.46
Median Wealth-to-Income	2.6054	2.5747	1.5052
Median Liquid Wealth-to-Income	1.3141	1.3408	1.5052
Hand-to-Mouth share %	8.35	9.01	3.65
Top 10% wealth share %	75.16	77.39	19.72

Table 5: Welfare Cost Comparison

Notes Table shows the welfare cost in terms of consumption equivalent compared to relevant benchmark models. *No GE* is a benchmark where firms use risk-neutral discounting. *One-Asset* is a benchmark, where households have no preference for housing and can only saving through investing in stocks. The production side and firm discounting is the same as in the baseline model. A detailed description of the *One-Asset* model can be found in Appendix C.

The first row in Table 5 shows how much the average household consumption in the baseline model would need to increase to make them indifferent between the baseline and the counterfactual economy. Column two shows that the general equilibrium effects do not hamper the household welfare much as the *No GE* benchmark is associated with 0.18% higher welfare in terms of consumption. The reason for this is that equilibrium effects have a two-fold effect on aggregate unemployment, which is the main driver of household income risk. On the one hand, volatile discounts imply more volatile unemployment, meaning that the degree of uninsurable income risk increases by more in recessions in the baseline economy. On the other hand, due to precautionary saving effects, the average risk-free rate in the baseline economy is higher than time discount β, leading to on average higher unemployment and higher average degree of income risk in the *No GE* benchmark. Overall, the first effect dominates and households are worse off in the baseline economy. Even though the *No GE* model was not recalibrated, its' implied cross-sectional moments are very comparable to the baseline economy, suggesting that the welfare effects are not due to changes in the distribution. The last column shows that partial equilibrium effects are huge - household consumption would need to increase by 13.27% on average to make them indifferent between the baseline and the *One-Asset* economy. The second and third rows show that unemployment level and volatility are comparable in both economies, meaning the huge welfare difference is not due to households experiencing more frequent or larger shocks. Instead, this must be due to either larger nondurable consumption responses to shocks, the fact that households dislike the variation in c/h or both. Typically, nondurable consumption becomes responsive to shocks when households hold low liquid wealth and feature a hand-to-mouth behavior. Row six shows that the share of hand-to-mouth households is larger in the baseline economy (8.35% versus 3.65%), but comparable in magnitude to *One-Asset* model.[25] Overall, business cycle costs of housing illiquidity are huge and are due to both illiquidity causing larger responses of nondurable consumption to shocks and due to such responses in consumption are more costly in terms of utility.

5.2 Implications for Policy

The fact that the interaction between household discounts and the labor market amplifies the aggregate unemployment volatility in a model, where households face liquidity frictions, provides novel implications for policies that affect wage rigidity. Examples of such could be the minimum wage laws or the level and cyclicality of unemployment insurance. Minimum wage regulations make wages more rigid for low-paid workers and determine whether firms respond to negative shocks by lowering average wages or by laying people off. Unemployment insurance policies on the other hand affect worker bargaining power. For example, countercyclical unemployment insurance increases worker outside option and, therefore, their bargaining power in recessions, which results in higher wage rigidity. Rigid wages then mean that firms respond to shocks by adjusting their employment instead of renegotiating wages. This is also a typical channel, through which the cyclical movement in unemployment gets amplified in search and matching models with risk-neutral households (Hall and Milgrom, 2008; Manovskii and Hagedorn, 2008).

In models with risk-averse workers unemployment volatility and the degree of wage rigidity also affect household welfare. From the aggregate point of view, rigid wages are associated with countercyclical labor share, which means that the labor side is effectively insured from aggregate fluctuations at the cost of the capital owners. From the individual household point of view, rigid wages mean that unemployment rises more in recessions and therefore, the individual unemployment shocks become more likely. Whether aggregate or individual concerns dominate the welfare calculations depends on how painful the individual shocks are and that depends on whether households have enough liquid assets to smooth their consumption.

[25] I follow Kaplan and Violante (2014) and define households as hand-to-mouth if their liquid wealth is less than half of periods income. Kaplan and Violante (2014) show that up to 35% of households are hand-to-mouth.

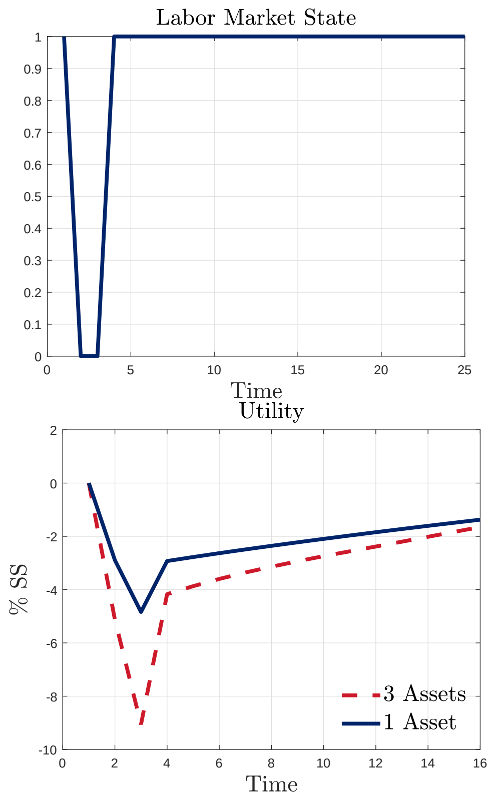


Figure 7: Impulse Response Functions to a Two-Period Unemployment Shock

Notes: Figure plots response of individual consumption and utility to a two-period unexpected unemployment shock. Lines represent the average response from a simulated panel of 30000 households that are simultaneously hit with the same shock. Aggregate states are fixed at the steady state values. Distribution of households at the time of the shock is consistent with the distribution in the baseline calibration.

Figure 7 compares the impulse responses of consumption and utility to a two-period unexpected unemployment shock in the baseline model and in the one-asset model with CRRA utility.[26] The middle and the right panels show that individual consumption falls by 7ppt more in the baseline economy and that unemployment shocks are much more painful in terms of utility. The consumption response is more pronounced because households hold a large fraction of their wealth in illiquid houses and, as shown in Section 5.1, are more likely to be hand-to-mouth. In response to unemployment shocks, they either decrease their consumption by a lot or adjust their holdings of the illiquid asset. Such large deviations are especially painful in terms of utility, since it is derived from the consumption of both housing service and nondurables, and this relative ratio gets distorted as households go through unemployment.

In order to study what this implies for policies, I calculate the household welfare in the baseline calibration, where I vary the parameter ζ, which controls the wage rigidity. Figure 8 shows the

[26]One asset model is a counterfactual model where households can only save in shares of the representative firm. Parameter values used in the one-asset model are the same as in the baseline calibration

result. The right panel shows the unemployment volatility in the baseline and in the one-asset model, where lower values on the x-axis are associated with more rigid wages. The middle panel shows that the economy with more rigid wages is associated with a lower variance of the aggregate utility, which illustrates the aggregate insurance view that more stable wages insure the labor side from aggregate fluctuations. The left panel shows the average variance of the individual utilities, which illustrates the role of individual shocks. In a one-asset model, households have enough liquid assets and are effectively insured against their individual shocks and therefore, the variance of individual utilities follows the same pattern as the aggregate welfare measure in the middle panel. In contrast to that, the baseline model with indivisible houses contains a trade-off. Too flexible wages mean that the labor side is heavily affected by variation in aggregate productivity, while too rigid wages mean that too many households need to go through unemployment state in recessions. The intermediate values of wage rigidity are associated with the largest welfare. An obvious extension for future work would be the endogenize wage determination and to study whether the government should insure the dividend risk or the labor income risk in a setting where government policies rely on distortionary taxation.

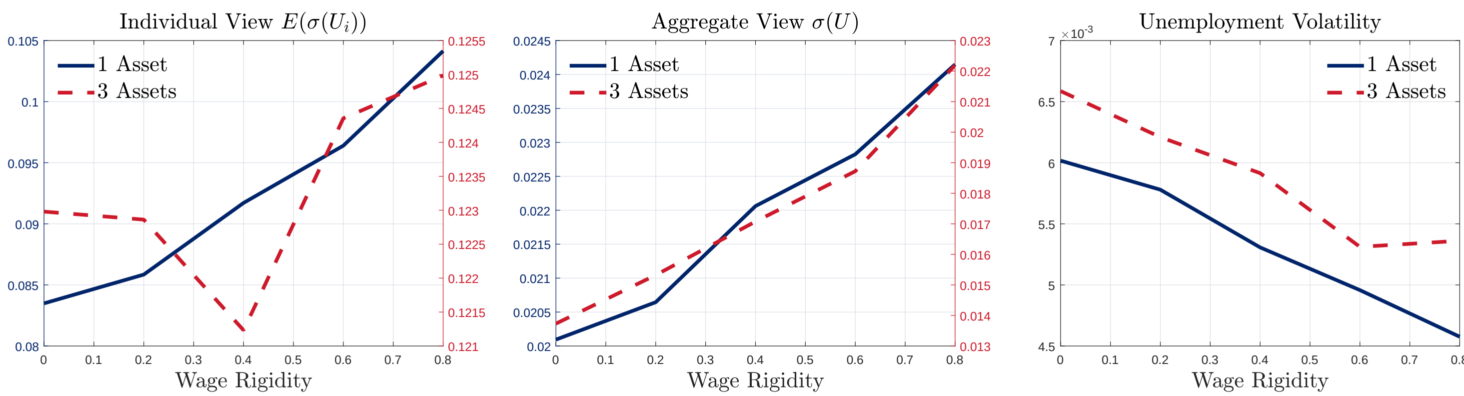


Figure 8: Welfare and Wage Rigidity

Notes: Figure shows two measures of welfare for different values of wage rigidity. Left: average variance of the individual utility, middle: variance of the aggregate utility, right: unemployment volatility. Welfare is calculated from a simulation of a panel of 30,000 households. Distribution of households in the simulation is consistent with the distribution underlying distribution in each model.

6 Evidence of Household Preferences

In the quantitative model the degree of complementarity, controlled by parameter α, was calibrated endogenously to match the cross-sectional data moments about household asset portfolio allocation. While informative, such inference is indirect and not independent of other assumptions about the model. To provide model-independent evidence of the nonseparability of household preferences, in this section, I directly estimate the intratemporal elasticity of substitution between housing and nondurable consumption.

To estimate it I use the canonical framework of Katz and Murphy (1992) and use a static model without housing illiquidity as an auxiliary model. I assume that household utility is described by 19 and is maximized subject to the budget constraint $\omega = p^c c + p^h h$, where ω is the household wealth and p^c and p^h are the prices of nondurable good and houses, respectively.

$$U(c,h) = \left((1-\phi)c^{\frac{\sigma-1}{\sigma}} + \phi h^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{19}$$

Using the optimality conditions of this misspecified model enables me to derive the regression equation, which I estimate using OLS. The parameter of interest is the intratemporal elasticity of substitution σ. The optimal allocation to nondurables and houses is not responsive to relative price changes when the two goods are complements ($\sigma < 1$), and changes by more than one for one when the goods are substitutes ($\sigma > 1$).

$$log(c_i/h_i) = \sigma log\left(\frac{1-\phi}{\phi}\right) + \sigma log(p_i^h/p_i^c) + \epsilon_i \tag{20}$$

I obtain variation in nondurable to housing ratio and house prices from the cross-sectional US data. I measure c_i as the per-person household expenditure on food, which I take from the Current Population Survey (CPS). To measure p^h I use county-level variation in house and rent prices from Zillow. Lastly, I measure h_i as the square footage of the living space from the Corelogic data, which contains the universe of the US housing transactions. More details about data construction can be found in Appendix E.3. Measuring h_i using square footage rather than house values captures the physical quantity rather than the value of housing consumed and overcomes the potential endogeneity problem if house price appears on both sides. To further alleviate the endogeneity concern I instrument the house price with its lag and control for county-level per capita income and population density. Lastly, it is not unreasonable to assume that there are little differences in food prices across counties.[27]

[27] DellaVigna and Gentzkow (2019) provide evidence that supermarket chains in the US set nearly uniform prices across states. However, to the extent that food prices co-move with the house prices, not accounting for differences in food prices may lead to a downward bias in the estimate of σ.

	Dependent variable: ln(c/h)				
	Data				Model
	(1)	(2)	(3)	(4)	(1)
log House Prices	0.172***	0.330***	0.239***	0.337***	0.34265***
	(0.038)	(0.058)	(0.071)	(0.064)	(0.0006)
log Per Capita Income	−0.109	−0.038	0.024	−0.030	–
	(0.100)	(0.083)	(0.097)	(0.081)	
log Population Density	0.014	−0.044***	−0.021	−0.040***	–
	(0.016)	(0.015)	(0.014)	(0.014)	
Constant	−3.619***	−1.916**	−4.455***	−2.005**	−0.28532***
	(0.788)	(0.896)	(0.734)	(0.882)	(0.000)
Observations	144	124	145	147	26400000
R^2	0.186	0.289	0.159	0.233	0.409
Adjusted R^2	0.168	0.271	0.141	0.216	0.409
Note:			*p<0.1; **p<0.05; ***p<0.01		

Table 6: Estimates of Intra-temporal Substitution

Notes: Table shows the estimates of intretemporal elasticity of substitution using different measures of house prices. Column 1 using Zillow House Value Index (ZHVI), Column 2: Median price per square feet, column 3: Zillow Rent Index (ZRI), column 4: Zillow Rent Index (ZRI) per square feet. Observations are weighted by county population. Regression using model simulated data also controls for the individual unemployment state.

Table 6 summarizes the regression results using different measures of house prices. Column one uses the Zillow house price index, column two uses prices per square foot, and columns three and four use the Zillow rent index and the same index per square foot. Overall results point to $\sigma < 0$ and that nondurables and houses are complements. However, this auxiliary model is misspecified as it ignores housing illiquidity and can lead to bias in estimates. For instance, we know that due to liquidity frictions and indivisibilities in housing people change their house infrequently and such changes are typically large.[28] If there are few households close to the threshold of changing their house, then small changes in the c/h ratio in response to the relative price change could be due to illiquidity frictions and the estimate of σ would be downward biased. On the other hand, if there was a large number of households close to such a threshold, then one would observe a lot of variation in c/h when the relative prices change and the misspecified model would underestimate the degree of complementarity. Overall, the direction of bias is ex-ante unknown. To better understand the direction of bias and how realistic the model calibration is I estimate the misspecified model on the data simulated from the true calibrated model. Results reported in the last column of table 6 are in the ballpark of the empirical estimates.

[28] In Appendix, E I document that this is indeed the case in the PSID data.

7 Conclusion

In this paper, I ask how housing illiquidity matters for household risk aversion and their saving behavior. To this end, I develop a model where illiquid houses are modeled both as assets and as consumption goods. The main idea comes from noting that households have nonseparable preferences between consumption of nondurables and houses and therefore, the nondurable-to-housing ratio enters their stochastic discount factor explicitly because, intuitively, households want to equate the nondurable-to-housing ratio to the relative price of the two goods but cannot achieve that due to illuidity and indivisibility in the housing market. Under such formulation, the relative risk aversion becomes a function of the nondurable-to-housing ratio, and such variation drives asset prices. I show both qualitatively and quantitatively that illiquidity is an important driver of household saving behavior, asset prices, and aggregate dynamics.

Quantitatively, I show that this channel is strong and consistent with otherwise challenging asset pricing facts: countercyclical market price of risk and stable risk-free rate. This behavior is qualitatively different from the standard one-asset model, in which the market price of risk is procyclical and the risk-free rate is volatile. These qualitative differences then translate into large quantitative differences in aggregate dynamics. Relative to the model without illiquidity, business cycles become 18% more volatile, and unemployment volatility doubles. In this sense, the housing illiquidity model of household discounts bears similarities with the habits model. The main difference is that the house illiquidity asset pricing channel is quantitatively important in a model with wealth accumulation and after matching the wealth and consumption profile of the marginal investor.

In the last part of the paper, I show that the fact that households dislike variation in their nondurable consumption relative to consumption of housing services has important an bearing on welfare and policy. I show that the welfare costs of business cycles in a model with illiquid houses are 13.27% greater than in the standard one-asset model and that labor market policies that provide individual as opposed to aggregate insurance are more important in the illiquid housing model. In contracts, the aggregate insurance motive is dominant in the one-asset model.

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Appendix

This Appendix consists of five parts. Appendix A contains the proofs of propositions in paper Section 2, Appendix B contains a full description of the quantitative model together with the equilibrium definition. Appendix C discusses the numerical implementation, Appendix D presents an analytical Search-and-Matching model to show how uncertainty arises endogenously in this class of models. Appendix E contains description of the data used in Sections 4 and 6.

A Proofs

Risk-free rate

Risk-free rate R^f is defined as $1/E_t(M_{t,t+1})$. Under log-normality assumptions about c_t and r_t^h, we can rewrite $E_t(M_{t,t+1})$ in terms of current quantities and future shocks

$$
\begin{aligned}
&\mathbb{E}_t(M_{t,t+1}) = e^{-\gamma g}\left(1-\phi+\phi(R_t^H)^{\alpha-1}\right)^{\frac{\alpha-\gamma}{\alpha-1}} \mathbb{E}_t\left(e^{-\gamma\epsilon_{t+1}^c}\left(1-\phi+\phi(e^{(\alpha-1)\epsilon_{t+1}^r})^{\alpha-1}\right)^{\frac{\alpha-\gamma}{\alpha-1}}\right)\\
&\mathbb{E}_t(M_{t,t+1}) = e^{-\gamma g}\left(1-\phi+\phi(R_t^H)^{\alpha-1}\right)^{\frac{\alpha-\gamma}{\alpha-1}} \cdot\\
&\left(\mathbb{E}_t\left(e^{-\gamma\epsilon_{t+1}^c}\right)\mathbb{E}_t\left((1-\phi+\phi(e^{(\alpha-1)\epsilon_{t+1}^r})^{\alpha-1})^{\frac{\alpha-\gamma}{\alpha-1}}\right)+\rho\sigma_t(e^{-\gamma\epsilon_{t+1}^c})\sigma_t(\left(1-\phi+\phi e^{(\alpha-1)}\epsilon_{t+1}^r\right)^{\frac{\alpha-\gamma}{\alpha-1}}))\right)
\end{aligned}
$$

To isolate the role of ϵ_{t+1}^c and ϵ_{t+1}^r, assume that the two shocks are uncorrelated, $\rho = 0$. Then

$$
\mathbb{E}_t(M_{t,t+1}) = e^{-\gamma g}\left(1-\phi+\phi(R_t^H)^{\alpha-1}\right)^{\frac{\alpha-\gamma}{\alpha-1}}\left(\mathbb{E}_t\left(e^{-\gamma\epsilon_{t+1}^c}\right)\mathbb{E}_t\left((1-\phi+\phi(e^{(\alpha-1)\epsilon_{t+1}^r})^{\alpha-1})^{\frac{\alpha-\gamma}{\alpha-1}}\right)\right)
$$

Finding $\mathbb{E}_t\left((1-\phi+\phi(e^{(\alpha-1)\epsilon_{t+1}^r})^{\alpha-1})^{\frac{\alpha-\gamma}{\alpha-1}}\right)$ is still non trivial. Since ϵ_{t+1}^r is distributed normally, $\phi(e^{(\alpha-1)\epsilon_{t+1}^r})$ is log-normal with parameters $ln(\phi)+(\alpha-1)\mu_r$ and $(\alpha-1)^2\sigma_r^2$.

Then $1-\phi+\phi e^{(\alpha-1)\epsilon_{t+1}^r}$ follows a three-parameter log-normal [29] distribution such that $\mathbb{E}(1-\phi+\phi e^{(\alpha-1)\epsilon_t^r}) = \mathbb{E}(\phi e^{(\alpha-1)\epsilon_t^r})+1-\phi$ and $Var(1-\phi+\phi e^{(\alpha-1)\epsilon_t^r}) = Var(\phi e^{(\alpha-1)\epsilon_t^r})$ with parameters $(\mu_x, \sigma_x, 1-\phi)$. μ_x and σ_x relate to μ_r and σ_r in the following way:

$$
\begin{aligned}
&e^{(\alpha-1)\mu_r+(\alpha-1)^2\frac{\sigma_r^2}{2}}+(1-\phi) = e^{(\alpha-1)\mu_x+(\alpha-1)^2\frac{\sigma_x^2}{2}}\\
&(e^{\sigma_r^2}-1)e^{2\mu_r+\sigma_r^2} = (e^{\sigma_x^2}-1)e^{2\mu_x+\sigma_x^2}
\end{aligned}
$$

Using this result, we can derive the risk-free rate in terms of μ_x and σ_x.

$$
R^f = e^{\gamma g}\left(1-\phi+\phi(R_t^H)^{\alpha-1}\right)^{\frac{\alpha-\gamma}{1-\alpha}} e^{-\gamma^2\sigma_c^2/2} e^{-(\frac{\alpha-\gamma}{1-\alpha}\mu_x+\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2/2)}
$$

[29] More information on three-parameter log-normal distributions can be found in Sangal and Biswas (1970).

$$ln(R^f) \equiv r^f = \gamma g - \gamma^2\sigma^2/2 + \frac{\alpha-\gamma}{1-\alpha}\left(ln(1-\phi+\phi(R^H_t)^{\alpha-1}) - \mu_x\right) - \left(\frac{\alpha-\gamma}{1-\alpha}\right)^2 \sigma_x^2/2$$

$\sigma_t(M_{t,t+1})$: assuming that ϵ^c_t and ϵ^r_t are independent, we can express the variance of the stochastic discount factor using the formula $Var(AB) = E(A^2)E(B^2) - E(A)^2E(B)^2$.Applying this we get:

$$Var_t(M_{t,t+1}) = e^{-2\gamma g}(1-\phi+\phi R^{\alpha-1})^{2\frac{\alpha-\gamma}{1-\alpha}}\left(e^{2\gamma^2\sigma^2}e^{2\frac{\alpha-\gamma}{1-\alpha}\mu_x}e^{2\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2} - e^{\gamma^2\sigma^2}e^{2\frac{\alpha-\gamma}{1-\alpha}\mu_x}e^{\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2}\right)$$

$$= e^{-2\gamma g}(1-\phi_\phi R_t^{\alpha-1})^{2\frac{\alpha-\gamma}{\alpha-1}}e^{2\frac{\alpha-\gamma}{1-\alpha}\mu_x}e^{\gamma^2\sigma_c^2\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2}\left(e^{\gamma^2\sigma_c^2\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2} - 1\right)$$

$$\sigma_t(M_{t,t+1}) = e^{-\gamma g}(1-\phi_\phi R_t^{\alpha-1})^{\frac{\alpha-\gamma}{\alpha-1}}e^{\frac{\alpha-\gamma}{1-\alpha}\mu_x}e^{\gamma^2\sigma_c^2/2\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2/2}\sqrt{\left(e^{\gamma^2\sigma_c^2\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2} - 1\right)}$$

$$ln(\sigma_t(M_{t,t+1})) = -\gamma g + \frac{\alpha-\gamma}{\alpha-1}(ln(1-\phi+\phi R_t^{\alpha-1}) - \mu_x) + \gamma^2\sigma_c^2/2 + \left(\frac{\alpha-\gamma}{\alpha-1}\right)^2\frac{\sigma_x^2}{2} + \frac{1}{2}ln(e^{\gamma^2\sigma_c^2+\left(\frac{\alpha-\gamma}{1-\alpha}\right)^2\sigma_x^2} - 1)$$

Proof of Proposition 1

Proof. Here is highlight three extreme cases when housing illiquidity is irrelevant for household perception of risk and the stochastic discount factor.

$$M_{t,t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{1-\phi+\phi(R^H_{t+1})^{\alpha-1}}{1-\phi+\phi(R^H_t)^{\alpha-1}}\right)^{\frac{\alpha-\gamma}{1-\alpha}}$$

Consider each of the three cases separately.

- Case 1. $\alpha = \gamma$. In this case the exponent on the second factor becomes 0 and the second factor is always equal to 1.

- Case 2. $\sigma_r^2 = 0$. It is easier to see if SDF is rewritten in terms of primitives.

$$M_{t,t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{1-\phi+\phi(R^H_{t+1})^{\alpha-1}}{1-\phi+\phi(R^H_t)^{\alpha-1}}\right)^{\frac{\alpha-\gamma}{1-\alpha}} = e^{\gamma g}e^{\gamma\epsilon^c_{t+1}}\frac{1-\phi+\phi e^{(\mu_r+\epsilon^r_{t+1})(\alpha-1)}}{1-\phi+\phi e^{(\mu_r+\epsilon^r_t)(\alpha-1)}}$$

  With $\sigma_r^2 = 0$, $\epsilon_t = \epsilon_{t+1}$ and $1-\phi+\phi e^{(\mu_r+\epsilon^r_{t+1})(\alpha-1)} = 1-\phi+\phi e^{(\mu_r+\epsilon^r_t)(\alpha-1)}$. And SDF becomes $M_{t,t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$.

- Case 3. $\phi = 0$. In this case the second factor becomes $\left(\frac{1-\phi}{1-\phi}\right)^{\frac{\alpha-\gamma}{1-\alpha}} = 1$.

□

Proof of Proposition 2

Proof. Relative risk aversion is defined as the local curvature of the utility function $RRA = -\frac{CU_{cc}}{U_c}$. First, let's find the expression for RRA. For notational convenience define $\mathcal{R}(C, H) \equiv (1-\phi)C^{1-\alpha} + \phi H^{1-\alpha}$.

$$U = ((1-\phi)C^{1-\alpha} + \phi C^{1-\alpha} R^{\alpha-1})^{\frac{1-\gamma}{1-\alpha}}/(1-\gamma)$$
$$U_c = \mathcal{R}(\mathcal{C},\mathcal{H})^{\frac{\alpha-\gamma}{1-\alpha}} \left((1-\phi)C^{-\alpha} + \phi C^{-\alpha}C^{-\alpha}(R^H)^{\alpha-1} - \phi C^{1-\alpha}(R^H)^{\alpha-2}\frac{\partial R^H}{\partial C}\right) = \mathcal{R}(\mathcal{C},\mathcal{H})^{\frac{\alpha-\gamma}{1-\alpha}} \left(1 - \phi C^{-\alpha}\right)$$

$$U_{cc} = -\alpha C^{-\alpha-1}(1-\phi)\mathcal{R}(\mathcal{C},\mathcal{H})^{\frac{\alpha-\gamma}{\infty-\alpha}} + (\alpha-\gamma)\mathcal{R}(\mathcal{C},\mathcal{H})]^{\frac{\alpha-\gamma}{1-\alpha}-1}2\left((1-\phi)C^{-\alpha}\right)$$

Then

$$-\frac{CU_{cc}}{U_c} = \alpha - (\alpha-\gamma)\frac{1-\phi}{1-\phi+\phi(R^H)^{\alpha-1}}$$

$$\begin{aligned}
&RRA^{CES} > RRA^{CRRA} \\
&\alpha - (\alpha-\gamma)\frac{1-\phi}{1-\phi+\phi\frac{c}{h}^{\alpha-1}} > \gamma \\
&\alpha - \gamma > (\alpha-\gamma)\frac{1-\phi}{1-\phi+\phi\frac{c}{h}^{\alpha-1}}
\end{aligned}$$

- Case 1: $\alpha > \gamma$

$$1 > \frac{1-\phi}{1-\phi+\phi(R^H)^{\alpha-1}} \quad \forall R^H > 0$$

- Case 2: $\alpha < \gamma$

$$1 < \frac{1-\phi}{1-\phi+\phi\frac{c}{h}^{\alpha-1}} \quad \emptyset$$

□

Proof of Proposition 3

Proof.

$$\frac{\partial RRA^{CES}}{\partial R^H} = (\alpha-\gamma)(\alpha-1)\frac{(1-\phi}{1-\phi+\phi(R^H)^{\alpha-1})^2}\phi(R^H)^{\alpha-2}$$

$\frac{1-\phi}{1-\phi+\phi(R^H)^{\alpha-1}}\phi(R^H)^{\alpha-2}$ term is positive as long as $\phi > 0$ and R^H¿0. If $\phi = 0$, preferences collapse to CRRA and $RRA = \gamma$. If $\phi > 0$, the sign depends on $(\alpha-\gamma)(\alpha-1)$.

$$\begin{aligned} \frac{\partial RRA^{CES}}{\partial R^H} < 0 \text{ if } \alpha > \gamma \text{ and } \alpha < 1 \\ \alpha < \gamma \text{ and } \alpha > 1 \end{aligned}$$

■

Proof of Proposition 4

This proposition states that conditional variance of stochastic discount factor under housing illiquidity is larger independently of parameter values if shocks to nondurable consumption growth and consumption-to-housing ratio ϵ_t^c and ϵ_t^h are independent.

Proof.

Start by noting that $Var(X) > Var(Y)$ implies $\sigma(X) > \sigma(Y)$, where Var denotes the variance and σ the standard deviation.

$$\begin{aligned} &Var_t(M_{t+1}^{CES}) = C_t^{\gamma}(1-\phi+\phi(R_t^H)^{\alpha-1})^{\frac{\alpha-\gamma}{\alpha-1}}Var_t\left(C_{t+1}^{-\gamma}(1-\phi+\phi(R_{t+1}^H)^{\alpha-1})^{\frac{\alpha-\gamma}{1-\alpha}}\right) \\ &Var_t(M_{t+1}^{CRRA}) = C^{\gamma}Var_t(C_{t+1}^{-\gamma}) \end{aligned}$$

To ease the notation, denote by $A_t \equiv C^{-\gamma}$ and $B_t \equiv (1-\phi+\phi(R_t^H)^{\alpha-1})^{\frac{\alpha-\gamma}{\alpha-1}}$.

$$\begin{aligned} &Var_t(M_{t+1}^{CES}) > Var_t(M_{t+1}^{CRRA}) \\ &A_tB_tVar(A_{t+1}B_{t+1}) > A_tVar(A_{t+1}) \end{aligned}$$

Using the fact that ϵ_t^c and ϵ_t^r are independent, $Var(AB) = \mathbb{E}(A^2)\mathbb{E}(B^2) - \mathbb{E}(A)^2\mathbb{E}(B)^2$. Then

$$\begin{aligned} &A_tB_tVar(A_{t+1}B_{t+1}) > A_tVar(A_{t+1}) \\ &B_t\mathbb{E}_t(A_{t+1}^2)\mathbb{E}_t(B_{t+1}^2) - \mathbb{E}_t(A_{t+1})^2\mathbb{E}_t(B_{t+1})^2 > \mathbb{E}_t(A_{t+1}^2) - \mathbb{E}_t(A_{t+1})^2 \\ &\mathbb{E}_t(A_{t+1}^2)(B_t\mathbb{E}_t(B_{t+1}^2) - 1) > \mathbb{E}_t(A_{t+1})^2(B_t\mathbb{E}_t(B_{t+1})^2 - 1) \end{aligned}$$

By the property that $Var(X) \geq 0$, $\mathbb{E}_t(A_{t+1}^2) \geq \mathbb{E}_t(A_{t+1})^2$. This allows to conclude that

$$B_t\mathbb{E}_t(B_{t+1}^2) - 1 > B_t\mathbb{E}_t(B_{t+1})^2 - 1$$
$$\mathbb{E}_t(B_{t+1}^2)) > \mathbb{E}_t(B_{t+1})^2$$

Using the same variance property, $\mathbb{E}_t(B_{t+1}^2) \geq \mathbb{E}_t(B_{t+1})^2$ always holds.

■

CES without housing friction

This subsection show that $RRA^{CES} = RRA^{CRRA}$ when housing market friction is absent. Consider a static household maximization problem

$$\max_{c,h} = \frac{[(1-\phi)c^{1-\alpha} + \phi h^{1-\alpha}]^{\frac{1-\gamma}{1-\alpha}}}{1-\gamma} \text{ s.t. } w \geq p^h h + p^c c$$

Optimality conditions give:

$$\frac{c}{h} = \left[\frac{p^c}{p^h}\frac{\phi}{(1-\phi)}\right]^{-\frac{1}{\alpha}}$$

We can use this to substitute h out from the utility function

$$U = \frac{c^{1-\gamma}}{1-\gamma}\left[1-\phi+\phi\left(\frac{p^c}{p^h}\frac{\phi}{1-\phi}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{1-\gamma}{1-\alpha}}$$

Then

$$U_c = c^{-\gamma}\left[1-\phi+\phi\left(\frac{p^c}{p^h}\frac{\phi}{1-\phi}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{1-\gamma}{1-\alpha}}$$
$$U_{cc} = -\gamma c^{-\gamma-1}\left[1-\phi+\phi\left(\frac{p^c}{p^h}\frac{\phi}{1-\phi}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{1-\gamma}{1-\alpha}}$$

Assuming that household takes p^c and p^h as given the elasticity is

$$RRA^{CES} = -U_{cc}\frac{c}{U_c} = \frac{\gamma c^{-\gamma-1}c}{c^{-\gamma}} = \gamma = RRA^{CRRA}$$

B Quantitative Model

This Appendix contains a full description of the quantitative model. It begins with the household value functions, then describes the working of the housing and the labor markets and finishes with the definition of recursive competitive equilibrium.

B.1 Households

Household period problem consists of three sub periods. Sub period 1 describes the consumption-saving problem, sub period 2 is the choice between houses and other assets. Sub period 3 solves the liquid asset allocation problem into bonds and stocks based on the trade-off between return and risk. Superscripts E and U indicate the value functions for employed and unemployed households, respectively.

B.1.1 Sub period 3

Sub period 3 is a portfolio choice problem between stocks and bonds, based on return and risk. In this period household state consist of the cash-on-hand in sub period 3 (ω_3), houses (h) and the aggregate state vector X.

$$
\begin{aligned}
V_3^E(\omega_3, h, X) = &\max_{a'} \mathbb{E}((1-\sigma(1-\lambda^w(\theta)))V_1^E(\omega', h, X) + \sigma(1-\lambda_w(\theta)))V_1^U(\omega', h, X)|X)] \\
&\text{s.t.} \\
&\omega' = a'(p^{a\prime} + d') + b'(1+r^b) \\
&b' = \omega_3 - a'p^a \\
&b' \geq \underline{\mathrm{b}}
\end{aligned}
$$

$$
\begin{aligned}
V_3^U(\omega_3, h, X) = &\max_{a'} \mathbb{E}(\lambda^f(\theta)V_1^E(\omega', h, X) + (1-\lambda^f(\theta))V_1^U(\omega', h, X)|X) \\
&\text{s.t.} \\
&\omega' = a'(p^{a\prime} + d') + b'(1+r^b) \\
&b' = \omega_3 - a'p^a \\
&b' \geq \underline{b}
\end{aligned}
$$

In sub period 2 household list prices in the housing market determine whether the housing market transaction get executed. In the case when it is not executed, the savings that had been allocated to housing transaction go into savings in bonds in sub period 3 and are denoted as b^f. In such cases households solve a slightly modified problem. Employed:

$$
\begin{aligned}
V_3^{f,E}(b^f,\omega_3,h,X) =& \mathbb{E}((1-\sigma(1-\lambda_w(\theta)))V_1^E(\omega',h,X)+\sigma(1-\lambda^w(\theta))V_1^U(\omega',h,X)|X)\\
& \text{s.t.}\\
& \omega' = a'(p^{a\prime}+d')+(b'+b^f)(1+r^b)\\
& b' = \omega_3 - a'p^a\\
& b'+b^f \geq \underline{b}
\end{aligned}
$$

Unemployed:

$$
\begin{aligned}
V_3^{f,U}(b^f,\omega_3,h,X) =& \mathbb{E}(\lambda^f(\theta)V_1^E(\omega',h,X)+(1-\lambda^f(\theta))\sigma V_1^U(\omega',h,X)|X)\\
& \text{s.t.}\\
& \omega' = a'(p^{a\prime}+d')+(b'+b^f)(1+r^b)\\
& b' = \omega_3 - a'p^a\\
& b'+b^f \geq \underline{b}
\end{aligned}
$$

B.1.2 Sub period 2

In the 2nd sub-period household allocate the savings ω_2 into houses and the remaining wealth ω_3. Important to note that the bond choice in sub period 3 depends on the success of housing transaction. If the transaction is not successful, the wealth that was supposed to be spent on adjusting goes into bonds.[30] The b^f notation captures this idea.

Employed household buying

$$
\begin{aligned}
V_2^{E,buy}(\omega_2,h,X) =& \max_{p^b,h'\in H}\Big(\lambda^b(p^b,\Delta h',p^h)V_3^E(\omega_3',h',X)+(1-\lambda^b(p^b,\Delta h',p^h))V_3^{f,E}(b^f,\omega_3',h,X')\Big)\\
& \text{s.t.}\\
& \omega_2 = hp^h+p^b+\omega_3'\\
& \omega_3' \geq \underline{b}\\
& b^f = p^b
\end{aligned}
$$

[30]Under this assumption the outcome of housing market transaction does not affect the stock choice and the consumption Euler equation holds.

Employed household selling

$$V_2^{E,sell}(\omega_2,h,X) = \max_{p^b,h'\in H} \Big(\lambda^b(p^s,-\Delta h')V_3^E(\omega_3',h',X) + (1-\lambda^b(p^s,\Delta h'))V_3^{f,E}(b^f,\omega_3,h,X)\Big)$$
$$\text{s.t.}$$
$$\omega_2 = hp^h - p^s + \omega_3'$$
$$\omega_3' \geq \underline{b}$$
$$b^f = -p^s$$

Unemployed household buying

$$V_2^{U,buy}(\omega_2,h,X) = \max_{p^b,h'\in H} \Big(\lambda^b(p^b,\Delta h',p^h)V_3^U(\omega_3',h',X) + (1-\lambda^b(p^b,\Delta h',p^h))V_3^{f,U}(b^f,\omega_3',h,X')\Big)$$
$$\text{s.t.}$$
$$\omega_2 = hp^h + p^b + \omega_3'$$
$$\omega_3' \geq \underline{b}$$
$$b^f = p^b$$

Unemployed household selling

$$V_2^{U,sell}(\omega_2,h,X) = \max_{p^b,h'\in H} \Big(\lambda^b(p^s,-\Delta h')V_3^U(\omega_3',h',X) + (1-\lambda^b(p^s,\Delta h'))V_3^{f,U}(b^f,\omega_3,h,X)\Big)$$
$$\text{s.t.}$$
$$\omega_2 = hp^h - p^s + \omega_3'$$
$$\omega_3' \geq \underline{b}$$
$$b^f = -p^s$$

and $V_2^E(\omega_2,h,X)$, $V_2^U(\omega_2,h,X)$ are defined by

$$V_2^E(\omega_2,h,X) = \max_{buy,sell}\{V_2^{E,buy}(\omega_2,h,X), V_2^{E,sell}(\omega_2,h,X)\}$$
$$V_2^U(\omega_2,h,X) = \max_{buy,sell}\{V_2^{U,buy}(\omega_2,h,X), V_2^{U,sell}(\omega_2,h,X)\}$$

B.1.3 Sub period 1

In the first sub-period households make consumption-saving decisions. Variable ω_2 is the total household carried from sub period 1, y is income, h - houses, ω - financial wealth defined as $\omega = (p^a + d)a + b(1 + r^b)$. For employed households $y = w$, for unemployed: $y = \mu w$

Employed:

$$
\begin{aligned}
V_1^E(\omega, h, X) = &\max_{c,\omega_2'} U(c,h) + V_2^E(\omega_2', h, X) \\
&\text{s.t.} \\
&\omega + hp^h + w = c + \omega_2' \\
&\omega_2' \geq \underline{b}
\end{aligned}
$$

Unemployed:

$$
\begin{aligned}
V_1^U(\omega, h, X) = &\max_{c,\omega_2'} U(c,h) + V_2^U(\omega_2', h, X) \\
&\text{s.t.} \\
&\omega + hp^h + \mu w = c + \omega_2' \\
&\omega_2' \geq \underline{b}
\end{aligned}
$$

B.2 Housing Market

Housing market is a non-Walrasian market, where households need to search for counter parties when they want to adjust their home ownership. To make this two-sided search problem tractable I introduce a modeling device called real estate brokers. These brokers collect bids from households and then trade with each other to determined to aggregate price in the housing market. I assume that households do not need to sell their house in order to acquire a new one. Instead they can directly move to another segment by posting quantity Δh.
Households direct their search in submarkets indexed by prices and quantities. By free entry condition real estate brokers enter each sub market until their expected profits are 0.

$$
\underbrace{(p^b - p^h \Delta h')}_{\text{Broker Profit}} \underbrace{\alpha_b(\theta(p_b, \Delta h')}_{\text{Match Probability}} \geq \underbrace{\Delta h' \kappa_b}_{\text{Broker Cost}} \quad \text{Bids from buyers}
$$

$$
\underbrace{(p^h \Delta h' - p^s)}_{\text{Broker Profit}} \underbrace{\alpha_s(\theta(p_s, \Delta h')}_{\text{Match Probability}} \geq \underbrace{-\Delta h' \kappa_s}_{\text{Broker Cost}} \quad \text{Bids from sellers}
$$

Broker free entry condition into selling and buying sub markets determines the market tightness in every sub market (equations 1 and 2)

$$
\theta_b(p_b, \Delta h', p^h) = \alpha_b^{-1}\left(\frac{\kappa_b \Delta h'}{p_b - p^h \Delta h'}\right) \tag{1}
$$

$$
\theta_s(p_s, -\Delta h', p^h) = \alpha_s^{-1}\left(\frac{\kappa_s(-\Delta h')}{-p^h \Delta h' - p_s}\right) \tag{2}
$$

And substituting 1 and 2 into expressions for $\lambda_h(\theta_s)$ and $\lambda_h(\theta_b)$ gives matching probabilities for the household.

$$\lambda_h(\theta_b(p_b, \Delta h', p^h)) = \left(\frac{\kappa_b \Delta h'}{p^b - p^h \Delta h'} \right)^{\frac{\eta_h}{\eta_h - 1}} \tag{3}$$

$$\lambda_h(\theta_s(p_s, -\Delta h', p^h)) = \left(\frac{-\kappa_s \Delta h'}{-p^h \Delta h' - p^s} \right)^{\frac{\eta_h}{\eta_h - 1}} \tag{4}$$

B.3 Labor Market

Search in labor market is random. The production sector consists of the representative firm that posts vacancies taking the household stochastic discount factor as given.
The value of posting a vacancy V is:

$$V(X) = \max_{\theta} -\kappa + \mathbb{E}\left[M(X, X')(\lambda^f(\theta)J(X') + (1 - \lambda^f(\theta))V(X'))|X\right]$$

The value of a filled job J is:

$$J(X) = z - w + \mathbb{E}\left[M(X, X')(\sigma V(X') + (1 - \sigma)J(X'))|X\right]$$

by free entry:

$$V(X) = 0$$

and therefore

$$J(X) = z - w + (1 - \sigma)\mathbb{E}\left[M(X, X')J(X'))|X\right]$$

Competitive entry by firms implies:

$$\frac{\kappa}{(1 - \sigma)\lambda^f(\theta)} = E[(M(X, X')J(X'))|X] \tag{5}$$

Firms discount profit by using the weighted average of the individual discount factors of it's owners.

$$M(X, X') = \beta \int a'(\omega, h, e, X) \frac{u_c(c'(\omega', h', e', X'), h'(\omega, h, e, X))}{u_c(c(\omega, h, e, X), h)} d\lambda(\omega, h, e, X) \tag{6}$$

B.4 Equilibrium

Recursive competitive equilibrium is made by

- Household value functions $V_1^E(\omega, h, X)$, $V_1^U(\omega, h, X)$, $V_2^E(\omega_2, h, X)$, $V_2^U(\omega_2, h, X)$, $V_3^E(\omega_3, h, X)$, $V_3^U(\omega_3, h, X)$, $V_2^{E,buy}(\omega_2, h, X)$, $V_2^{U,buy}(\omega_2, h, X)$, $V_2^{E,sell}(\omega_2, h, X)$, $V_2^{U,sell}(\omega_2, h, X)$ and the associated policy functions $c^E(\omega,h,X)$, $c^U(\omega,h,X)$, $\omega_2^E(\omega,h,X)$, $\omega_2^U(\omega,h,X)$, $p^{E,s}(\omega_2,h,X)$, $p^{U,s}(\omega_2,h,X)$, $p^{E,b}(\omega_2,h,X)$, $p^{U,b}(\omega_2,h,X)$, $h'^{buy,E}(\omega_2,h,X)$, $h'^{buy,U}(\omega_2,h,X)$, $h'^{sell,E}(\omega_2,h,X)$, $h'^{sell,U}(\omega_2,h,X)$, $\omega_3'^{buy,E}(\omega_2,h,X)$, $\omega_3'^{buy,U}(\omega_2,h,X)$, $\omega_3'^{sell,E}(\omega_2,h,X)$, $\omega_3'^{sell,U}(\omega_2,h,X)$, $b'^E(\omega_3,h,X)$, $b'^U(\omega_3,h,X)$, $a'^E(\omega_3,h,X)$, $a'^U(\omega_3,h,X)$.

- Firm value functions $V(X)$ and $J(X)$.
- Prices $p^a(X)$, $p^h(X)$, $\theta(X)$, $w(X)$.
- Aggregate unemployment u_t.
- Aggregate wealth distribution $\lambda(\omega,h,e,X)$[31].

such that

- $V_1^E(\omega,h,X)$, $V_2^E(\omega_2,h,X)$, $V_3^E(\omega_3,h,X)$, $V_2^{E,buy}(\omega_2,h,X)$, $V_2^{E,sell}(\omega_2,h,X)$ solve the employed workers problem and $V_1^U(\omega,h,X)$, $V_2^U(\omega_2,h,X)$, $V_3^U(\omega_3,h,X)$, $V_2^{U,buy}(\omega_2,h,X)$, $V_2^{U,sell}(\omega_2,h,X)$ solve the unemployed workers problem.
- Aggregate discount factor $M(X,X')$ satisfies equation 6.
- Free entry in the labor market ($V(X)=0$) pins down $\theta(X)$.
- stock price $p^a(X)$ satisfies the market clearing condition.

$$1 = \int a'(\omega,h,e,X)d\lambda(\omega,h,e,X) \tag{7}$$

- Aggregate house price $p^h(X)$ is determined by the market clearing condition between the real estate brokers.

$$\mathcal{D}(p^h,X) = \mathcal{S} \tag{8}$$

where

$$D(p^h,X) = \int h'\alpha_b(\theta(p_b,\Delta h',p^h)\lambda(w,h,e,X)\perp(h'\geq h) + \int h'\alpha_s(\theta(p_s,\Delta(-h'),p^h)\lambda(w,h,e,X)\perp(h'<h)$$

- Housing market tightness satisfy 2 and 1.
- Wealth distribution $\lambda(\omega,h,e,X)$ is induced by household policies for h, a,b p^s and p^b, an exogenous process s determining the labor market state and the aggregate state vector X. Aggregate unemployment evolves according to:[32]

$$u' = u(1-\lambda^w(\theta)) + \sigma(1-u) - \sigma(1-u)\lambda^w(\theta)$$

[31] e denotes the individual labor market state (employed/unemployed)

[32] The $-\sigma(1-u)\lambda_w(\theta)$ is needed to match the steady state level of unemployment in the quarterly calibration.

- Aggregate productivity follows an AR(1) process in logs: $ln(z_t) = \rho ln(z_{t-1}) + \epsilon_t$ with $\epsilon_t \sim N(0, \sigma^2)$.

B.5 From individual Euler to aggregate SDF

$$0 = a_{it+1}\left(1 - \beta \mathbb{E}_t\left[\frac{u'(c_{it+1}, h_{it+1})}{u'(c_{it}, h_{it})}\frac{d_{t+1} + p^a_{t+1}}{p^a_t}\right]\right)$$

Aggregate

$$\int a_{it+1} d\lambda = \int a_{it+1}\beta \mathbb{E}_t\left[\frac{u'(c_{it+1}, h_{it+1})}{u'(c_{it}, h_{it})}\frac{d_{t+1} + p^a_{t+1}}{p^a_t}\right] d\lambda$$

Market Clearing

$$\int \underbrace{a_{it+1} d\lambda}_{=1} = \int a_{it+1}\beta \mathbb{E}_t\left[\frac{u'(c_{it+1}, h_{it+1})}{u'(c_{it}, h_{it})}\frac{d_{t+1} + p^a_{t+1}}{p^a_t}\right] d\lambda$$

$$p^a_t = \int a_{it+1}\beta \mathbb{E}_t\left[\frac{u'(c_{it+1}, h_{it+1})}{u'(c_{it}, h_{it})}(d_{t+1} + p^a_{t+1})\right] d\lambda$$

C One-Asset Model

This section presents a One-Asset model considered in Section 5. Households maximize expected lifetime utility defined over nondurable consumption c_t. There are two uninsurable labor market states, employment and unemployment. Employed households receive wage income w_t and unemployed households receive unemployment insurance proportional to wages μw_t. Households can save in risky shares a that are traded at an equilibrium price p^a and yield dividends d_t. On the production side, the representative firm engages in random search in the labor market and takes the household discount factor as given in discounting its' future surpluses. Firms' productivity is determined by the aggregate technology z_t. Let V^E and V^U denote the value functions of employed and unemployed households, respectively. The recursive representation of households' problem is the following:

- Households:

Employed:

$$V^E(a,X) = \max_{c,a'} \mathbb{E}\left((1-\sigma(1-\lambda^w(\theta)))V^E(a',X') + \sigma(1-\lambda^w(\theta))V^U(a',X')|X\right)$$

s.t.

$$a(p^a + d) + w \geq c + p^a a'$$

Unemployed:

$$V^U(a,X) = \max_{c,a'} \mathbb{E}\left((1-\lambda^w(\theta))V^U(a',X'|X) + \lambda^w(\theta))V^E(a',X')|X\right)$$

s.t.

$$a(p^a + d) + \mu w \geq c + p^a a'$$

- Firms:

$$J(X) = z - w + \beta\mathbb{E}\left(M(X,X')(J(X')(1-\sigma) + V(X')\sigma|X\right)$$
$$V(X) = \max_{\theta} -\kappa + \mathbb{E}\left(M(X,X')(\lambda^f(\theta)J(X') + (1-\lambda^f(\theta))V(X'))|X\right)$$

- Equilibrium:
  - Asset market clearing determines the stock price p^a, $1 = \int a'(a,e,X)d\lambda(a,e)$
  - Firms free entry determines the labor market tightness θ, $V = 0$.
  - Representative firm uses household's SDF $M(X', X)$ to discount future profits

$$M(X,X') = \beta \int a'_i \frac{u_c(c'(a',e',X'))}{u_c(c(a,e,X))} d\Lambda(a,e,X) \tag{9}$$

  - Equilibrium wealth distribution λ is induced by household policy a', their labor income shocks and aggregate states. $\lambda' = \Gamma(\lambda, X)$. Aggregate unemployment evolves according to:

$$u' = u(1-\lambda^w(\theta)) + \sigma(1-u) - \sigma(1-u)\lambda^w(\theta)$$

  - Aggregate productivity exogenously according to an AR(1) process:

$$ln(z_t) + \rho ln(z_{t-1}) + \epsilon_t; \quad \epsilon_t \sim N(0,\sigma^2)$$

D Numerical Implementation

D.1 Solution Algorithm

Global model solution poses several important computational challenges. First, housing indivisiblility means that solution of the household problem using first-order conditions is not feasible. Second, the paper is about time-varying aggregate risk, which means that methods preserving non-linearities at the individual level, but using linear perturbation around aggregate states (Winberry (2018), Reiter (2009)) are not sufficient. Third, three markets need to be cleared numerically each period in the stochastic simulation.

The solution method used here is a version of Krusell-Smith method with a non-trivial market clearing, which means that the equilibrium prices in the stochastic simulation need to be found numerically and therefore need to enter the household problem as state variables. The algorithm is as follows:

1. Initialize parameters for the perceived laws of motion for the endogenous state variables

$$p_t = f_p(X_{t-1}, X_t) \tag{10}$$

$$\theta_t = f_\theta(X_{t-1}, X_t) \tag{11}$$

$$p_t^h = f_\theta(X_{t-1}, X_t) \tag{12}$$

2. Given perceived laws of motion for p_t, p_t^h and θ_t, solve individual problems:

   Household Problem

   (a) Define the grid for the household problem in the (ω, h, X) and (ω_2, h, X), (ω_3, h, X) and (b^f, ω_3, h, X) space.

   (b) For each value in the grids (ω, h, X) and (ω_2, h, X), (ω_3, h, X) and (b^f, ω_3, h, X) initialize the associated value functions.

   (c) Solve $F_1^E(\omega, h, X)$, $F_2^E(\omega_2, h, X)$, $F_3^E(\omega_3, h, X)$, $F_3^{f,E}(b^f, \omega_3, h, X)$, $F_1^U(\omega, h, X)$, $F_2^U(\omega_2, h, X)$, $F_3^U(\omega_3, h, X)$, $F_3^{f,U}(b^f, \omega_3, h, X)$ by finding a fixed point.

   Firms problem

   (a) Define the grid in (X) space

   (b) Solve for $J(X)$ by imposing the free entry condition $V(X) = 0$

3. Simulation: Initialize the grid for the histogram in (ω, h) space. Draw a sequence of aggregate shocks of length T. For T periods

(a) Given the household policies $a'(\omega, h, X)$, $h'(\omega, h, X)$, $b'(\omega, h, X)$ $p^s(\omega, h, X)$ and $p^b(\omega, h, X)$ calculate the house purchase and sell probabilities $\lambda_h(\theta_b)$ and $\lambda_h(\theta_s)$ for each value in (ω, h) space.

(b) Draw a vector of random shocks and determine, which housing transactions are successful given the implied $\lambda_b(\theta_b)$ and $\lambda_s(\theta_s)$. This gives the realized demand for houses $\hat{h}(\omega, h, X)$ and bonds b after transactions are completed $\hat{b}'(\omega, h, X)$.

(c) Using the wealth distribution and demand for stocks $a'(\omega, h, X)$ and houses $\hat{h}'(\omega, h, X)$, find p^a_t and p^h_t that satisfy asset and housing market clearing (equations 7 and 8). Record these p^a_t and p^h_t as data.

(d) Using p^a_t and p^h_t and predicted values $\hat{p}^a_{t+1}$, $\hat{p}^h_{t+1}$, $\hat{\theta}_t$ calculate the SDF defined in 6. Using this SDF, solve for θ_t from the free entry condition $V(X) = 0$. Record this θ_t as data and use it to calculate u_{t+1} as $u_{t+1} = u_t(1 - \lambda^w(\theta_t)) + \sigma(1 - u_t) - \sigma(1 - u_t)\lambda_w(\theta_t)$.

(e) Use Young's method to get the next period's wealth distribution. Calculate ω_{it+1} using $a'(\omega, h, X)$, $\hat{b}'(\omega, h, X)$ and the predictions for the next period prices and dividends $\hat{p_{t+1}}$ and $\hat{d_{t+1}}$ using the perceived laws of motion for p^a_t, p^h_t and θ_t.[33]

4. Using the simulated series for p^a_t, p^h_t, θ_t and a sequence of aggregate shocks z_t, estimate coefficients in 10, 11 and 12

5. Update regression coefficients using a dampening rule: $\beta_{xy} = (1 - \mu)\beta_{xy} + (\mu)\beta^{new}_{xy}$

6. Keep repeating 2-5 until prediction errors for p^a_{t+1}, p^h_{t+1} and θ_t are minimized

D.2 Numerical Implementation

In the numerical implementation I approximate the future prices using the first moments of the aggregate state variables, in which case $X = \{p^a, p^h, u, z\}$. The intuition is that household wealth distribution matters for the aggregate dynamics to the extent that it affects prices therefore, including the stock and house prices p^a, p^h as state variables helps to capture the relevant moments of the wealth distribution.

I solve the household problem using a value function iteration using golden section search. I use 190 grid points for ω_1, 190 for ω_2, 190 for ω_3, 5 for u, 11 for p^a, 10 for p^h, 6 for h and 4 for z. Aggregate productivity process (z) is discretized using Rouwenhorst method. In addition, there are 2 individual labor market states. Overall grids for sub periods one to three contain 5016000 points each. I also space the grids for ω_1, ω_2, ω_3 such that more points are clustered at the low wealth

[33] At this point the next period equilibrium distribution may not be consistent with the realized prices at $t + 1$ if the perceived and actual laws of motion differ too much and the algorithm will not converge in these cases. In the numerical implementation I ensure that this does not happen by updating the parameters for the perceived law of motion slowly.

levels. I select the highest value for the h grid to be such that the mass of households holding h^{max} is negligible in equilibrium. Specifically, the grid for h is $h \in \{0.5, 0.75, 1.5, 10, 20, 50\}$

To simulate the wealth distribution I specify a histogram on a denser grid for ω_1, containing 2200 points, which means that the simulated histogram contains 26400 points and use Youngs method to (Young, 2010) to simulate the wealth distribution. In the simulation T=1200 and I discard the first 300 periods when updating the regression coefficients. In each simulation I use the wealth distribution of the last period of the previous simulation as a starting point. Here histogram contains a relatively large number of points and values for p_t^a, p_t^h and θ_t need to be found numerically in each period, making the simulation computationally intensive. Two solutions help to increase the speed significantly: 1. I parallelize the simulation in time, such that I use the wealth distribution and aggregate states of the adjacent parallel block as starting points in the next block. 2. I take advantage of the fact that the relevant set of the state space decreases in the number of state variables. In the context of this paper it means that around 90% of the points in the pre-specified histogram have 0 mass in equilibrium therefore I simulate the economy by only keeping track of the points with non zero mass. This increases the speed of numerical market clearing and saves a lot of memory in the step that shifts the mass between the two periods.

I approximate p_t^a, p_t^h and θ with the following laws of motion:

$$p_t^a = exp(\beta_{01} + \beta_{11}ln(p_{t-1}^a) + \beta_{21}ln(p_{t-1}^h) + \beta_{31}ln(u_t) + \beta_{41}log(z_t) + \beta_{51}ln(z_t)log(u_t)) + \beta_{61}ln(z_t)ln(p_{t-1}^a)) + \epsilon_t$$

$$p_t^h = exp(\beta_{02} + \beta_{12}ln(p_{t-1}^a) + \beta_{22}ln(p_{t-1}^h) + \beta_{32}ln(u_t) + \beta_{42}log(z_t) + \beta_{52}ln(z_t)log(u_t)) + \beta_{62}ln(z_t)ln(p_{t-1}^a)) + \epsilon_t$$

$$\theta_t = exp(\beta_{03} + \beta_{13}lln(p_t^a) + \beta_{23}ln(p_t^h) + \beta_{33}ln(u_t) + \beta_{43}ln(z_t) + \beta_{53}ln(z_t)ln(u_t)) + \beta_{63}ln(z_t)ln(p_t)) + \epsilon_t$$

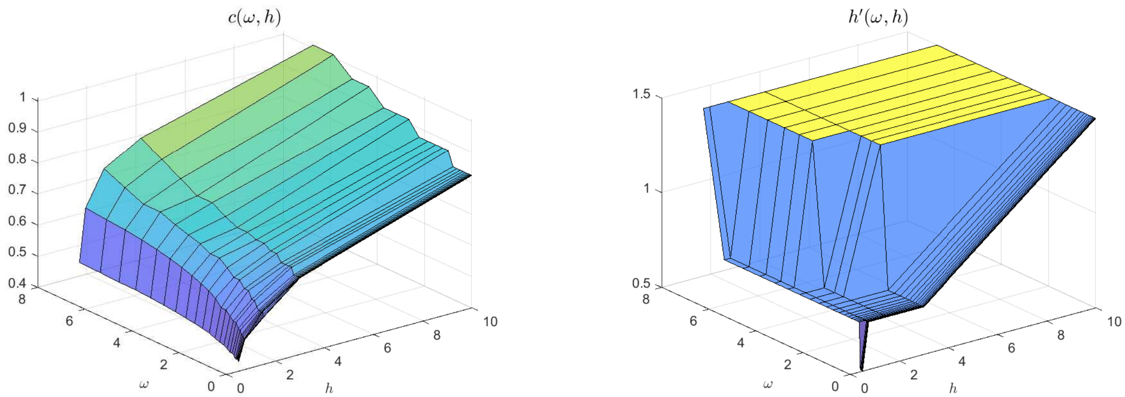
The following tables and figures contain information about the prediction errors. To formally test the precision I perform the fundamental accuracy test as in (Haan, 2010). I compare the difference between the equilibrium sequences and the artificial data generated using the approximated law of motion, simulated from T=1200 periods. I report the maximum and the mean differences series between the two sequences. Since numbers are differences in logs, they can be interpreted as percentage errors.

	Error mean	Error max
p_t^a	0.0005	0.0020
p_t^h	0.0087	0.0308
θ_t	0.0034	0.0093

Table A.1: Table reports fundamental prediction errors as (Haan, 2010). Error mean - Mean percentage difference, Error max - maximum percentage difference

D.3 Numerical Results

D.3.1 Policy Functions

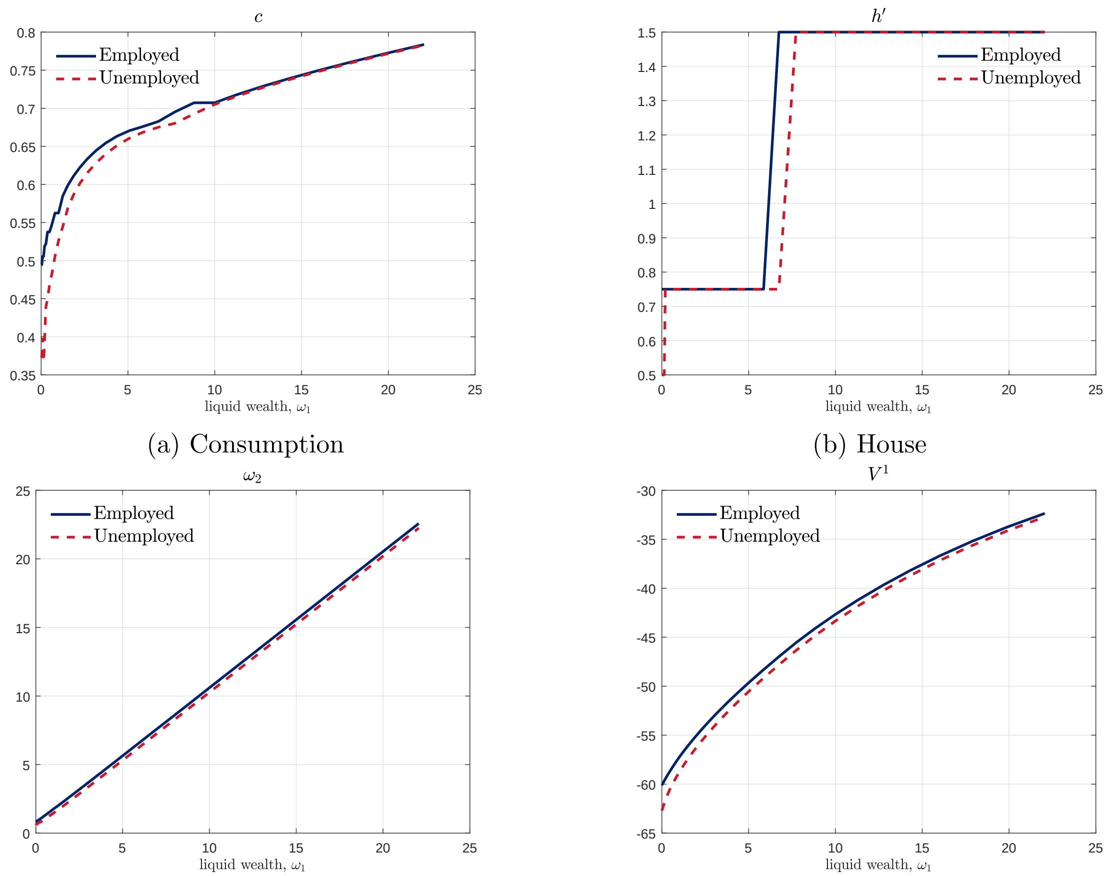


(a) Consumption

(b) House

Figure A.1: Policy Functions by Liquid Wealth and House

Notes: Figure shows consumption and housing policy functions of employed household by liquid wealth and house. All other state variables are at their steady state values



(a) Consumption

(b) House

(c) Saving

(d) Value function

Figure A.2: Policy Functions for Employed and Unemployed

Notes: Figure shows consumption and housing and savings policy functions and the value function for the first sub-period by employment state. Aggregate state variables are at their steady state value and $h = 0.75$, which is the house value chosen by the median wealth household.

E Data Construction

E.1 Panel Study of Income Dynamics (PSID), wealth and income data

I Panel Study of Income Dynamics household panel waves 2013-2017 to calculate wealth and consumption data. The sample consists of households, whose head is between 25 and 65 years old. In calculating the moments I used individual cross-sectional weights to have a sample representative of the US population in in the respective year. Table provides an overview of the household level balance sheet variables.

Assets	*House*, *Value of farm/business*, *Other Real estate assets*, *Value of checking/saving account*, *Stocks and Mutual funds*, *Vehicles*, *Other Assets*, *Annuity IRA account*
Liabilities	*Mortgages*, *Farm/business debt*, *Other real estate debt*, *Credit card debt*, *Student Loan Debt*, *Medical Debt*, *Other Debt*, *Family Loan Debt*

Table A.2: Components of Net Wealth in PSID Household Survey.

I calculate the household level wealth variables as follows:

- *Net Wealth*: Net Wealth is defined as assets minus liabilities. This variable is used to calculate wealth distribution.
- *Houses*: Housing wealth is defined as the assets held in the primary residence and other real estate. House share relative to total assets used in the calibration is constructed as the value of primary residence divided by total assets.
- *Stocks*: Stocks is defined as the sum assets held in stocks and mutual funds and business assets. Note that definition does not include stocks held through 401k accounts.[34] Stock concentration is defined as amount of stocks owned by a particular decile of the wealth distribution divided by the total value of stocks in the sample.
- *Income*: Household income is the total household yearly income. It includes Head's and Spouse's and other family unit members yearly income from labor, assets and the net profit from farm or business, transfers and social security. Wealth to income ratio is then constructed at the household level as a fraction of household net wealth to quarterly income. Quarterly income is simply one fourth of the yearly value. A large number of households have large wealth, but little income. In order to avoid extreme values coming from this I wincorize top and bottom 1% of the income variable before calculating the ratio.

[34] Including 401k accounts would result in a lower concentration of stock ownership, but 401k accounts are highly illiquid and do not fit the definition of stocks as defined in the model

E.2 Panel Study of Income Dynamics (PSID), consumption data

In order to have a sample, where expenditure categories are not changing and their number is fixed I use the household level panel waves 2005 to 2015.[35] Again I restrict the sample to households whose head is between 25 and 65 years old and use cross-sectional individual weights to have the moments consistent with the US population in the respective year.

Expenditure categories available in the sample include:

- *Child care*, *Clothing* ,*Education*, *Food*, *Health Care*, *Housing*, *Transportation*, *Vacation and Entertainment*

Non-Durable consumption: In the model where consumption is either non-durable goods or housing it is not obvious where the transportation expenditure fits. Part of it includes gasoline and commuting cost, but a large part of it includes expenditure on vehicles, which are durable goods. To avoid that I subtract the vehicle down payments and loan payments from the transportation costs. In the end non durable consumption measure includes expenditure on child care, clothing, education, food, health care, vacation and entertainment and the transportation expenditure mines expenditure on vehicles.

Housing consumption: Measurement of housing consumption requires converting the stock variable (housing wealth) into a flow (housing consumption). I measure the flow value of a house as the amount that the household would spend on renting the same property. Specifically for the households that are not renting I divide the value of their primary residence by the yearly price-to-rent ratio in their state and year. Price-to-rent ratio data at a state level comes from Zillow,[36] which uses the ratio of zillow price and rent estimates for each residential house that exists in the area. I match it to PSID households by state and year.[37] PSID housing expenditure includes both rent payments and utilities. The final measure of housing consumption is the sum housing expenditure and my measure of implied rent. In this way the measure captures housing consumption of both renters and home-owners.

E.3 Data Used to Estimate the EIS

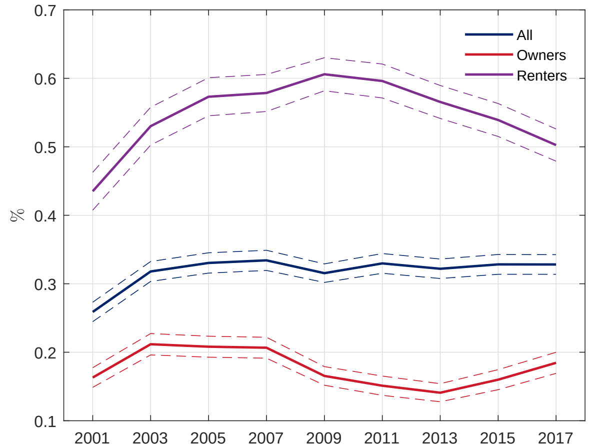
IPUMS-CPS

- The measure of consumption in section comes from the current population survey integrated public use microdata series (IPUMS-CPS) food security supplement. It provides average weekly food expenditure (FSULXPNSC) at the household level. I calculate the averages for each county-year using the cross-sectional individual weights.

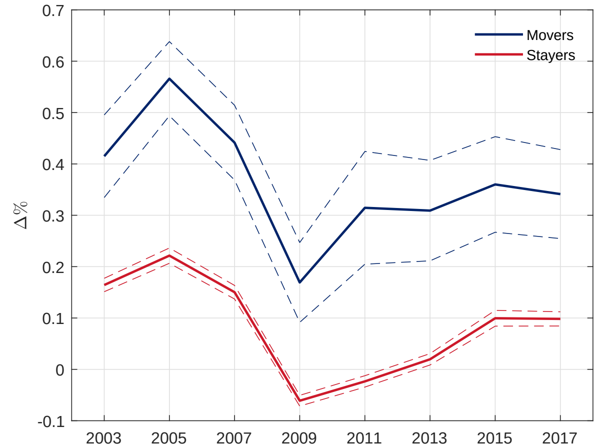
[35] Data on most of the expenditure categories starts in 1999, but data on vacation and entertainment is only available from 2005 wave. 2017 wave included a new expenditure category, computers and electronics. I exclude wave 2017 to have the stable list of categories over time

[36] https://data.world/zillow-data/price-to-rent-ratio

[37] PSID public data does not include more detailed geographical identifiers than the state level



(a) % adjusting



(b) Change in house value

Notes: Figures show the percent of households changing their primary residence (left) and the change in the value of the primary residence (right) between the consecutive PSID waves. This suggests that people change their residence infrequently. Right panel suggests that when such changes happen, the price difference between the two residences is too large to be explained by change in average house prices.

Zillow

I use Zillow indices to measure local house prices. Zillow publicly available data contains information for local house prices at the county level 2010-2019 period. I use the following indices:

- ZHVI: The Zillow House Value Index is an index that measures the typical home value in a given region. Data includes single-family, multi-family homes and condominiums and uses data of listed properties as well as rent estimates for properties that are not listed.

- ZRI: The Zillow Rent Index is an index that captures the typical market rent prices for a given segment and area. It includes single-family, multi-family homes and condominiums and includes and uses data of listed properties as well as rent estimates for properties that are not listed. ZRI also provides rent estimates per square feet. I use both ZRI and ZRI per square feet.

- Median List Prices per square feet: This index provides the median per square feet price of properties listed for rent in a given area. The index includes listings for single-family, multi-family homes and condominiums.

Corelogic

I use Corelogic property tax dataset to measure the average square feet of the apartment. The dataset covers 95% of the US counties and includes tax information on 154+ million parcels. It provides ample of property characteristics among which is the square feet information. The dataset covers years 2014-2016. When cleaning it I exclude the points where the year information is missing. The dataset covers most of the types of properties, such as warehouses, commercial properties, industrial units etc. For each county I restrict the sample to single family residence and condominiums, which are the two types classified as residential properties, and use the living square feet as the preferred size measure. The reason is two-fold. First, it excludes the garage or patio and similar parts of the unit that are not used for living, but constitute a large fraction of the total unit square feet. Second, this variable has fever missing values.

Controls

- Population Density: County level population density data comes from 2010 US census.
- Per Capital Income: Data comes from BEA regional accounts. Calculated as total personal income divided by county population.